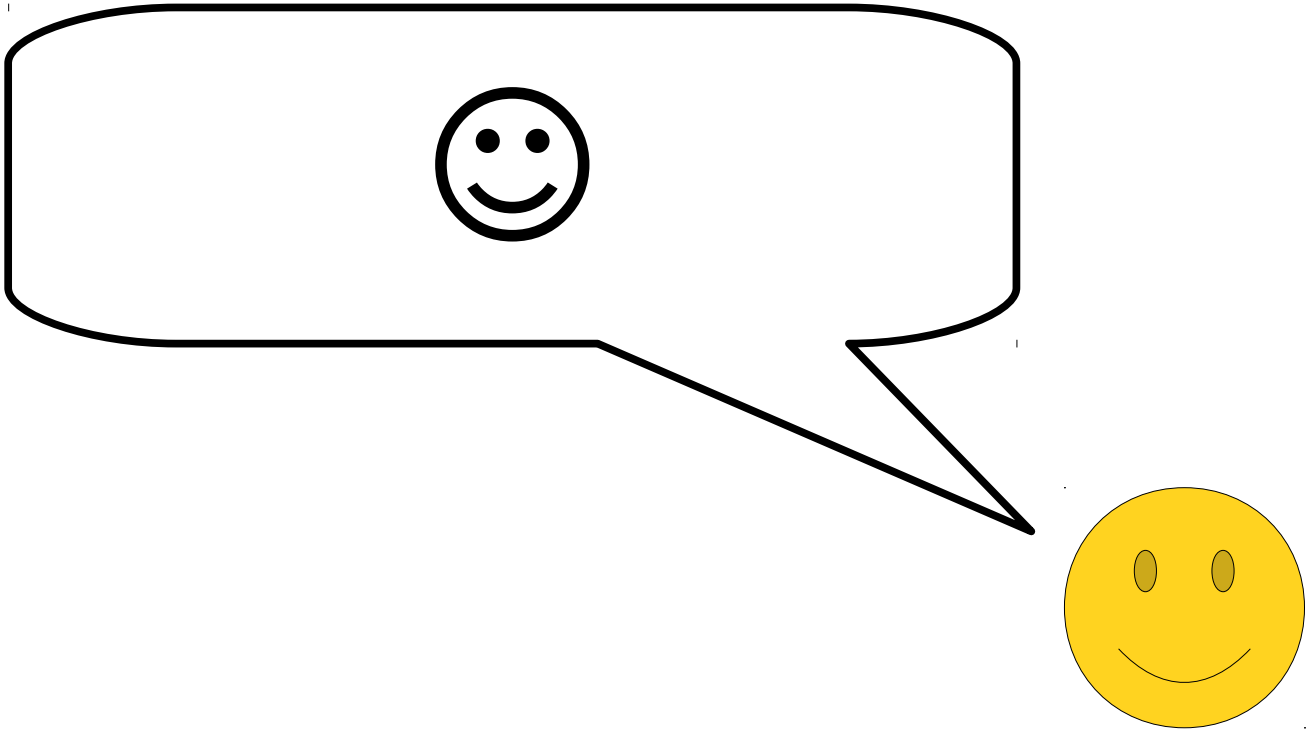
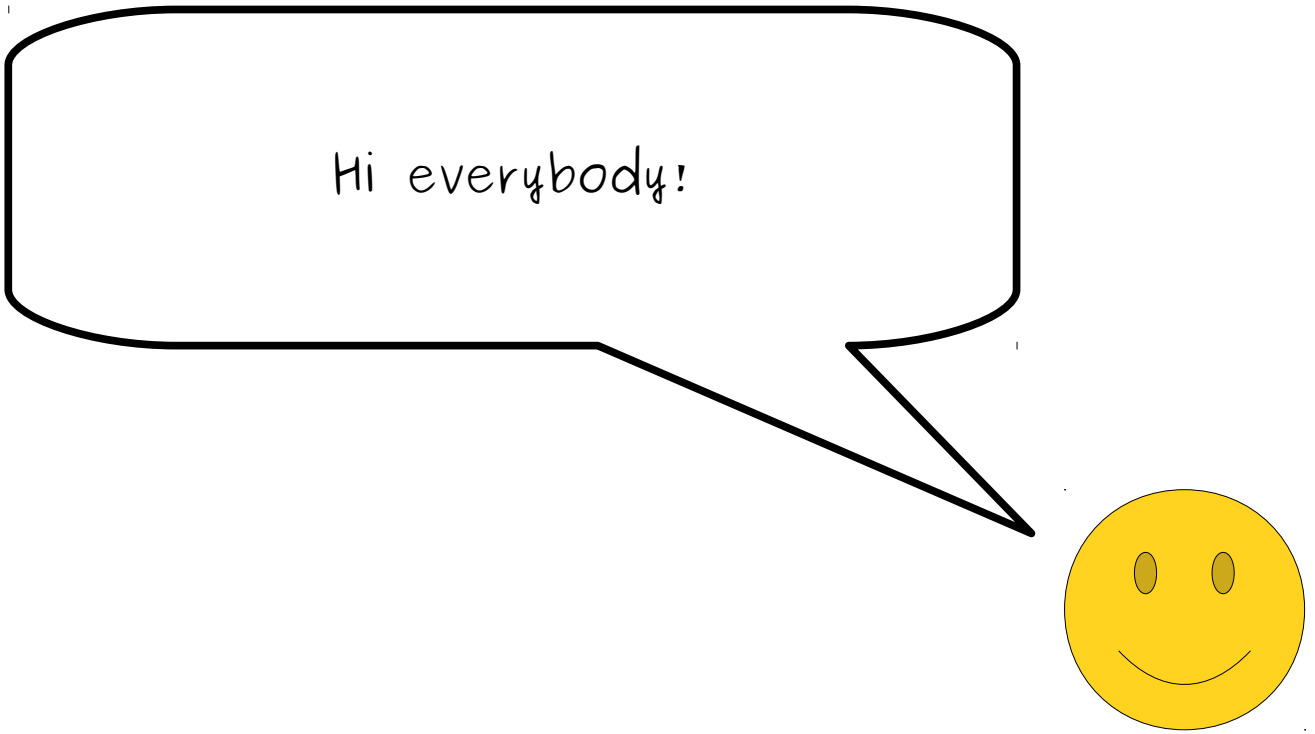
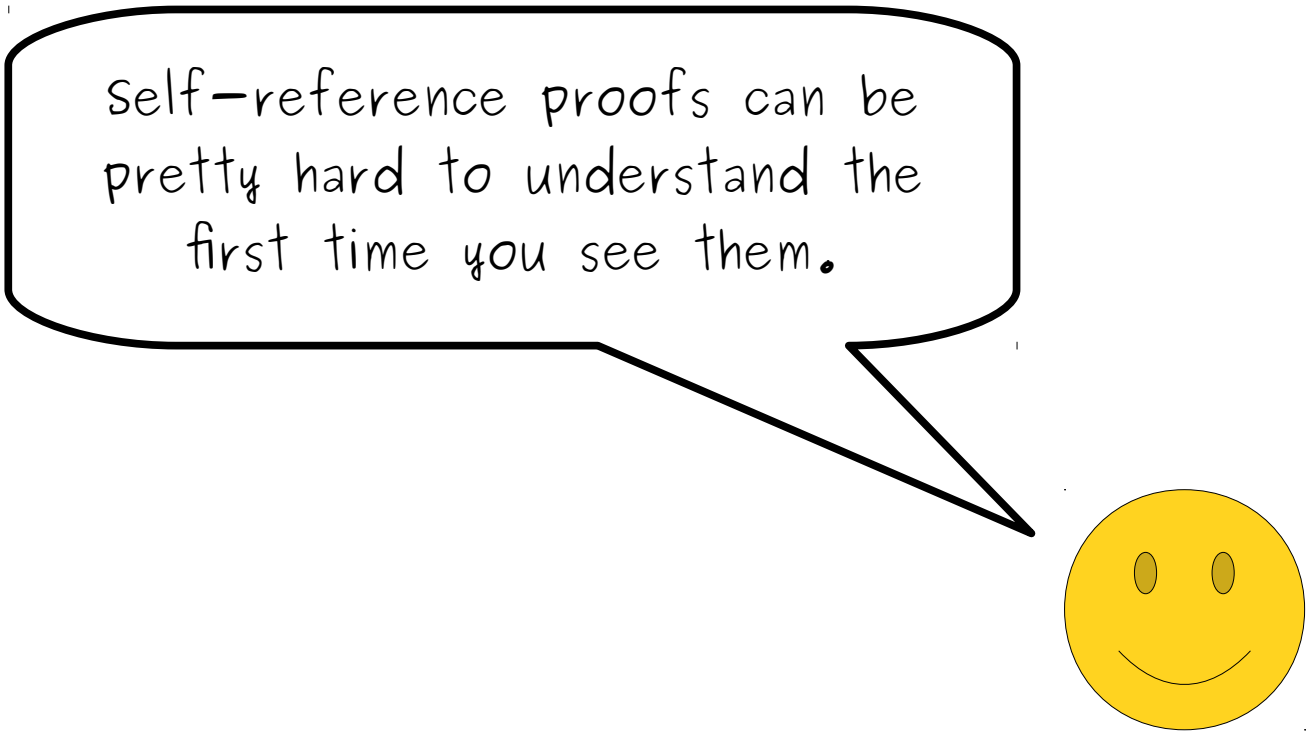
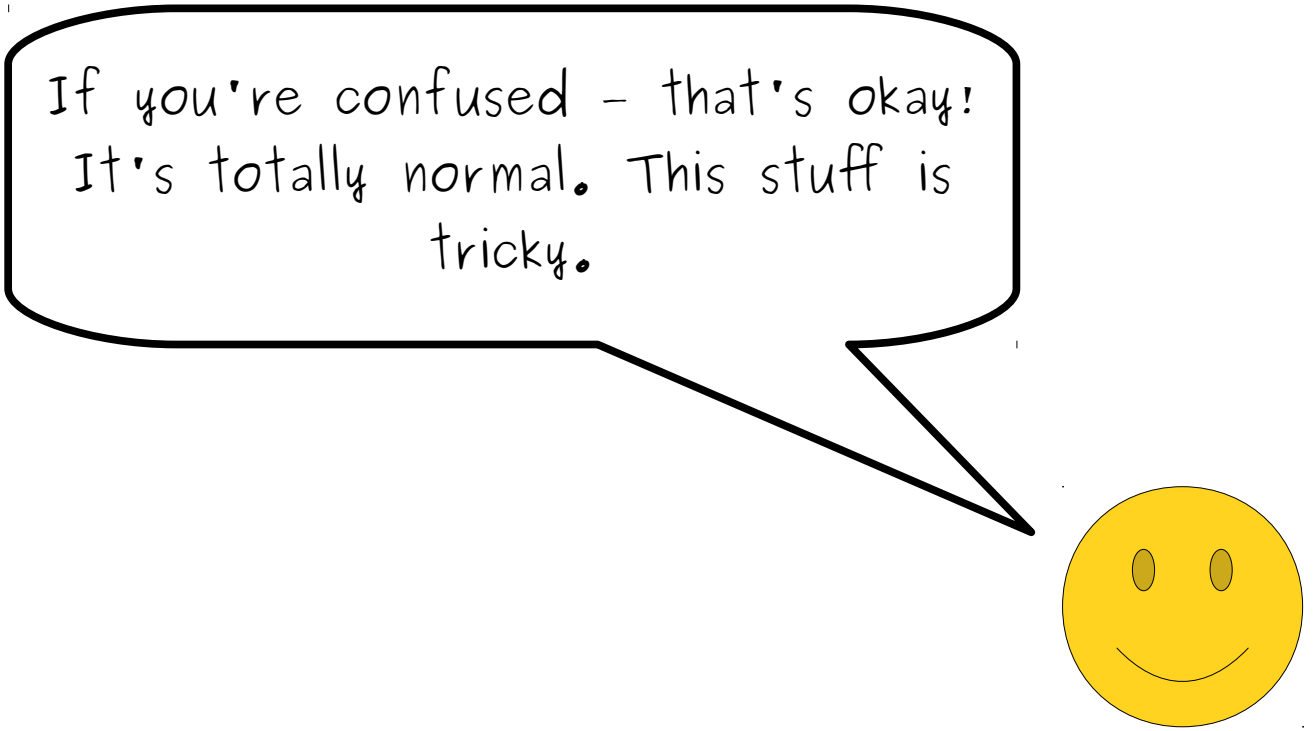
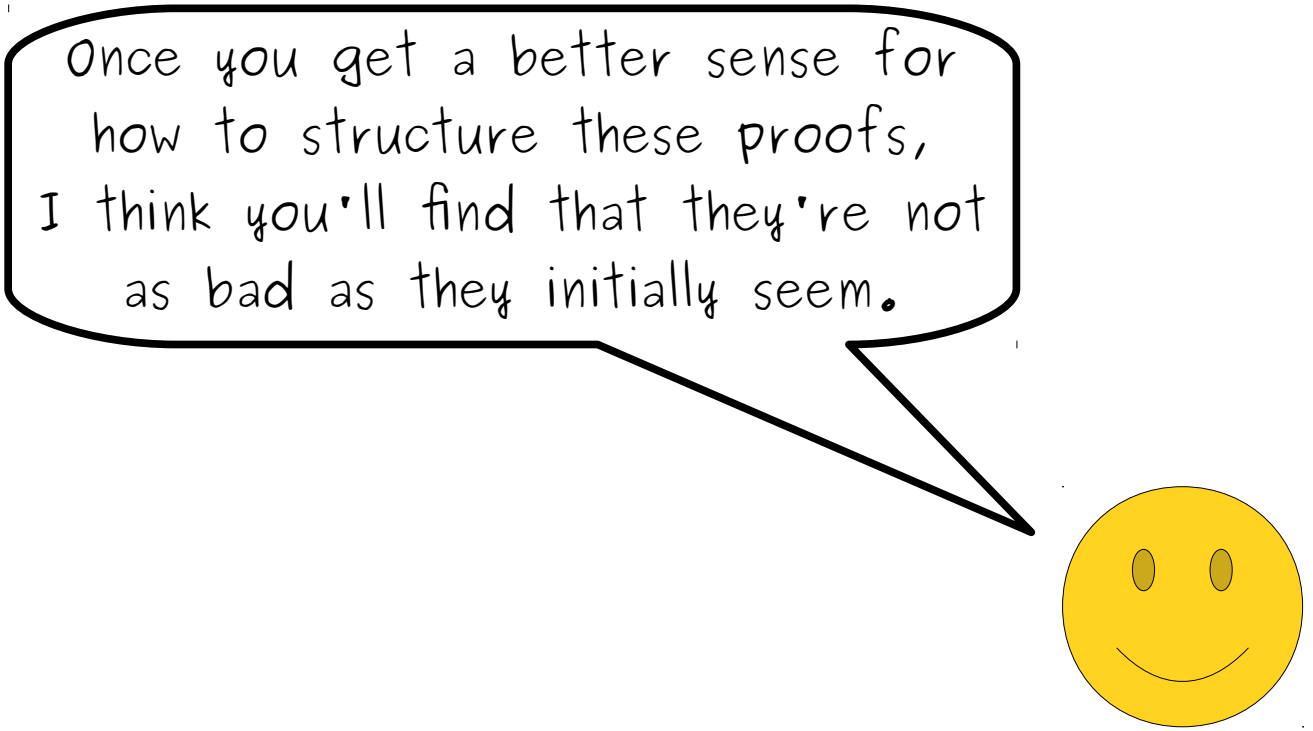
The Guide to Self-Reference




Hi everybody!


self-reference proofs can be pretty hard to understand the first time you see them.


If you're confused - that's okay!
It's totally normal. This stuff is
tricky.


Once you get a better sense for how to structure these proofs, I think you'll find that they're not as bad as they initially seem.

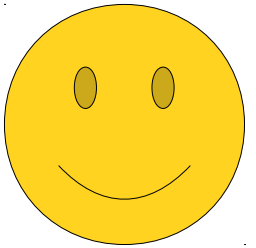
What does this program do?

```
bool willAccept(string program, string input) {
    /* … some im
}

int main() {
    string me = 
    string input = 

    if (willAccept(me, input)) {
        reject();
    } else {
        accept();
    }
}
```

This lecture slide was the first time that we really saw self-reference, and a lot of you got pretty tripped up by what was going on.



Try running this program on any input. What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

What does this program do?

```
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    /* … some
}

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    string me
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```
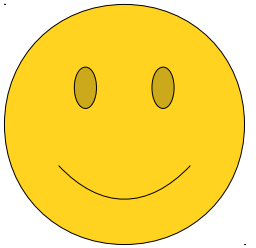

Part of the reason why this can be tricky is that what you're looking at is a finished product. If you don't have a sense of where it comes from, it's really hard to understand!



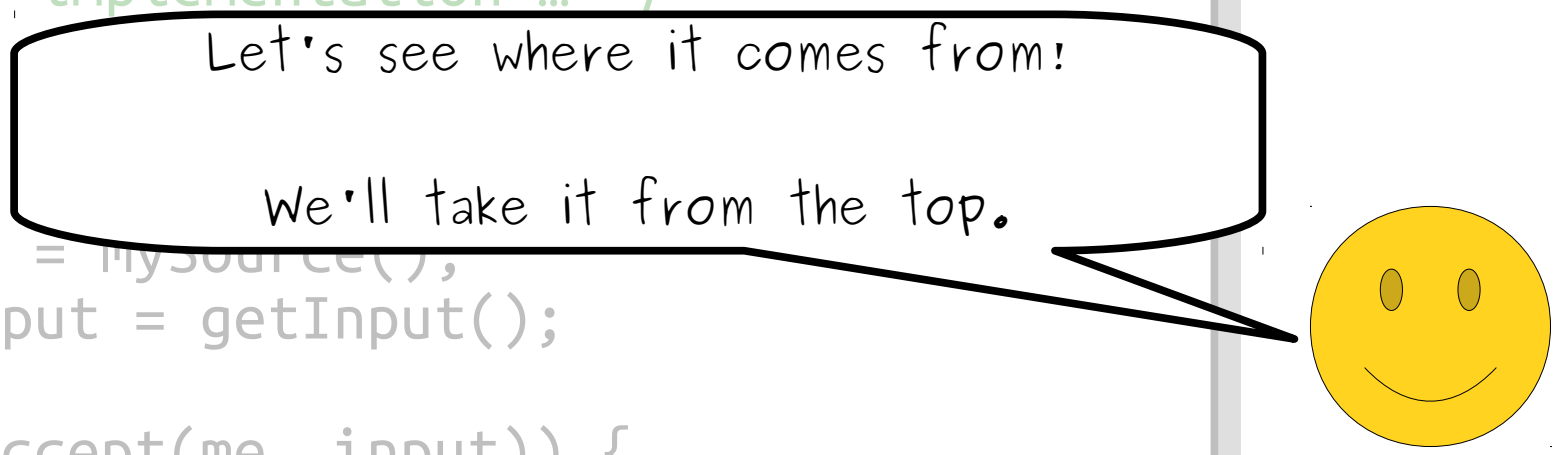
Try running this program on any input. What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

What does this program do?

```
bool willAccept(string program, string input) {
   /* … some implementation … */
}

int main() {
   string me = mySource();
   string input = getInput();

   if (willAccept(me, input)) {
      reject();
   } else {
      accept();
   }
}
```



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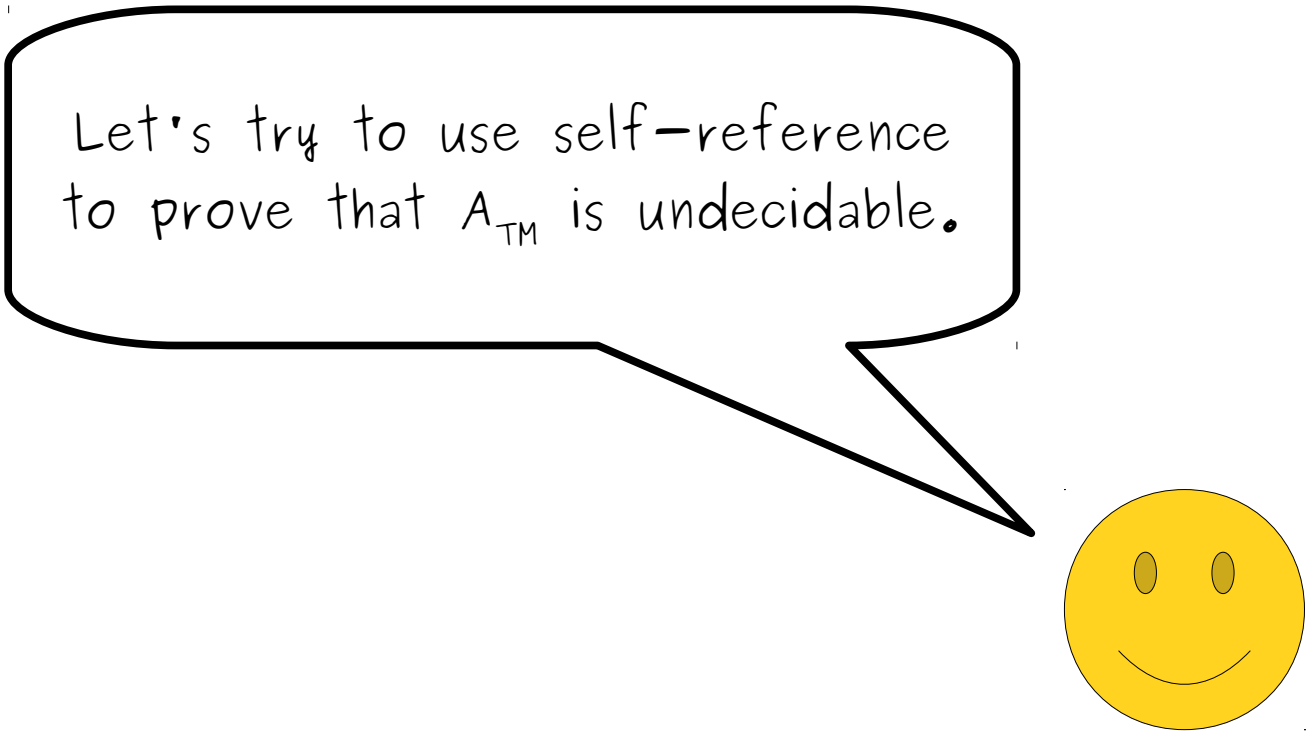
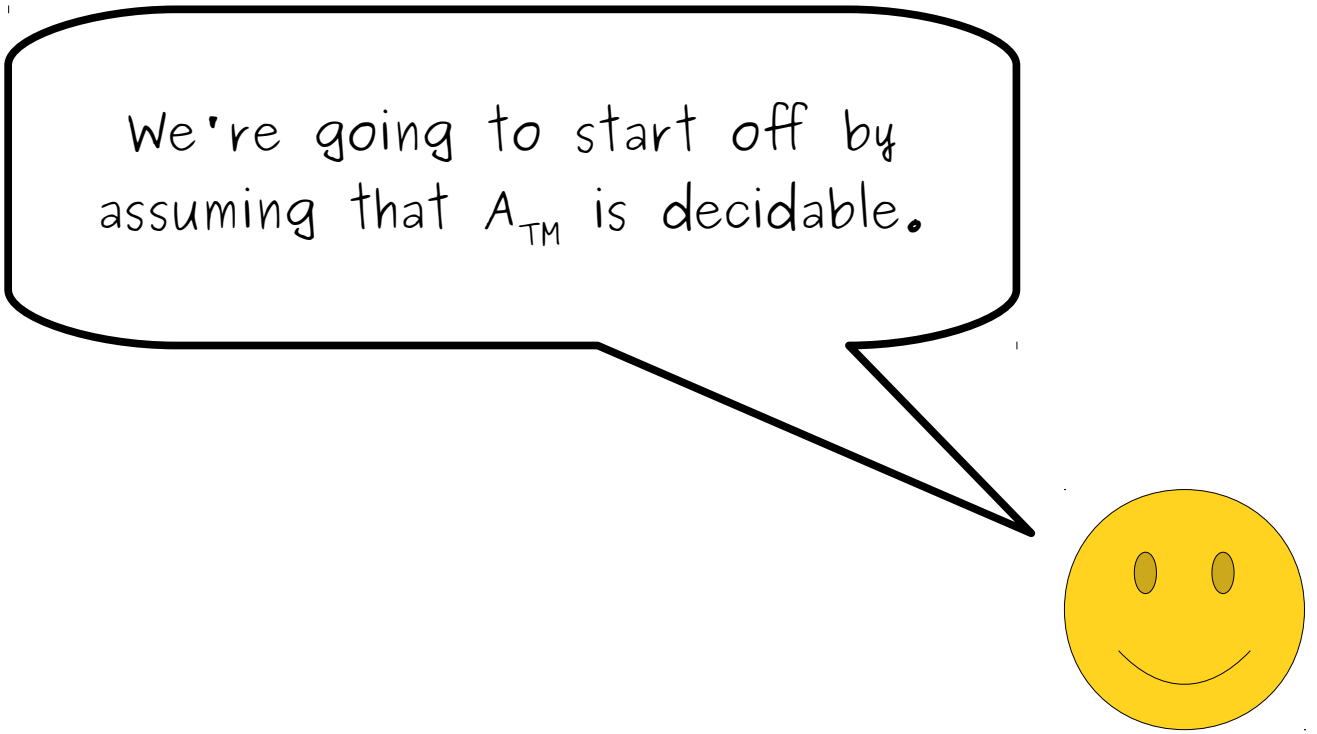

Let's try to use self-reference
to prove that A_{TM} is undecidable.


At a high level, we're going to do a proof by contradiction.

$A_{TM} \in \mathbf{R}$



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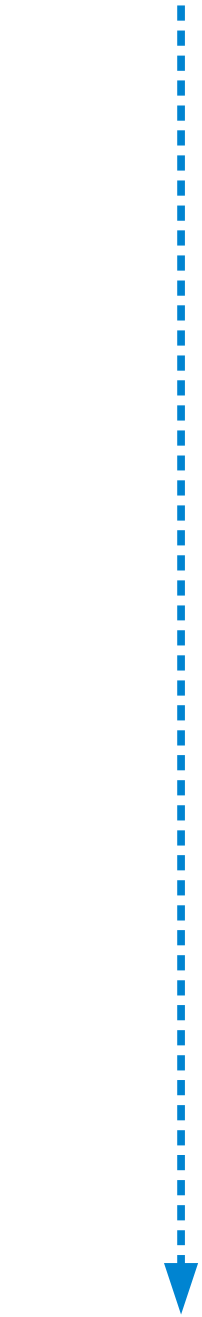
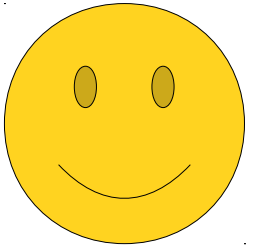


Contradiction!

$A_{TM} \in \mathbf{R}$

Contradiction!

If we can get a contradiction – any contradiction – we'll see that our assumption was wrong.



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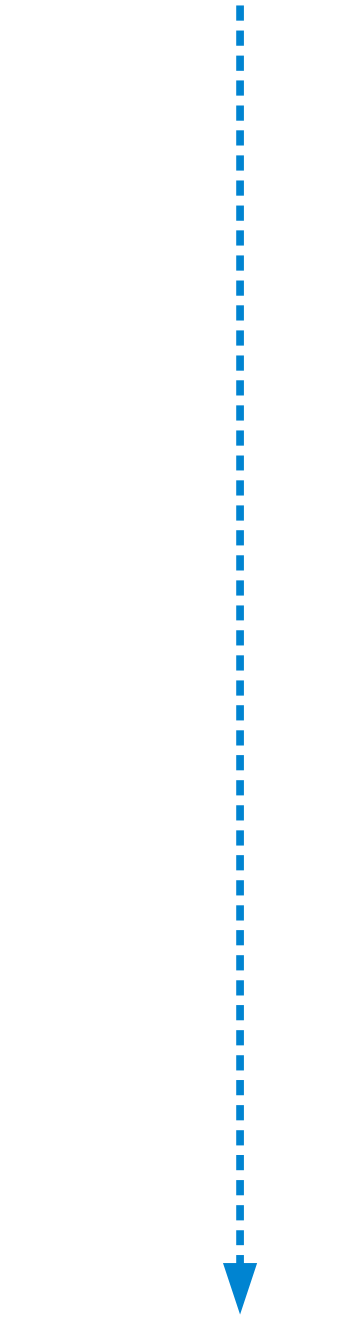

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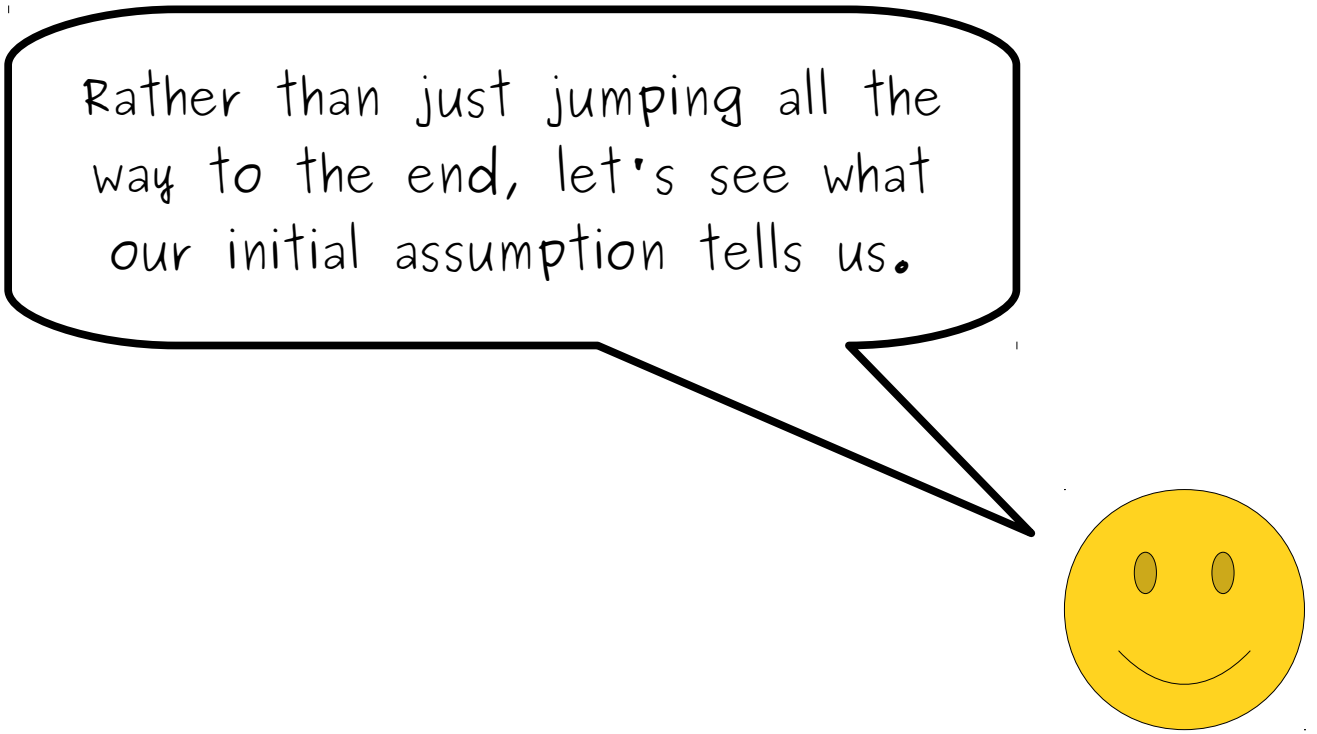

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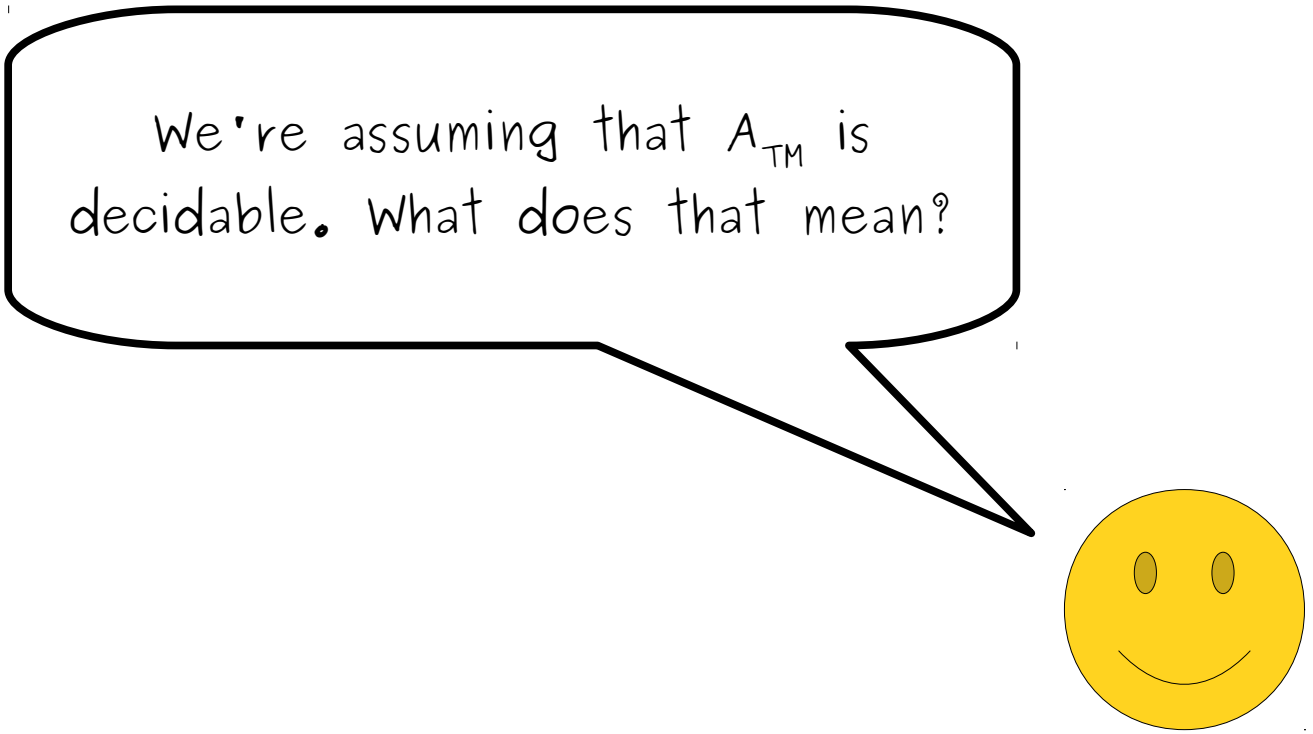

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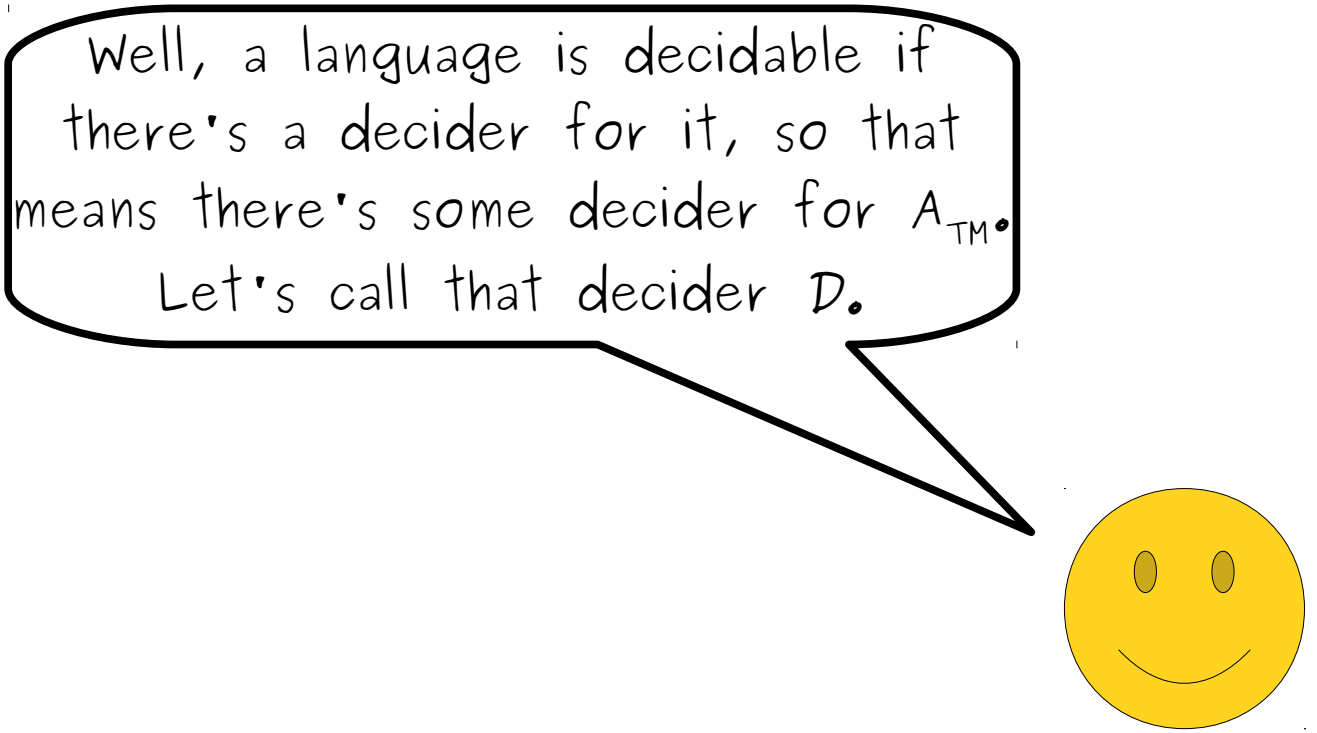
There is a decider D for A_{TM}



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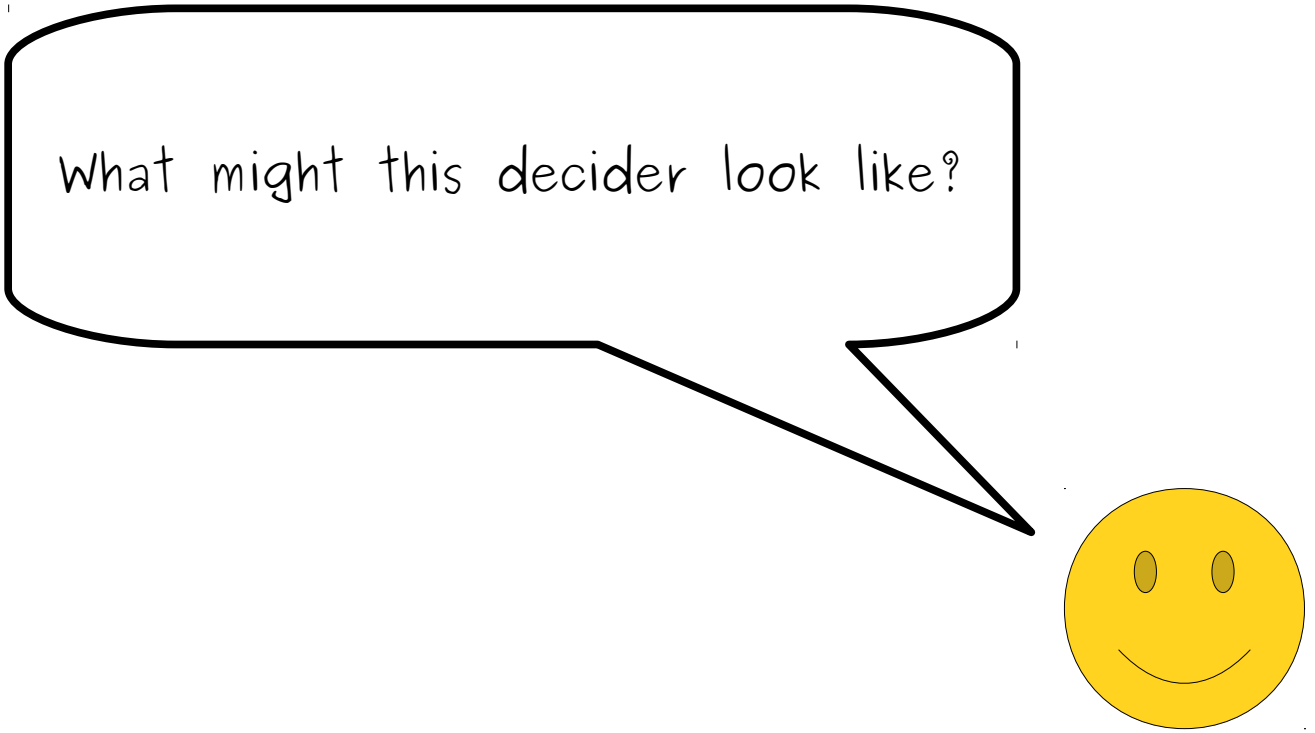

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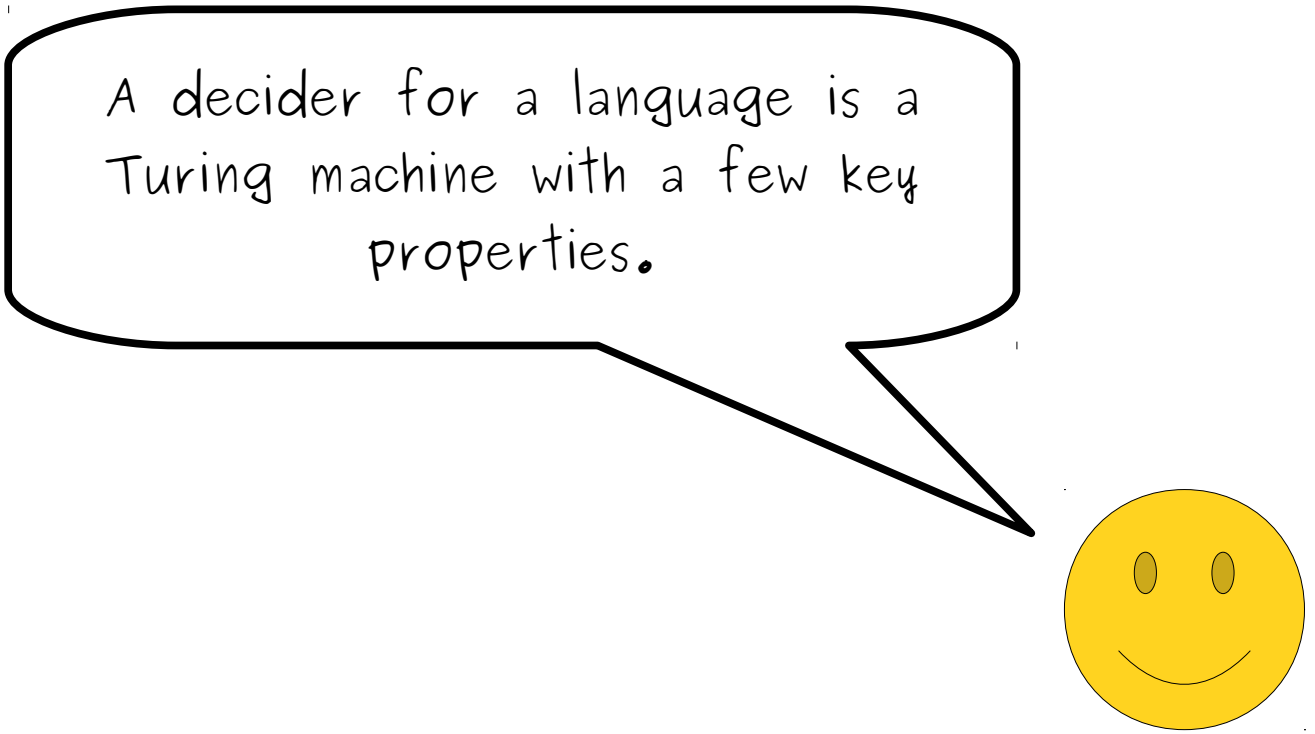

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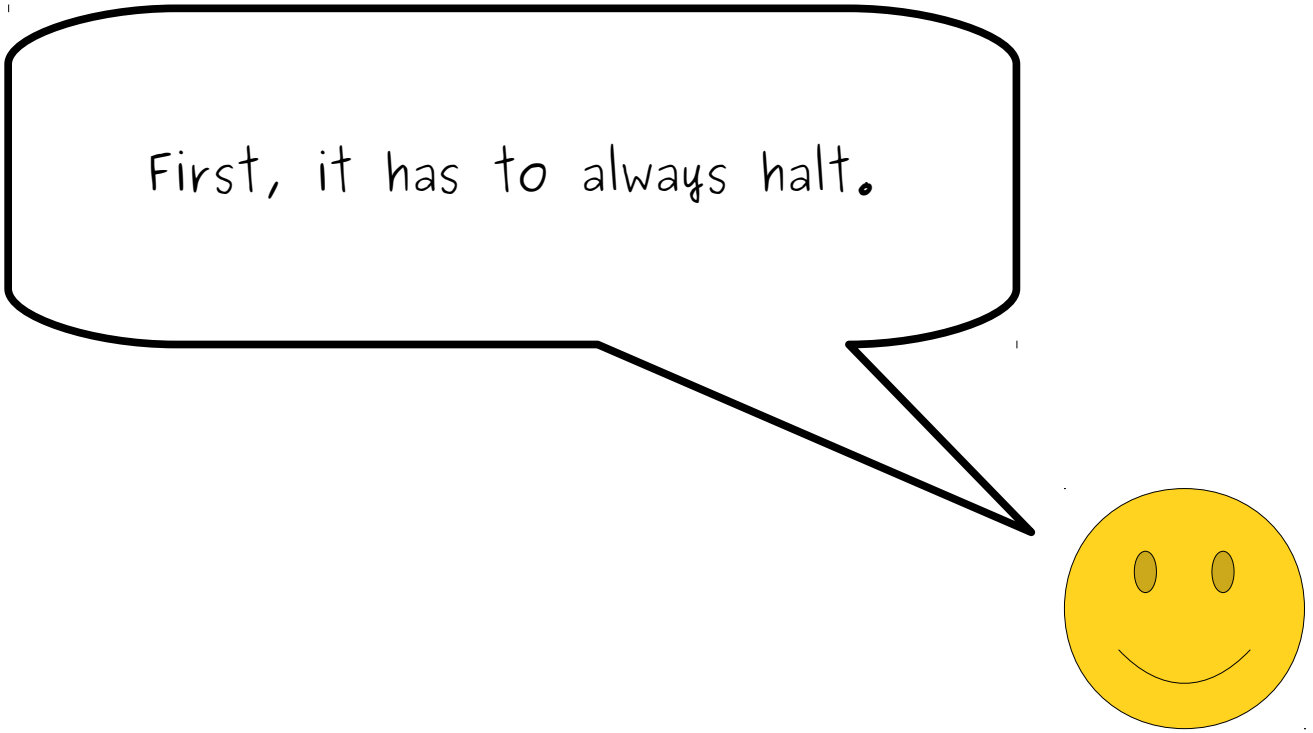

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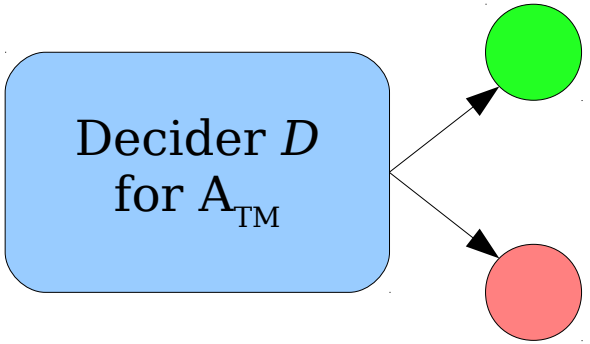

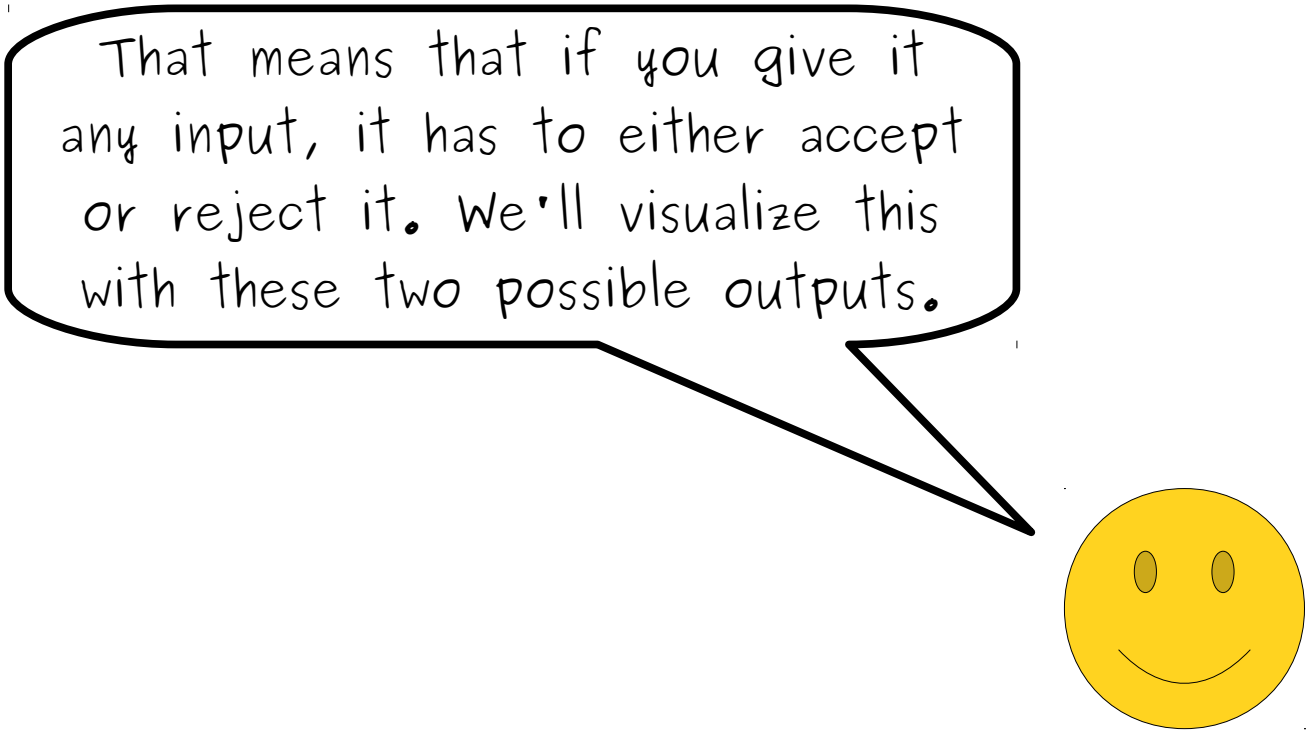

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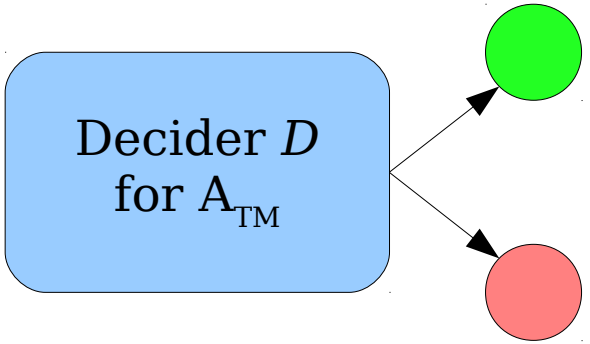

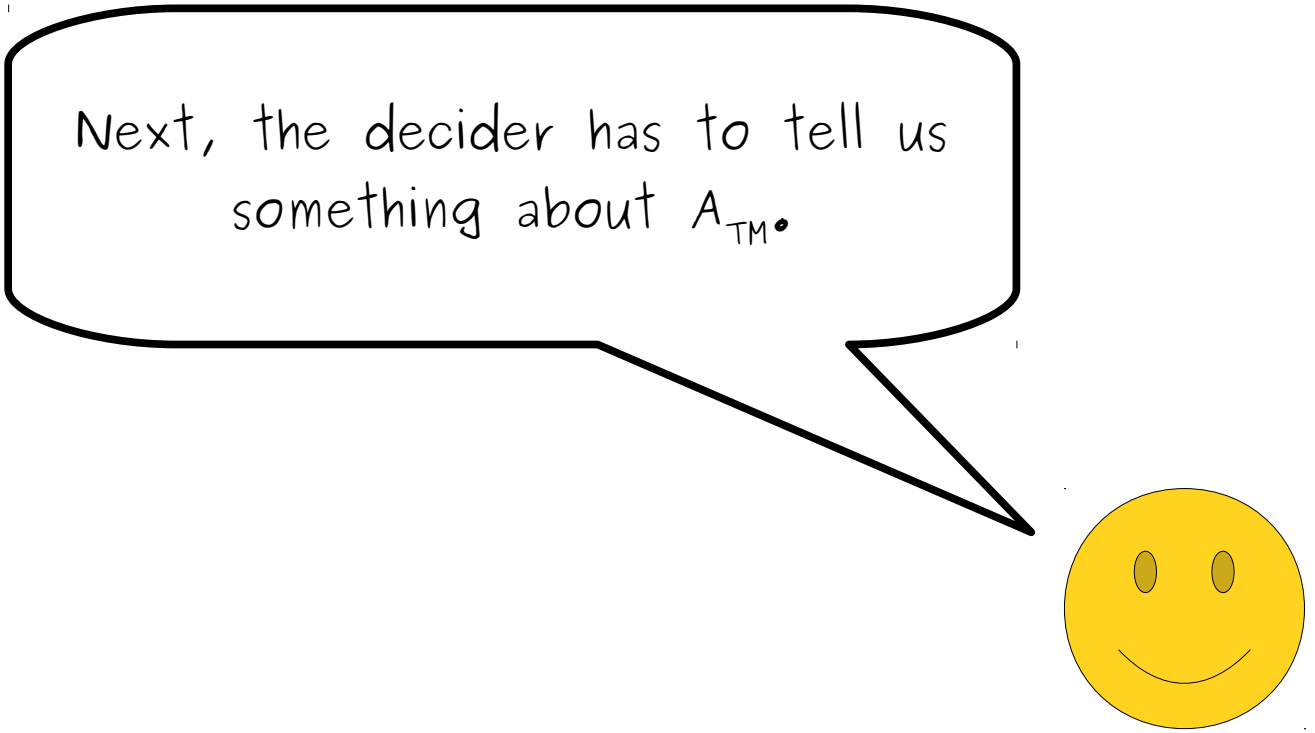

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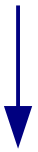

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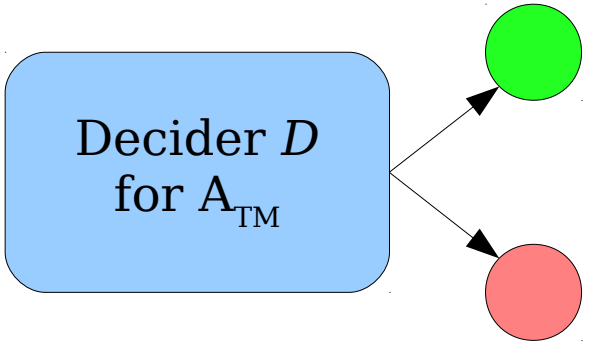
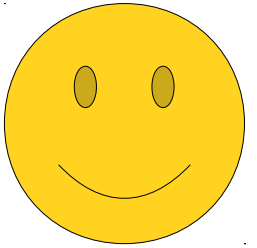
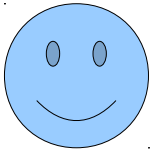


Next, the decider has to tell us something about A_{TM}.



As a reminder, A_{TM} is the language

$\{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w \}$



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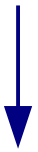

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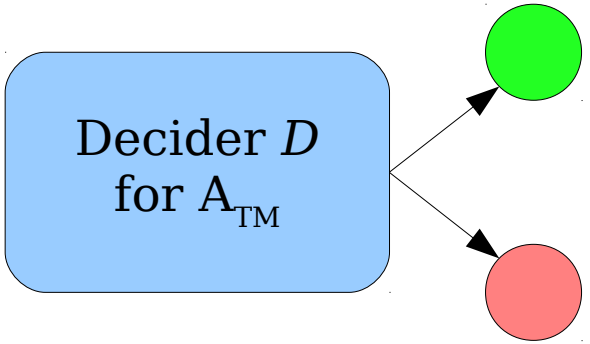
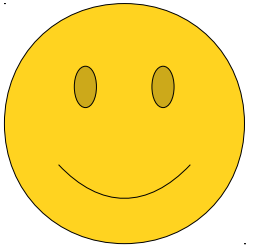


Specifically, the decider D needs to take in an input and tell us whether that input is in A_{TM}



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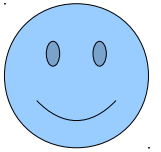

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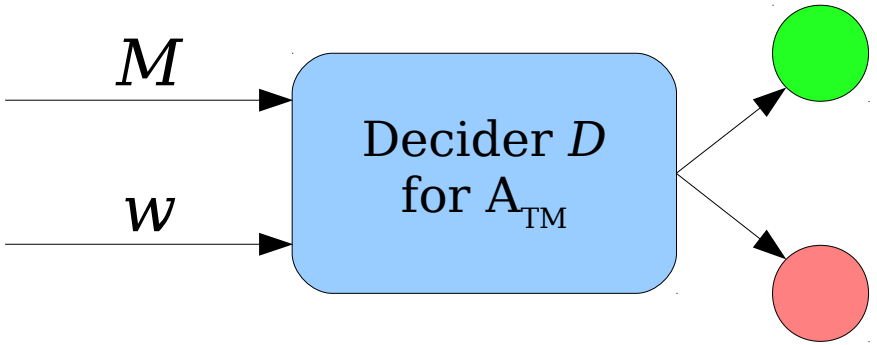
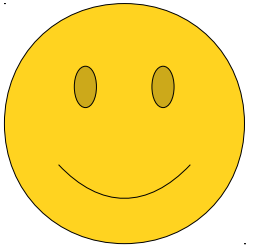


A_{TM} is a language of pairs of TMs and strings, so D will take in two inputs, a machine M and a string w.



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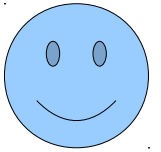

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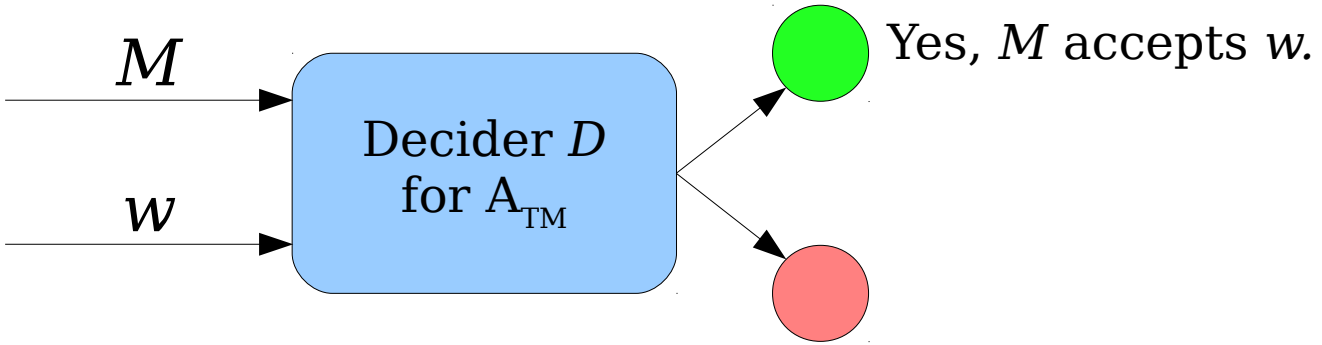

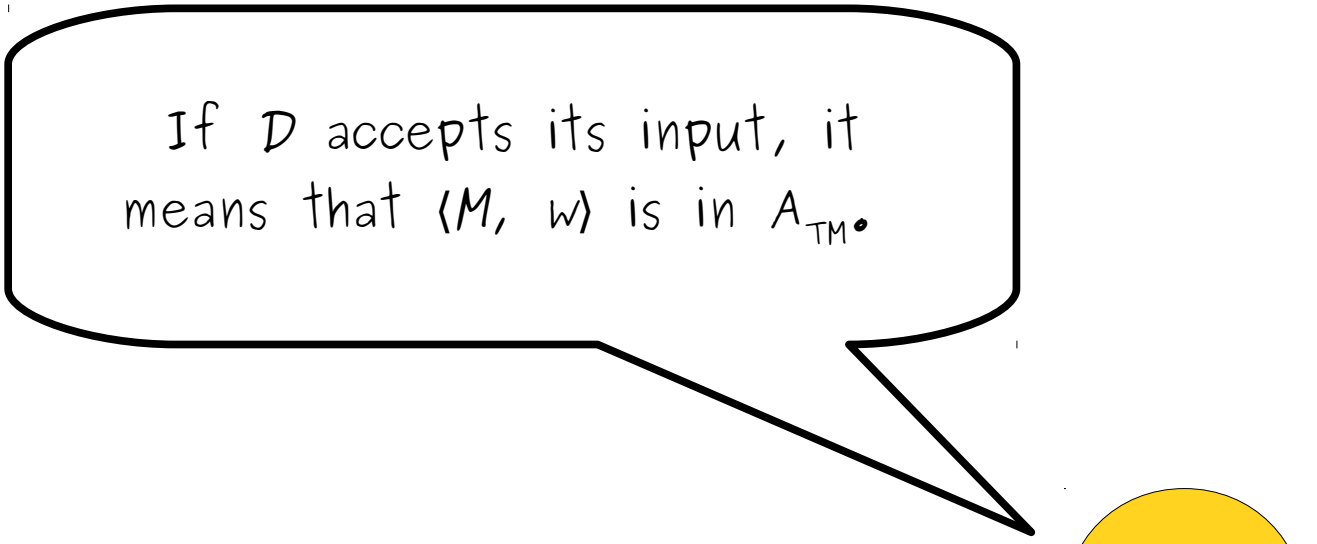

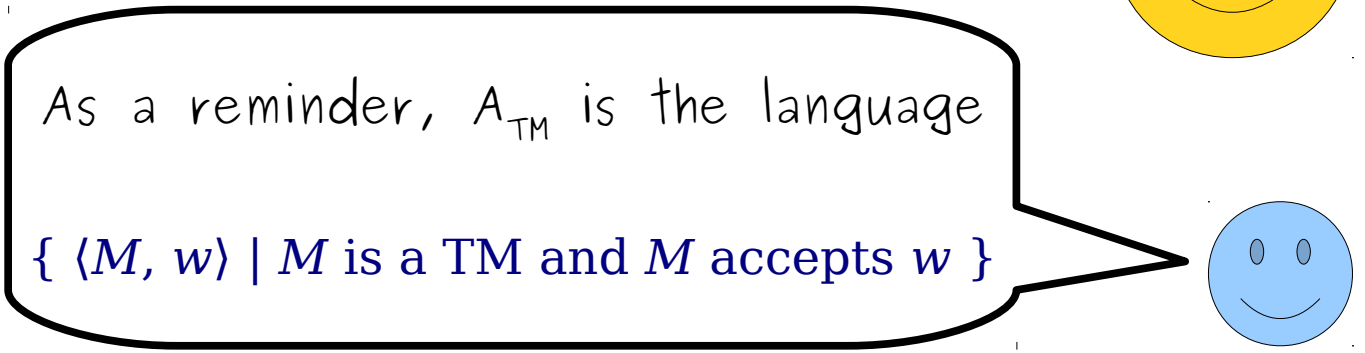

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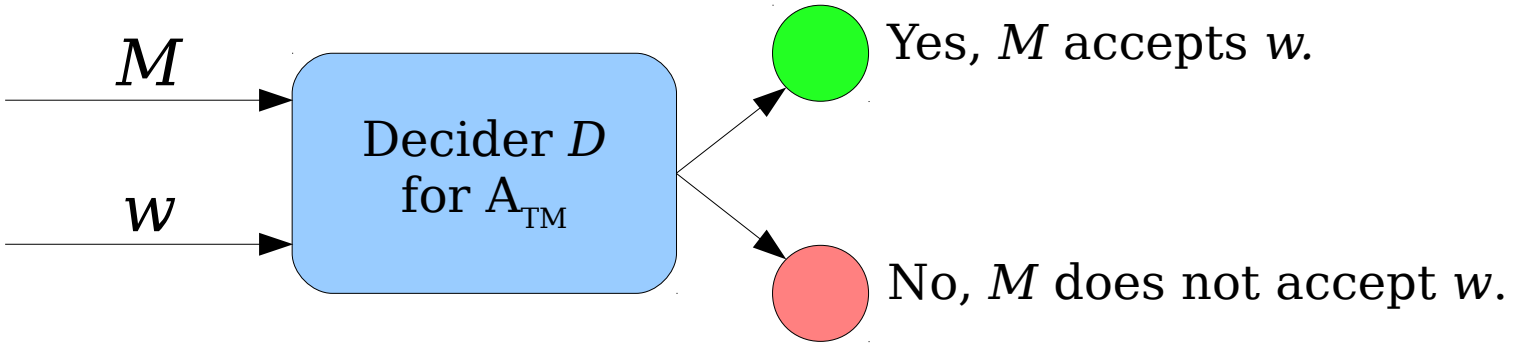
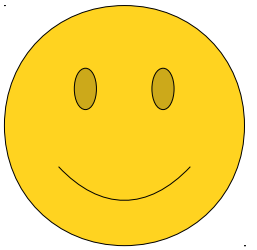


Otherwise, if D rejects its input, it means that M doesn't accept w.



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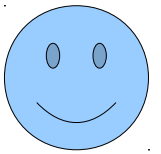

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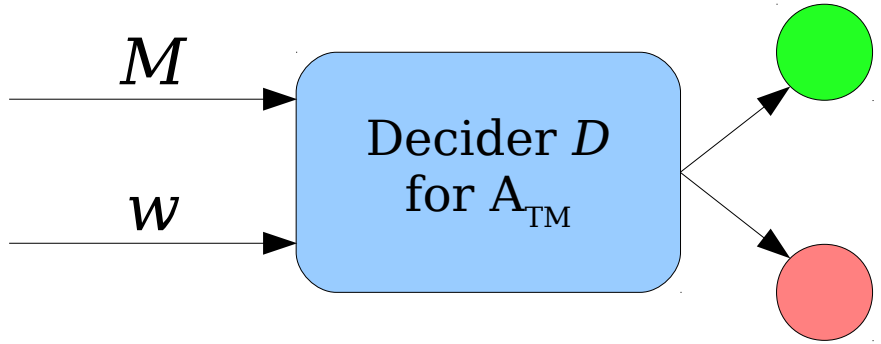

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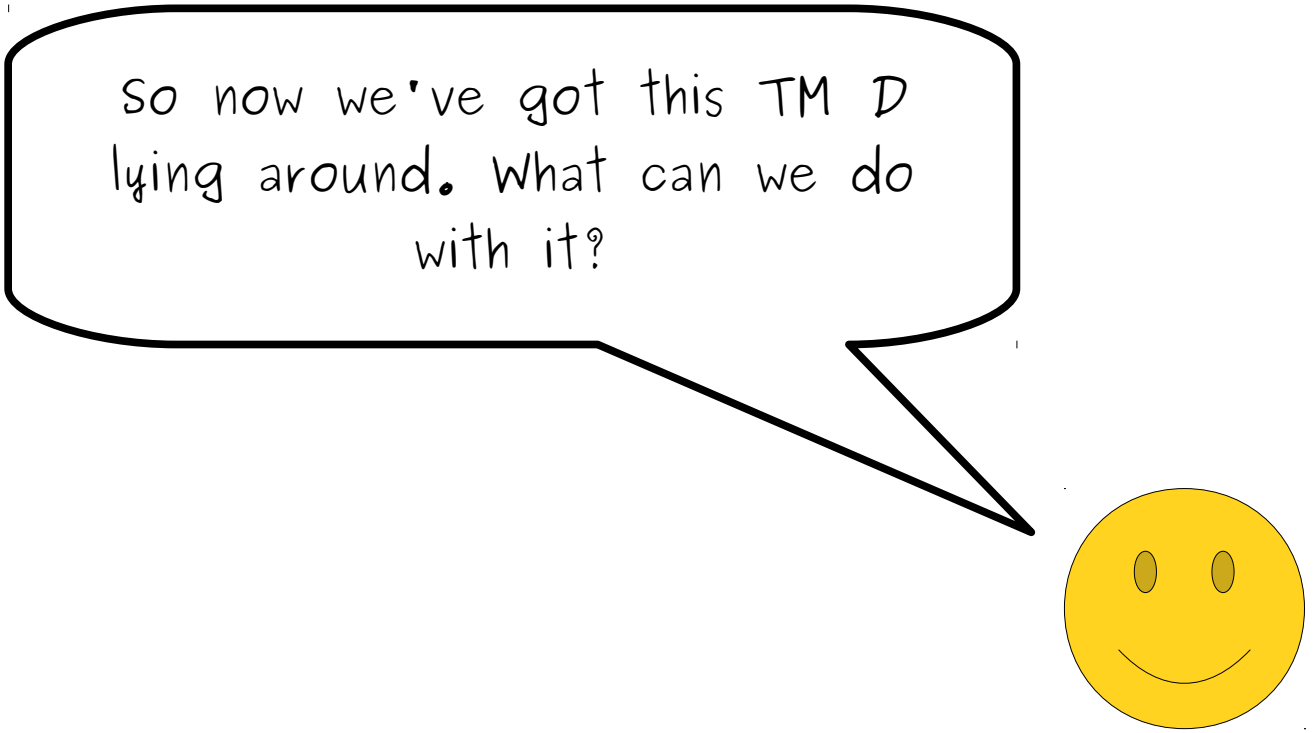

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We can write programs that use D as a helper method

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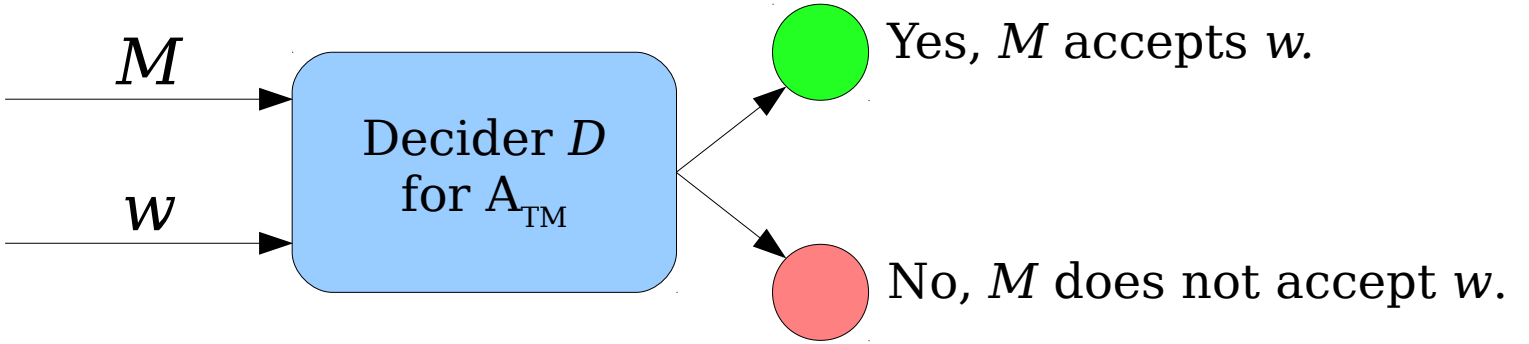

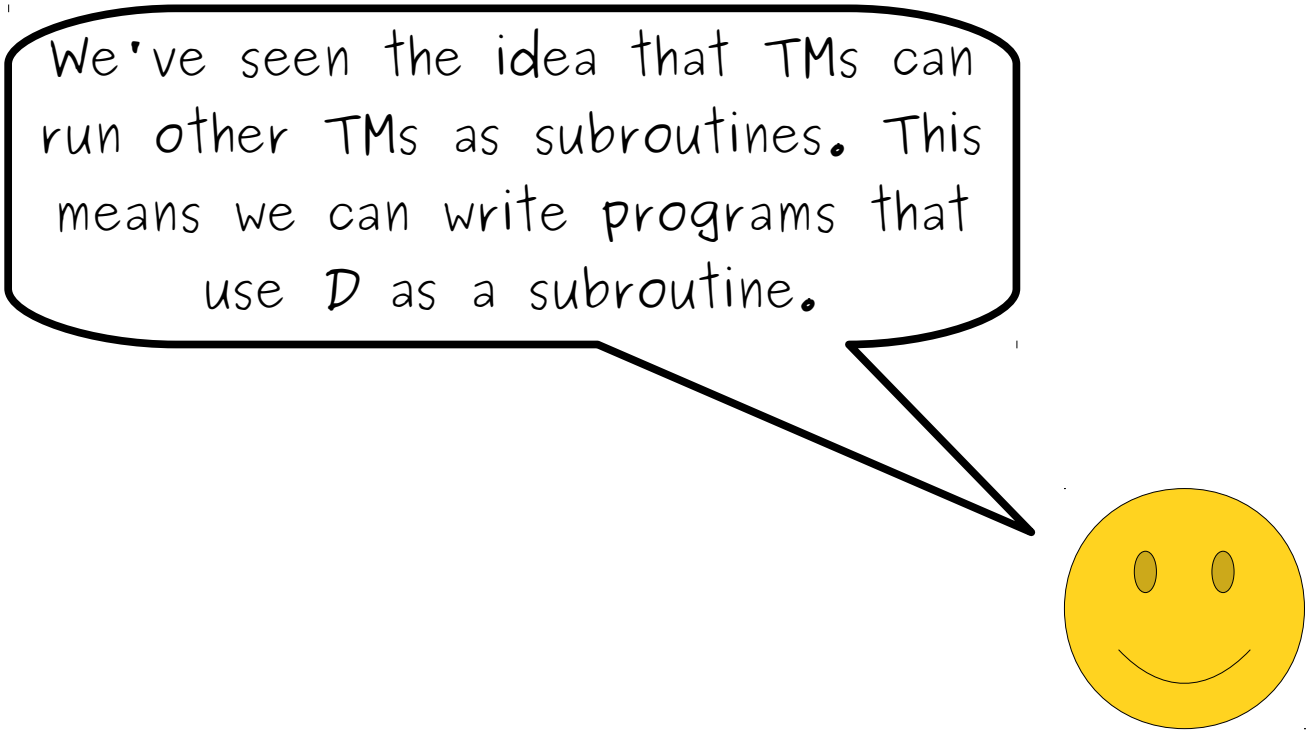

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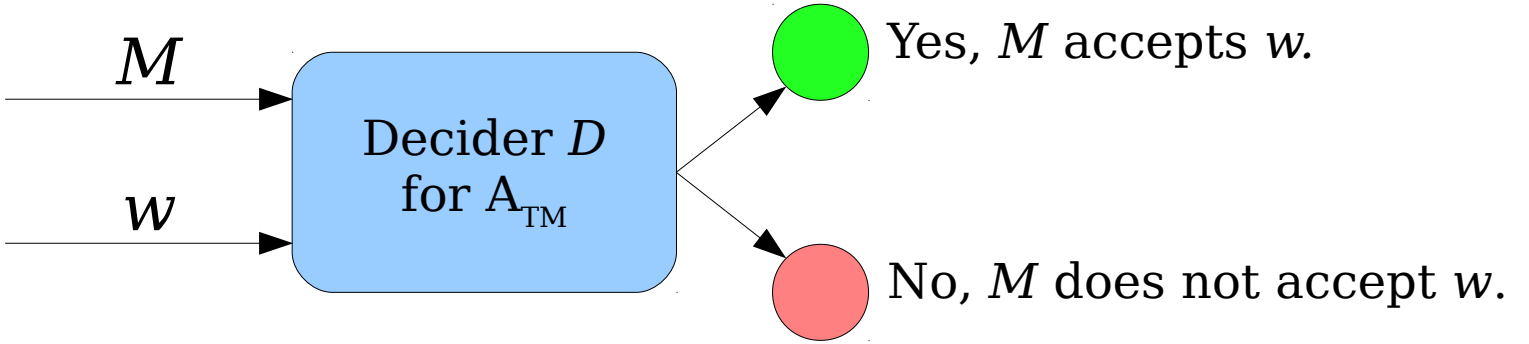


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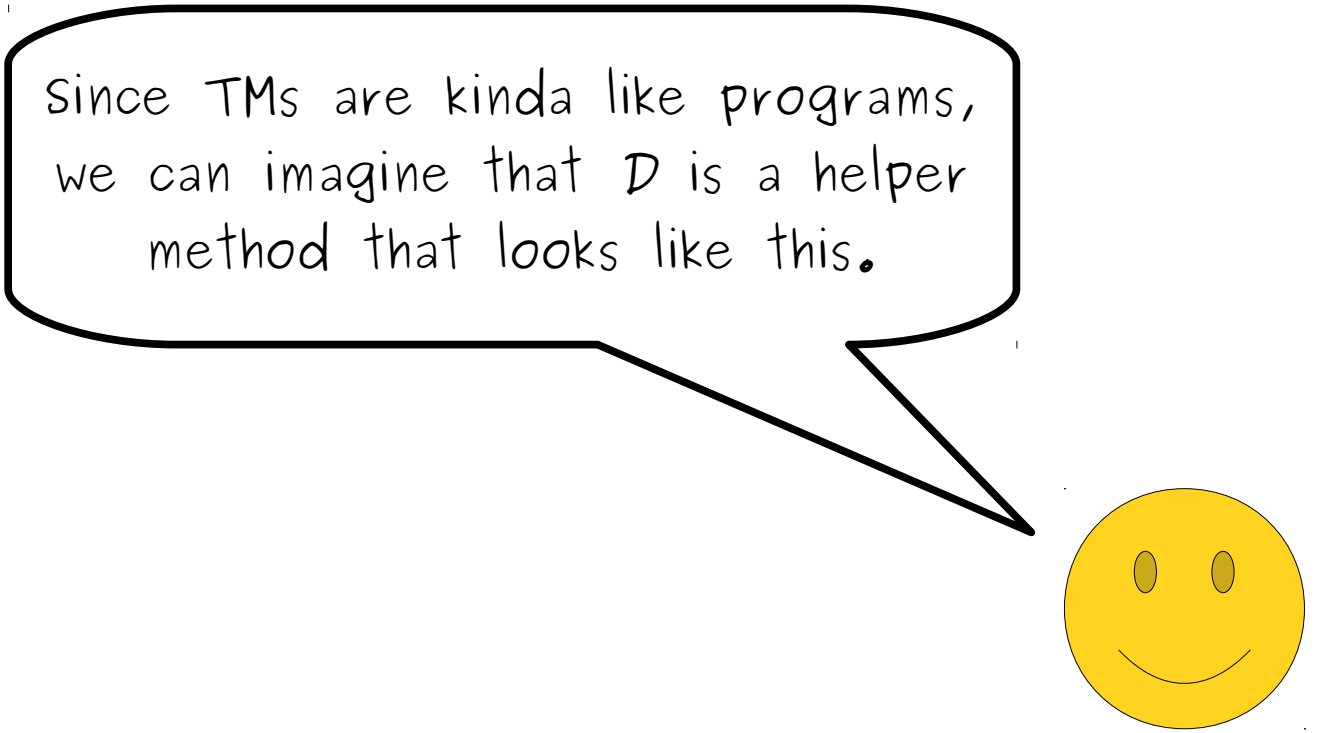

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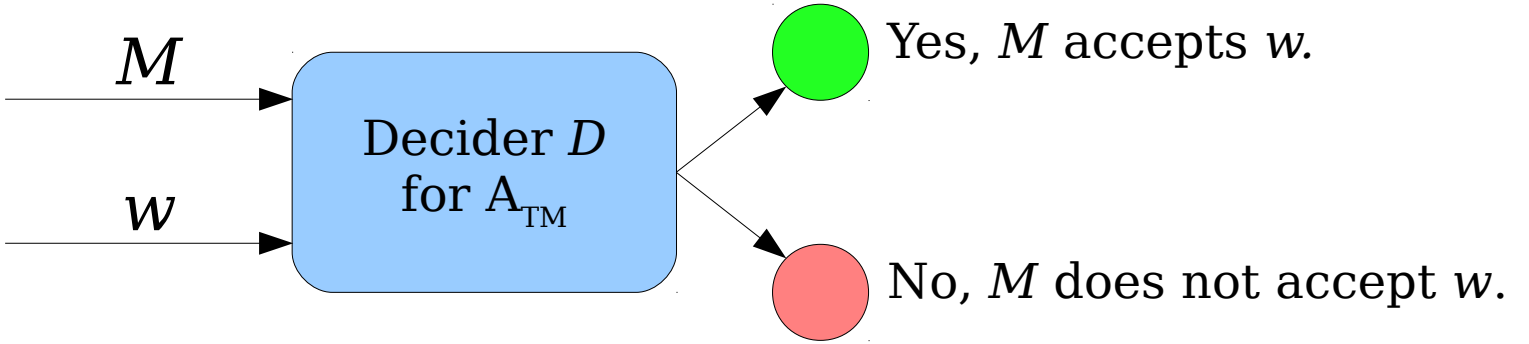


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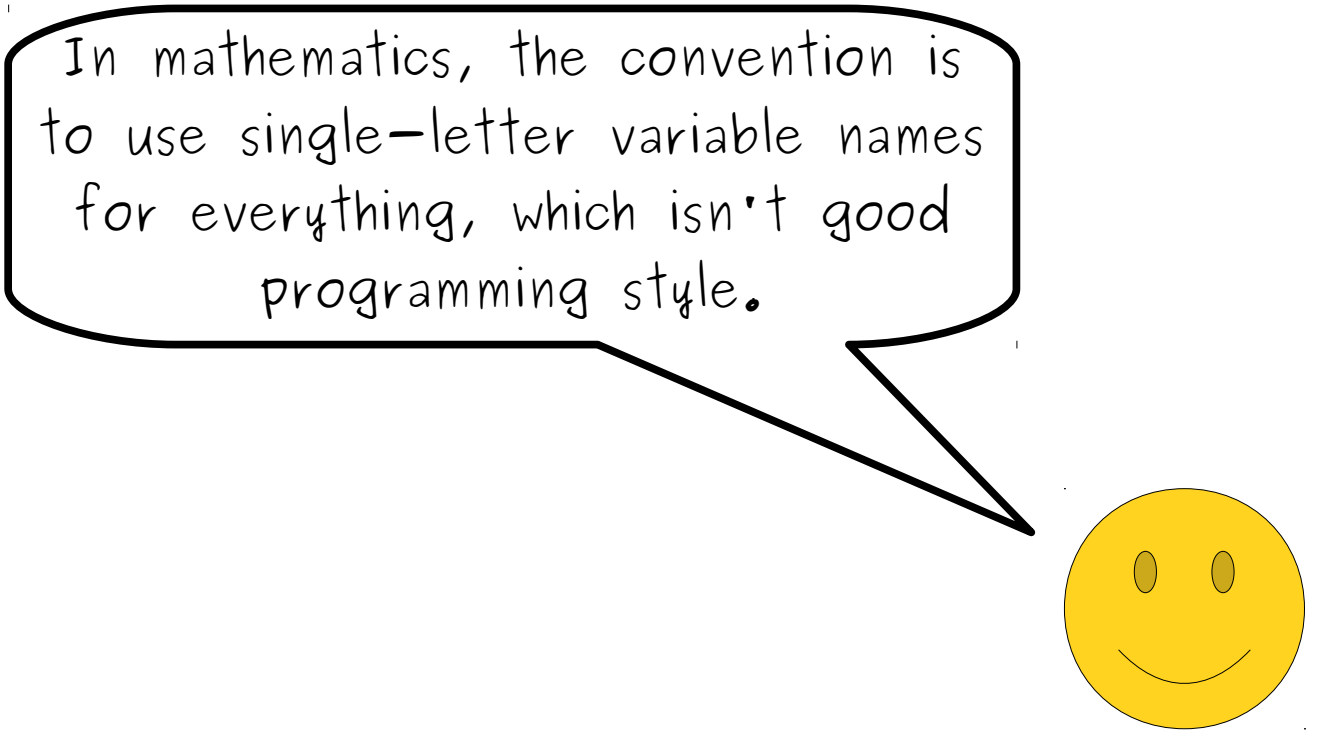

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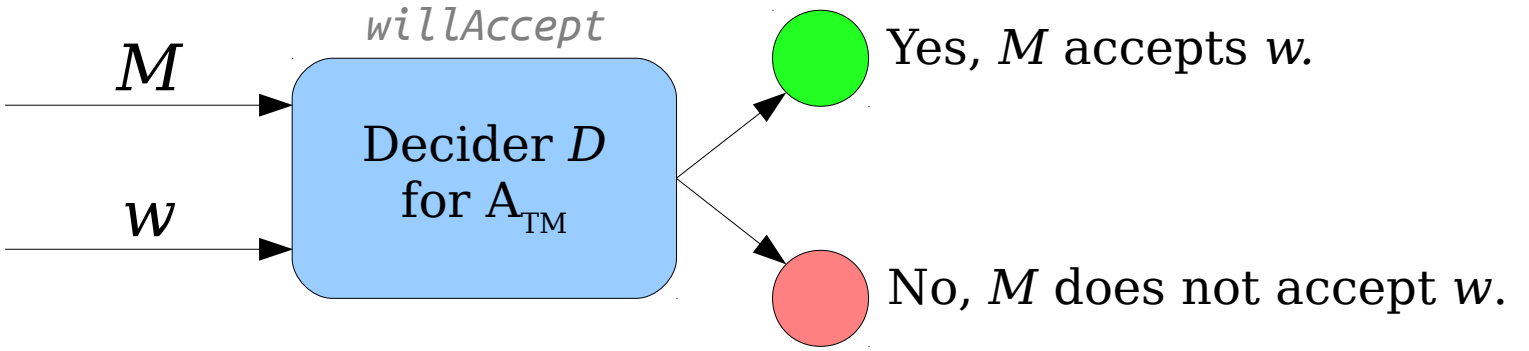


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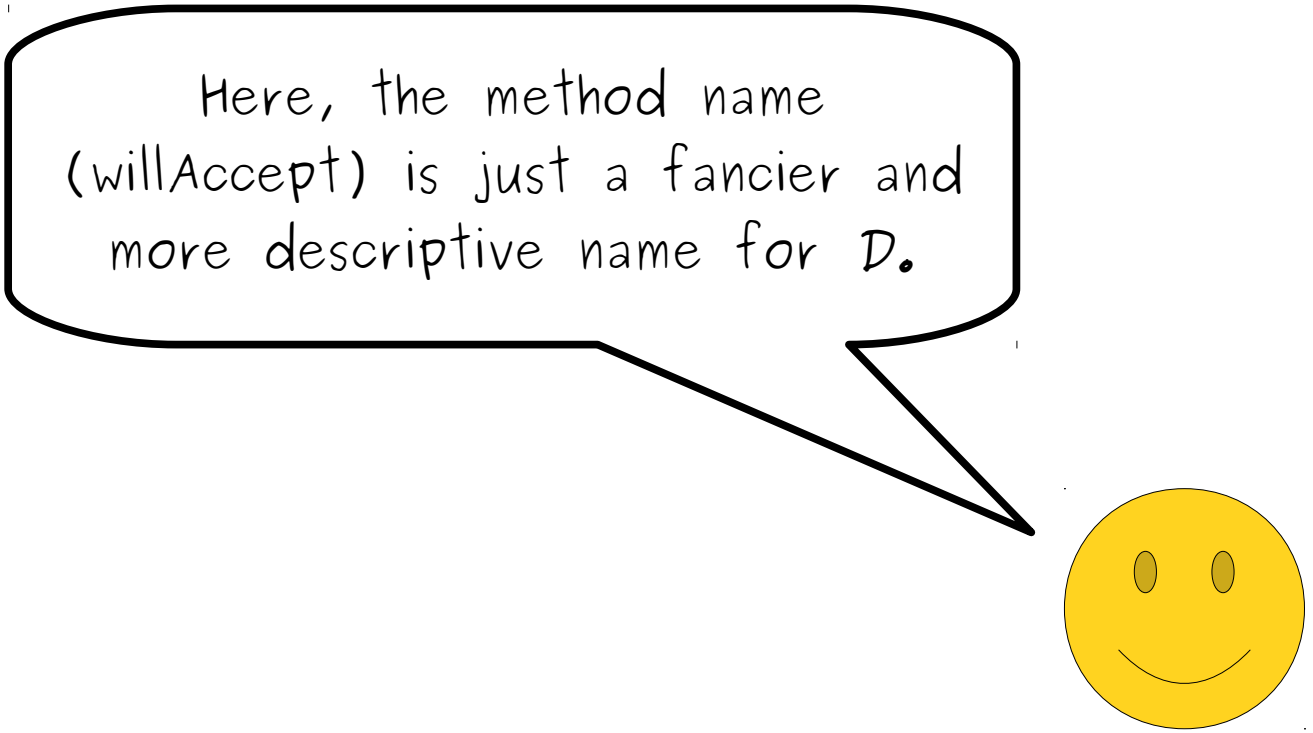

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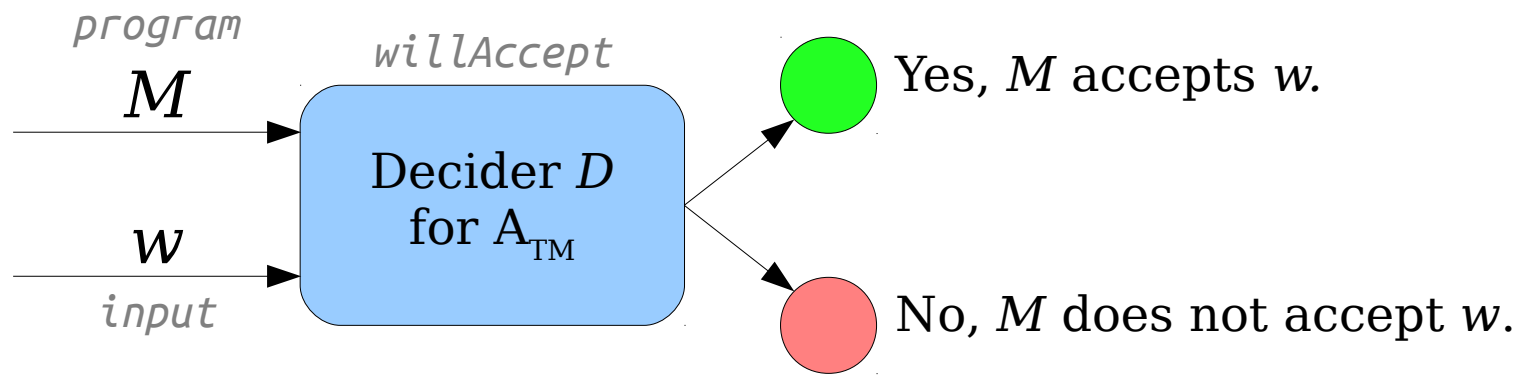

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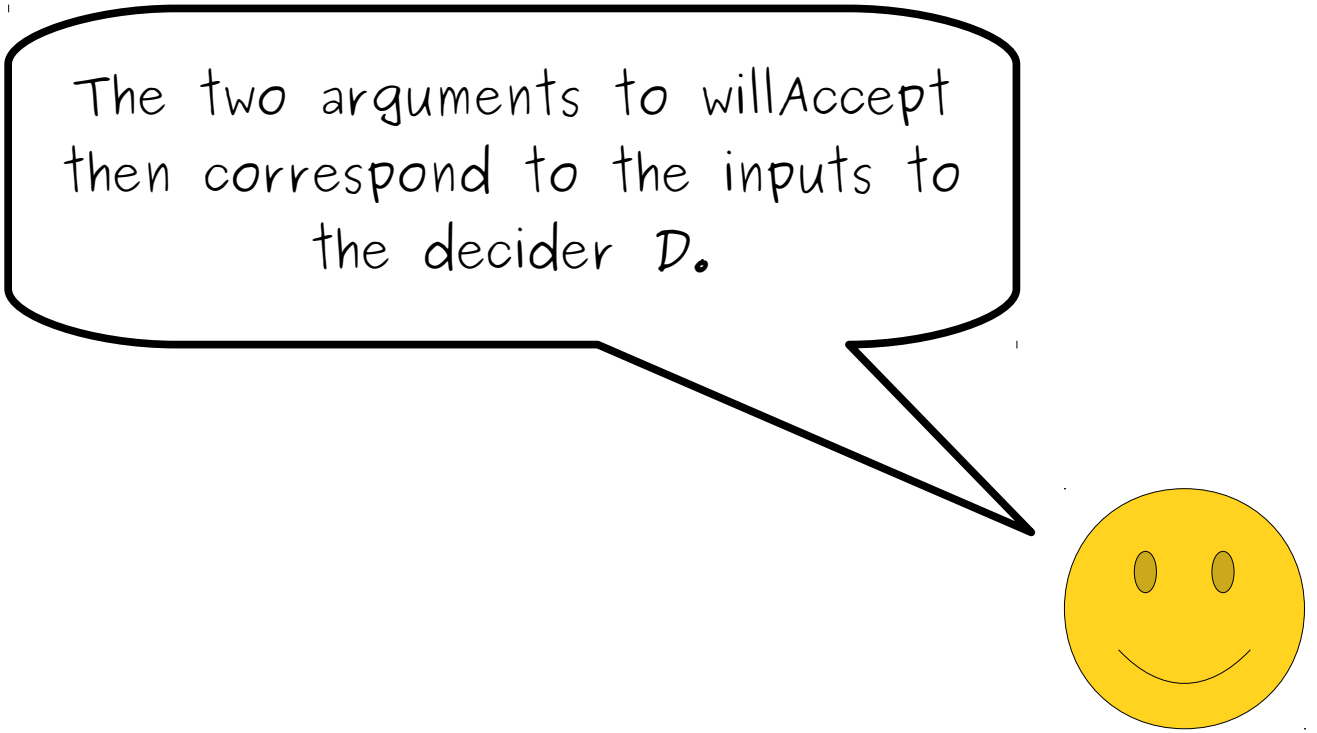
We can write programs that use D as a helper method



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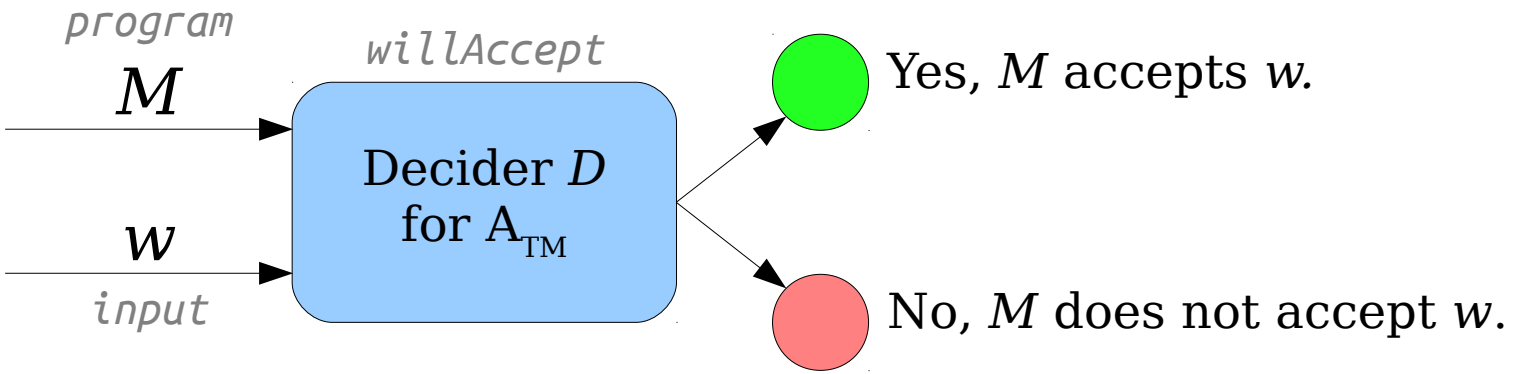


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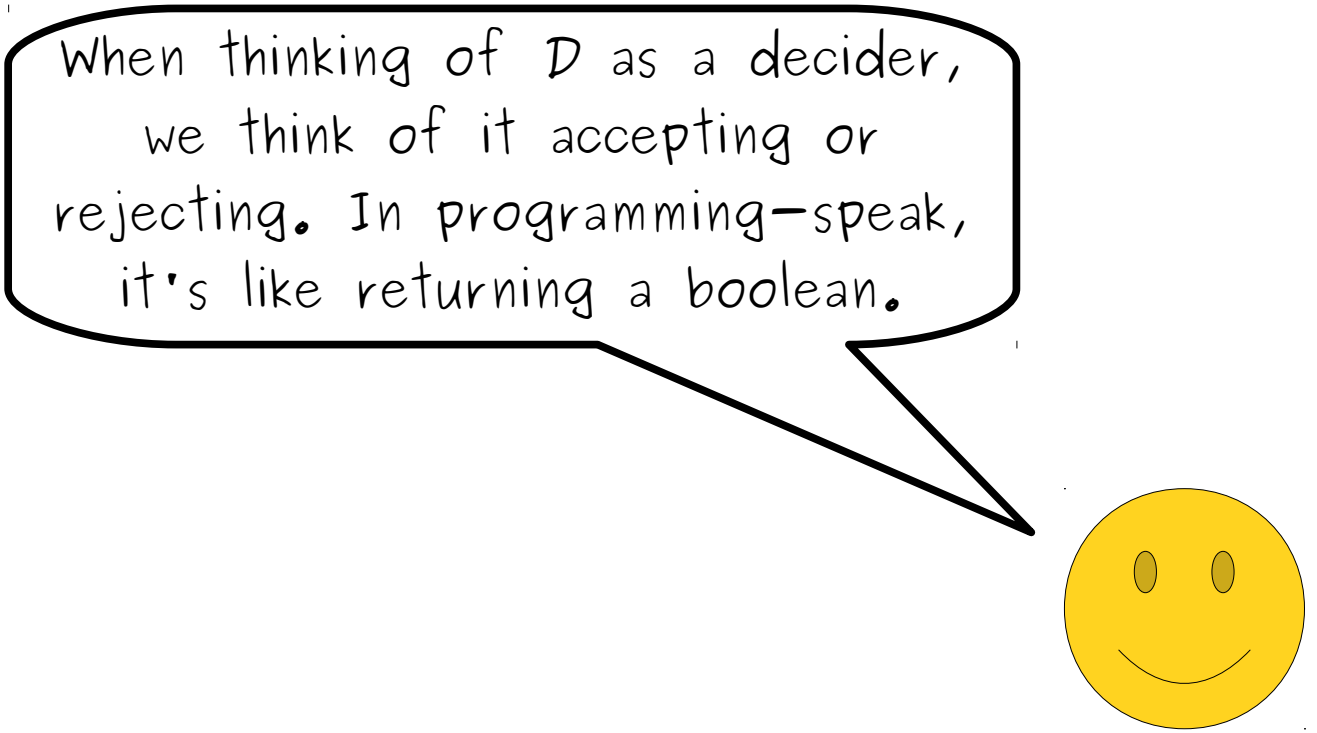

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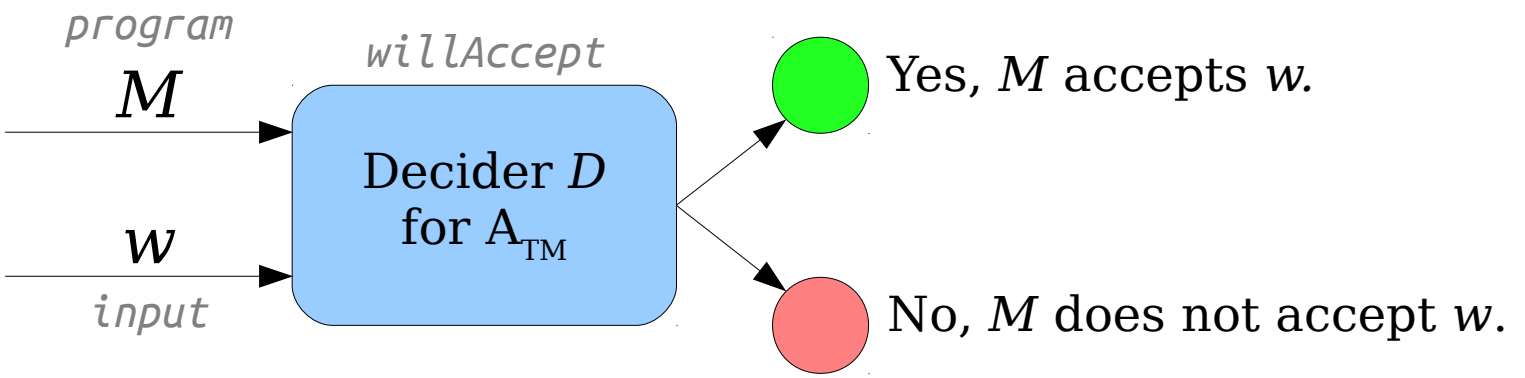


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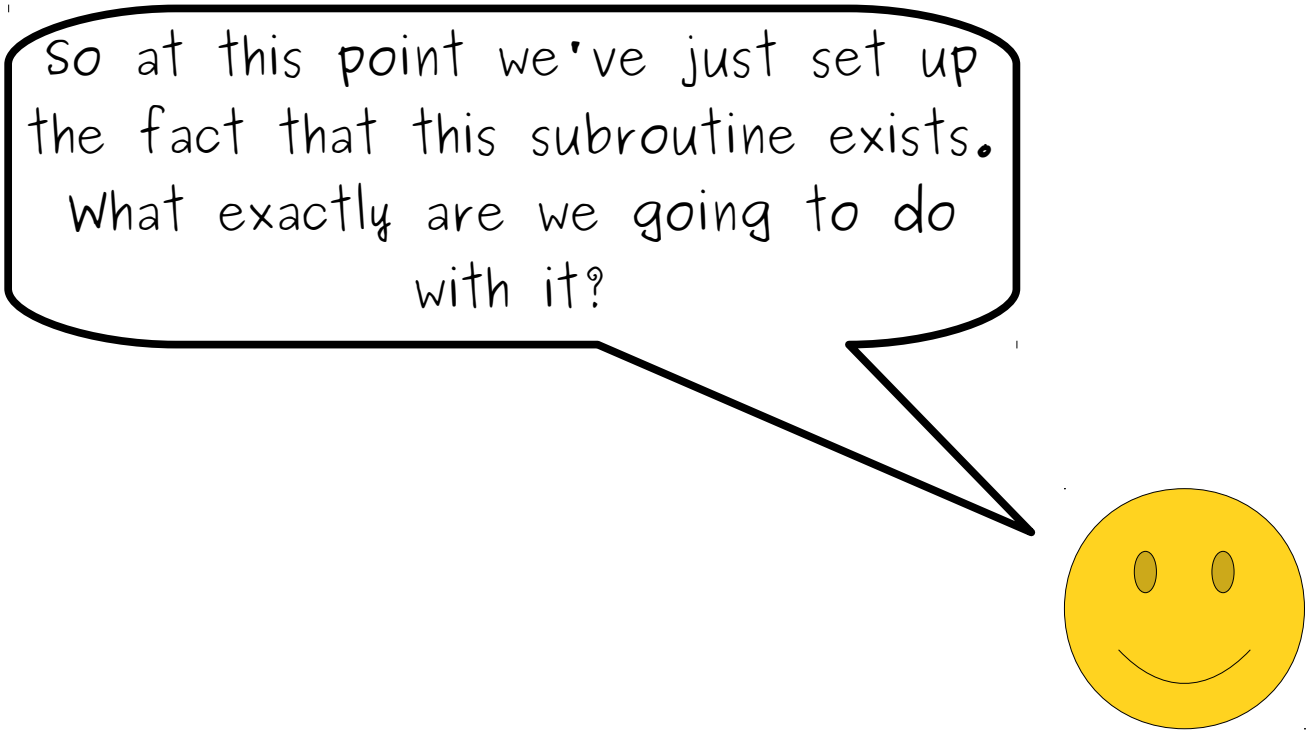

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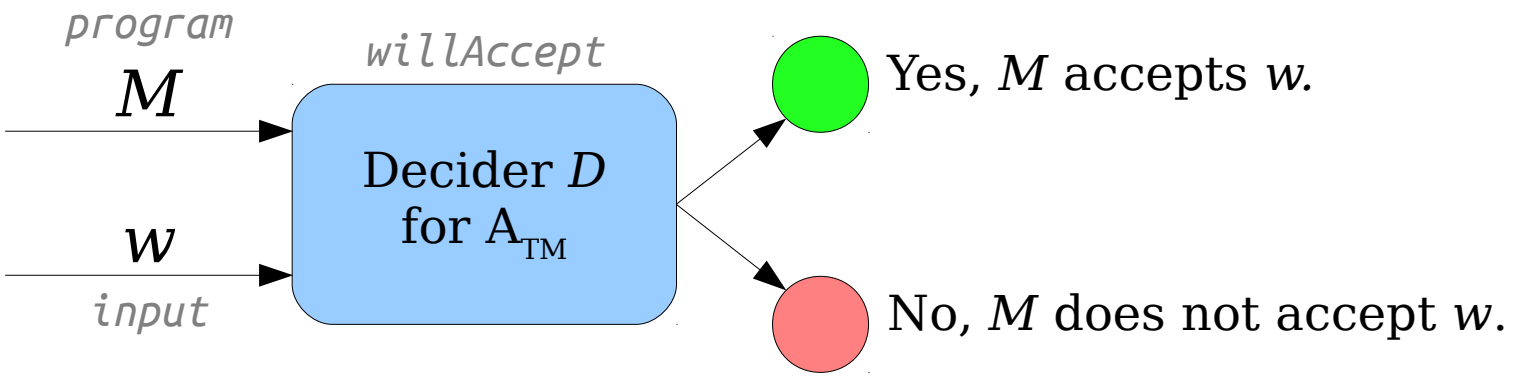


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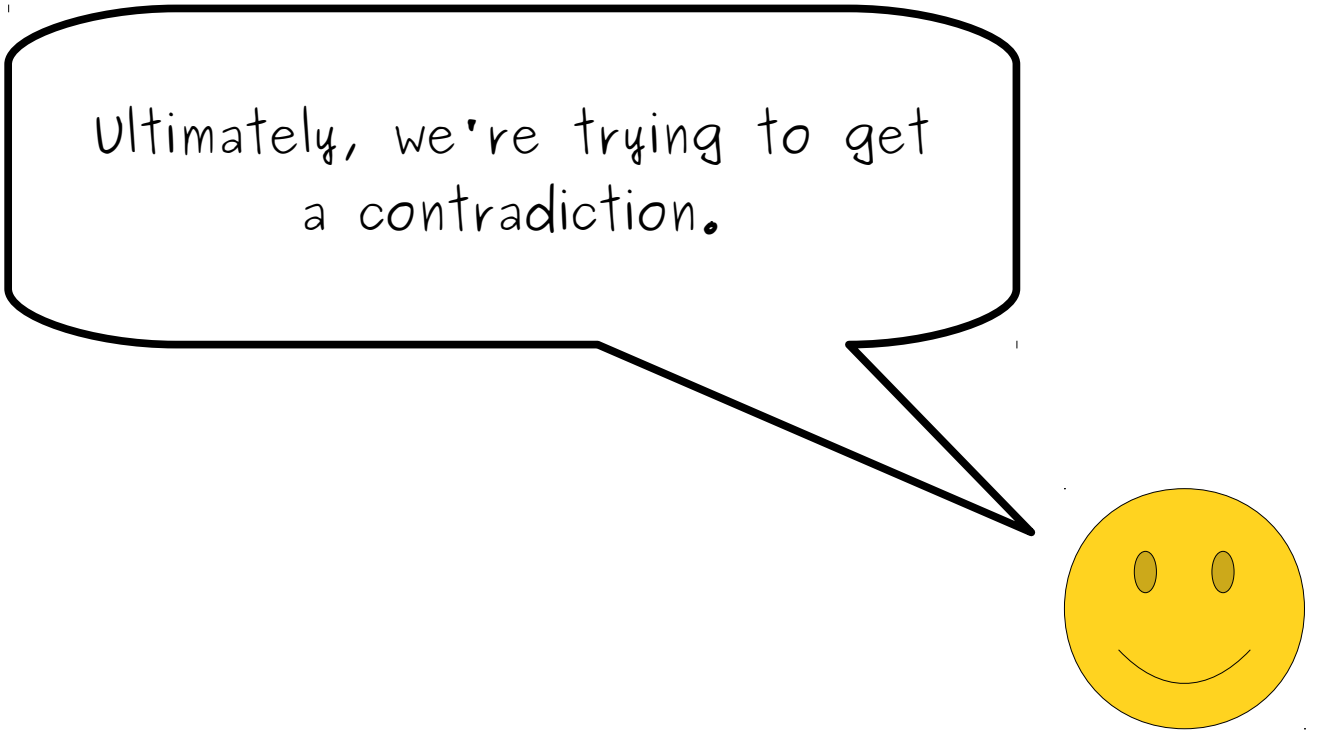

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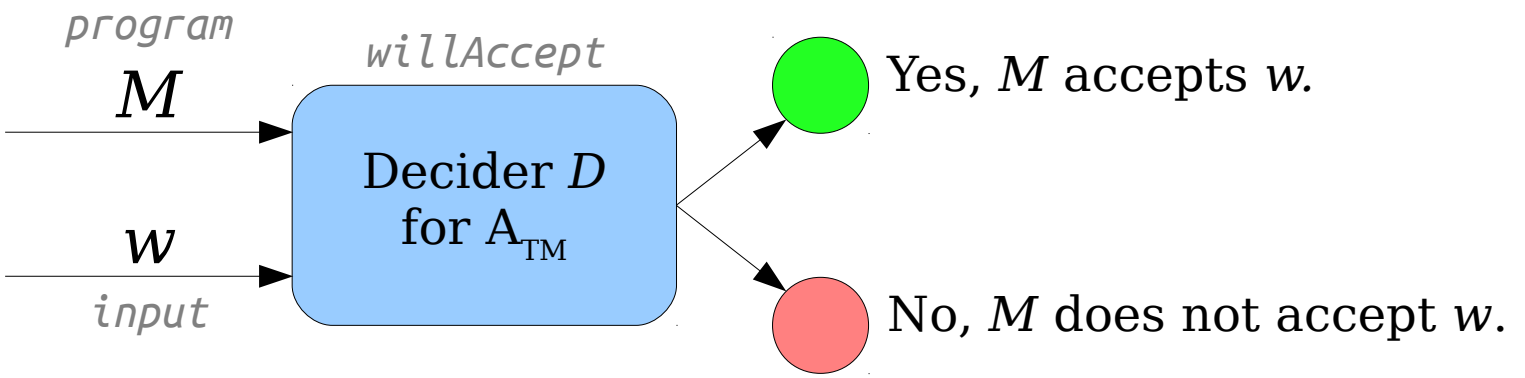


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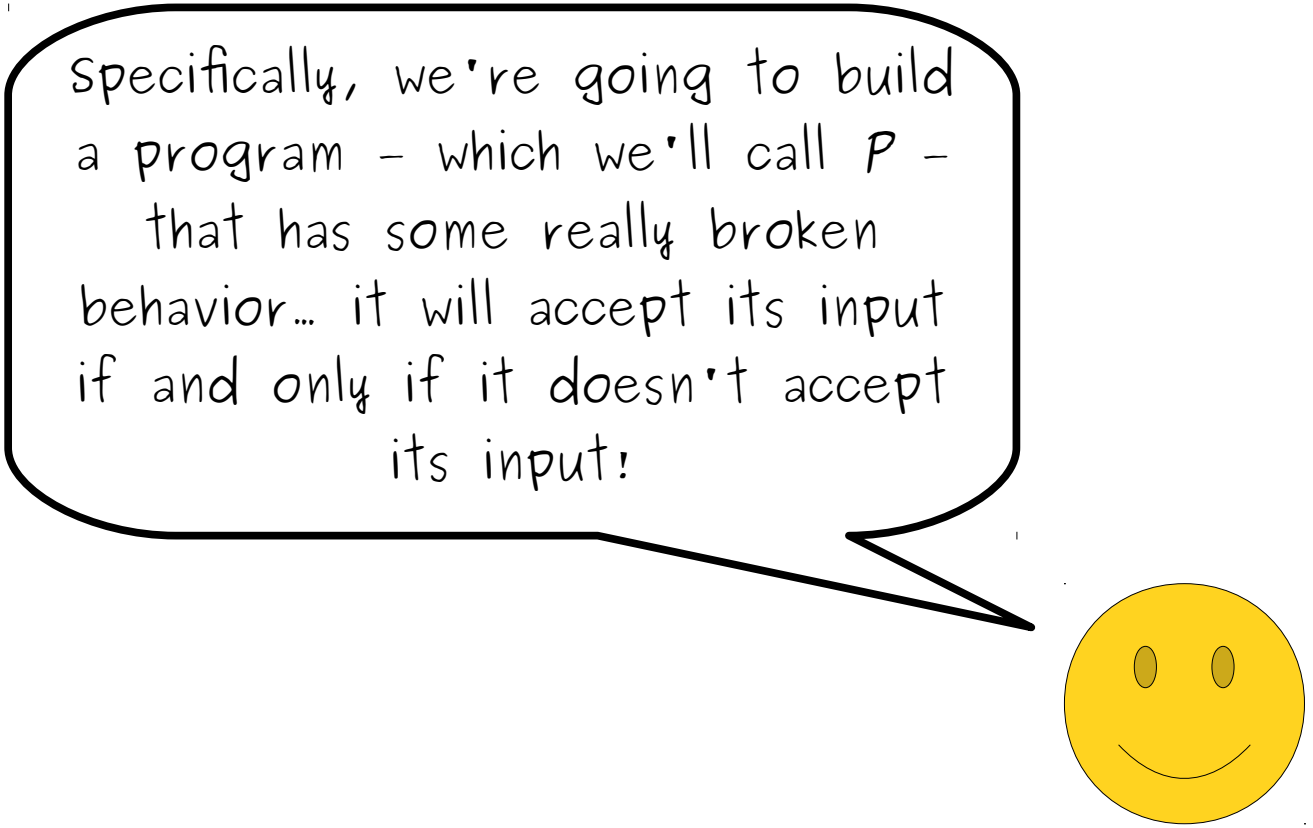

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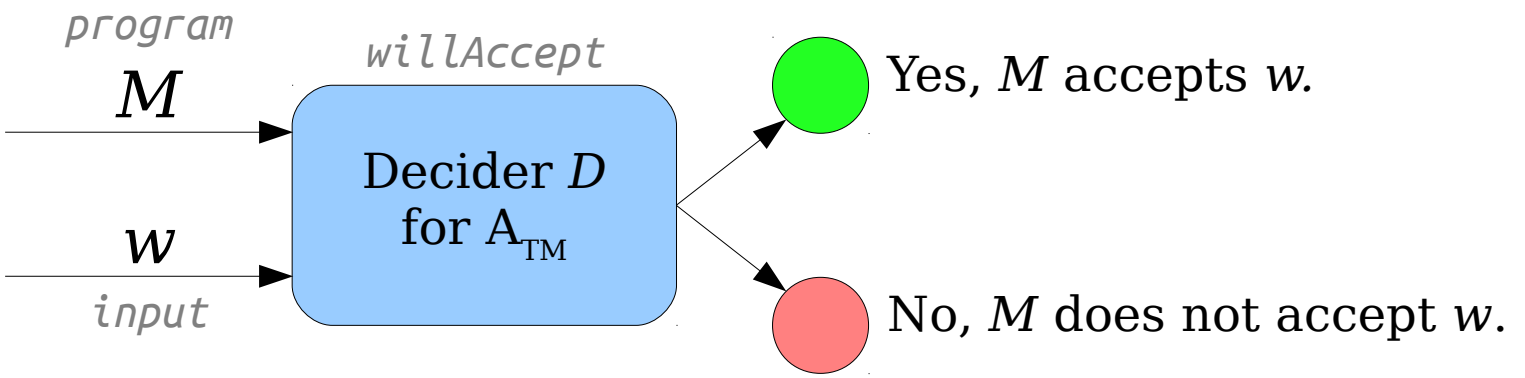


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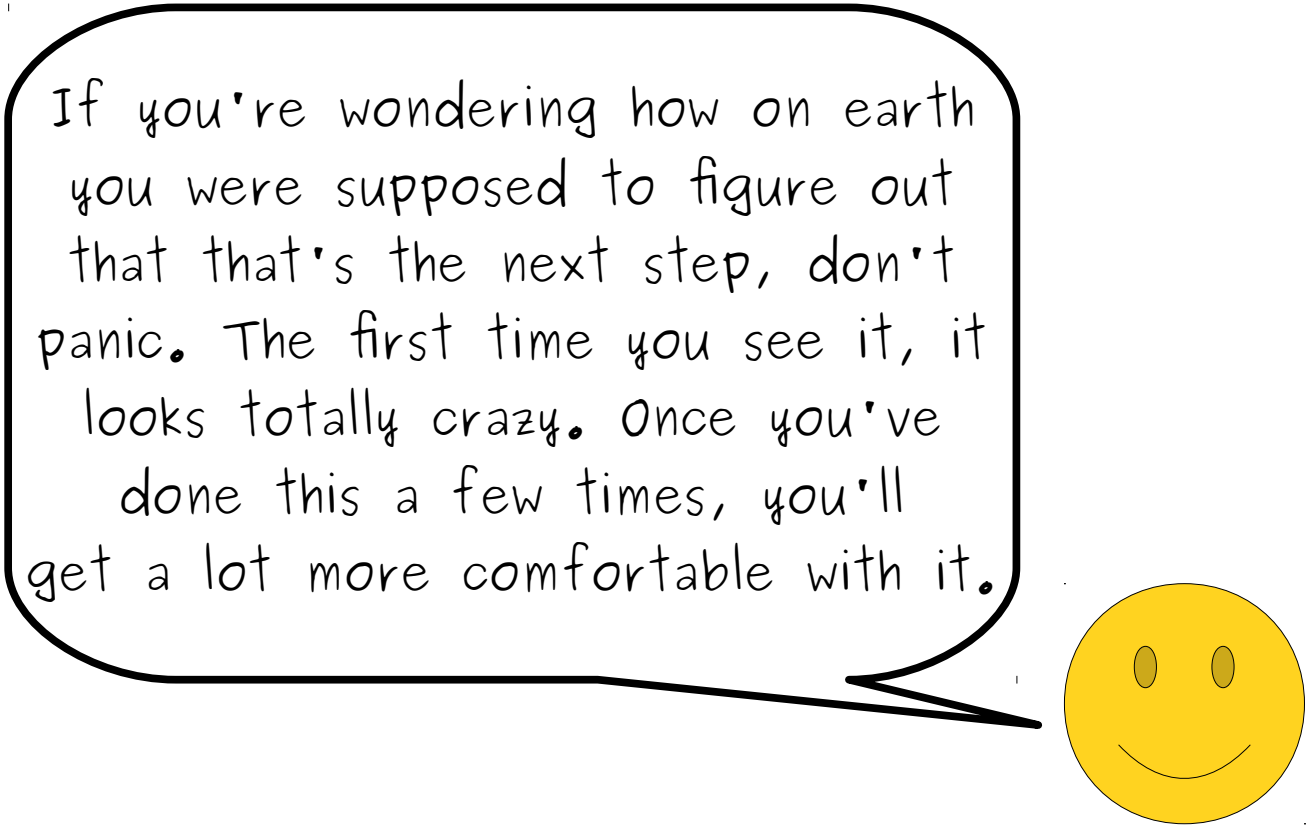

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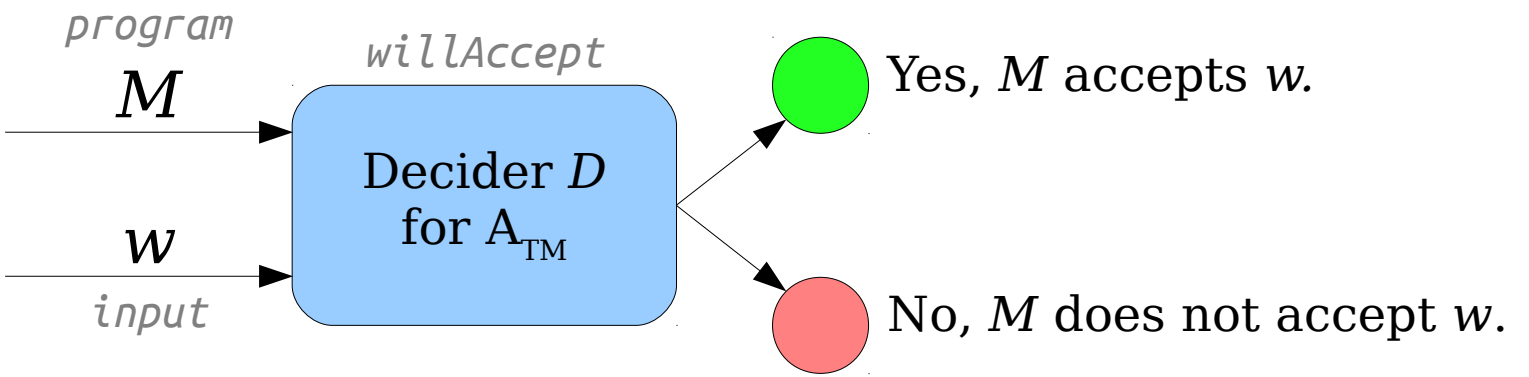


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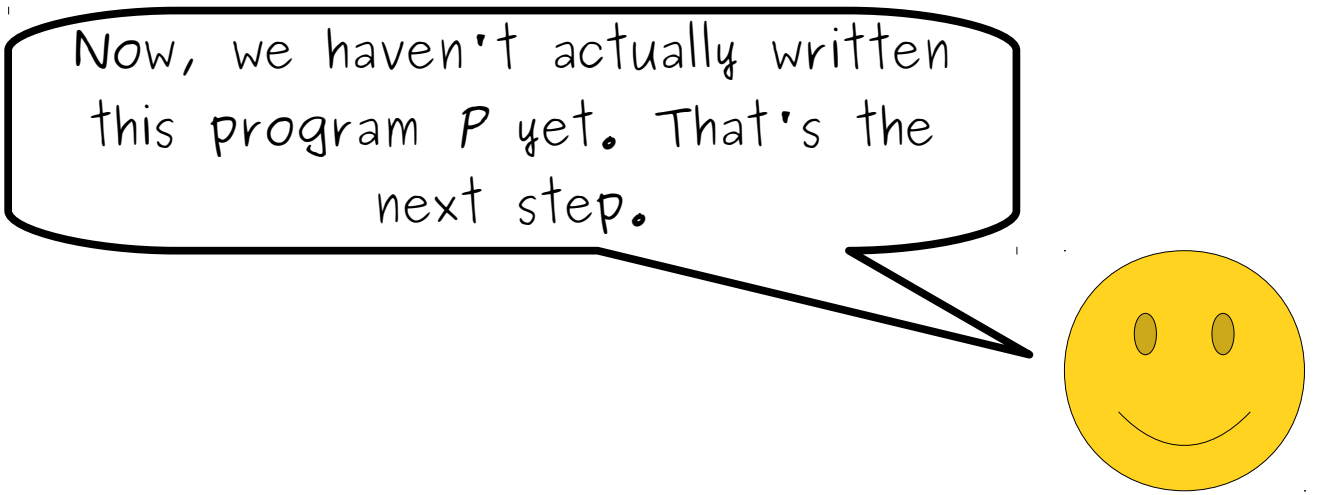

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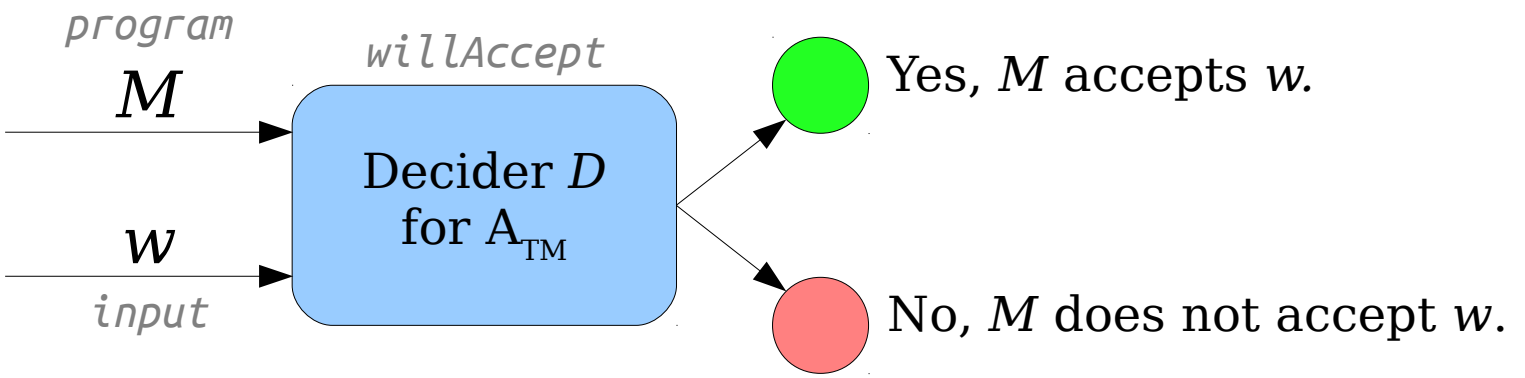


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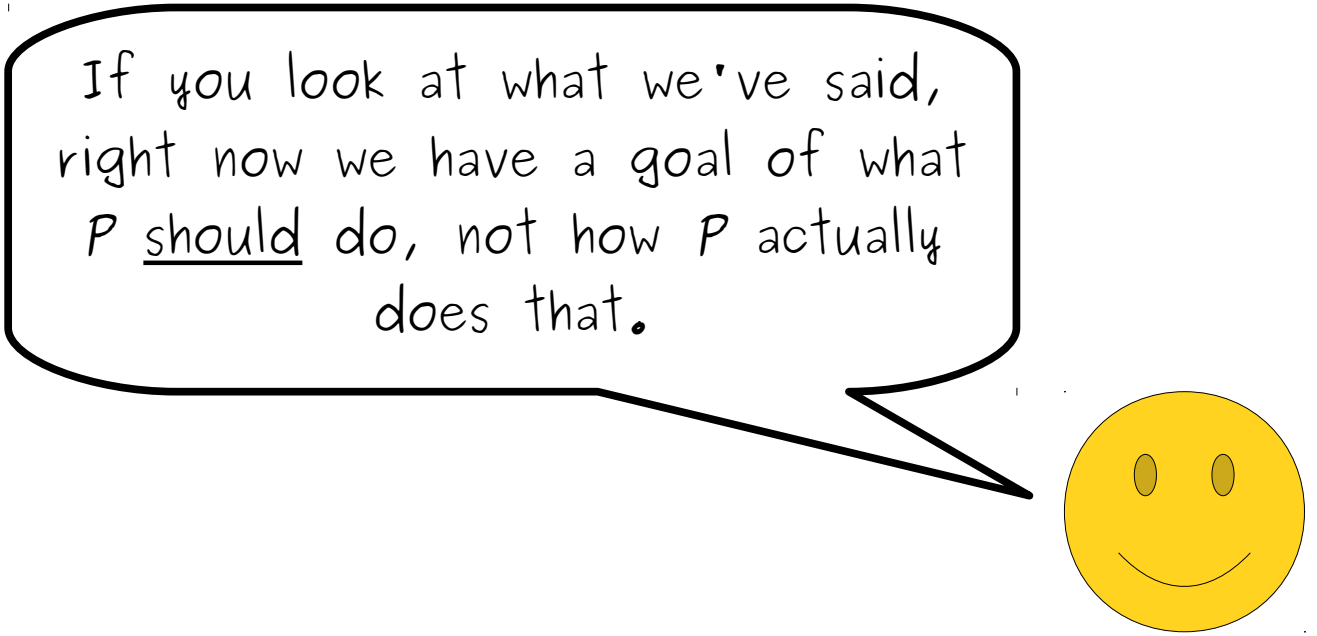

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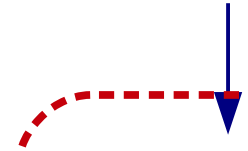

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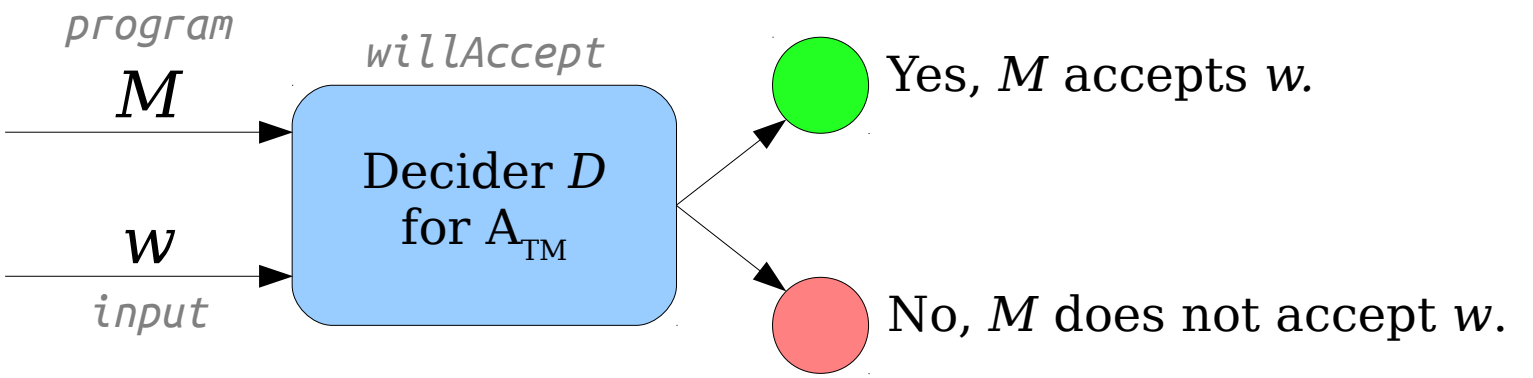


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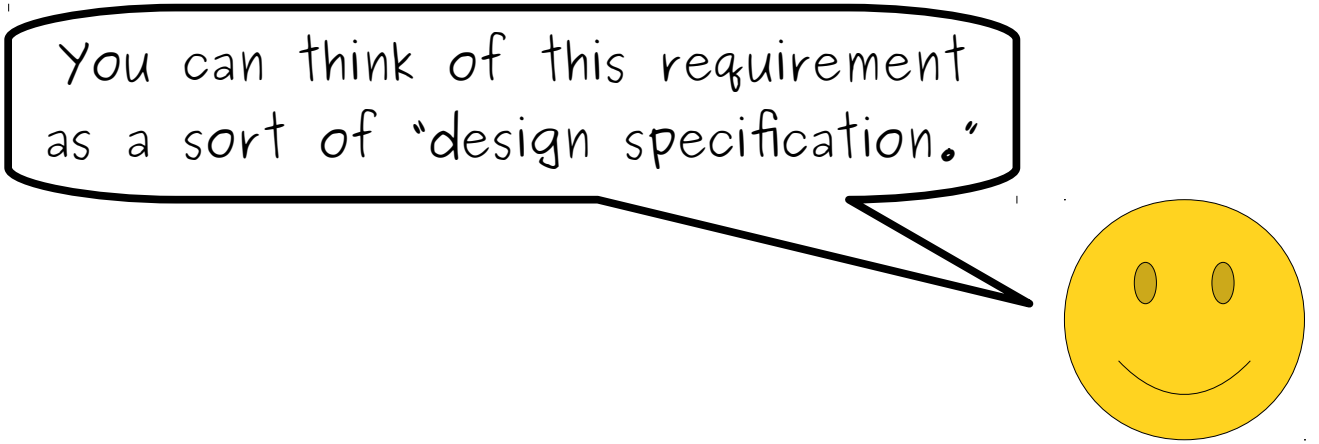

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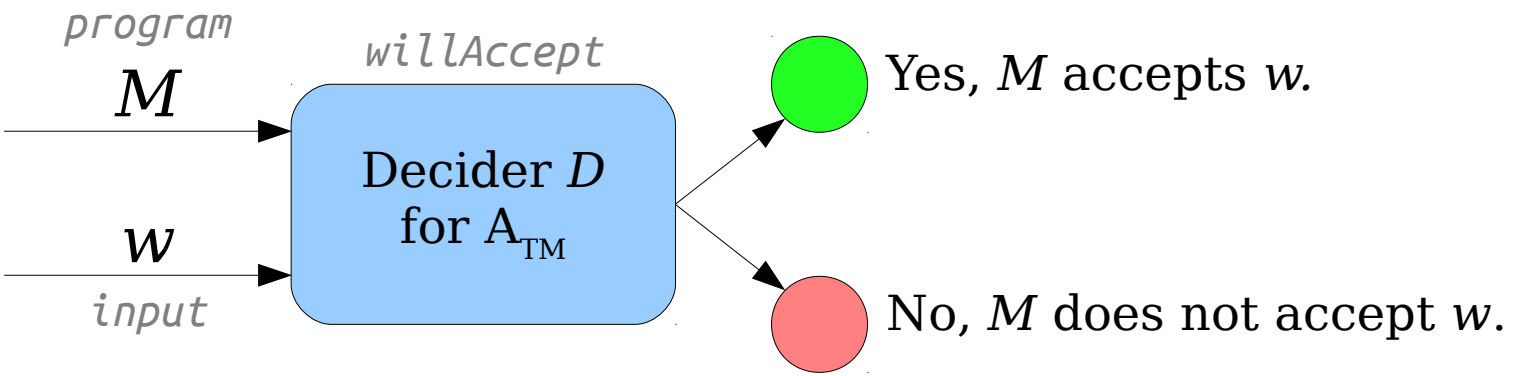


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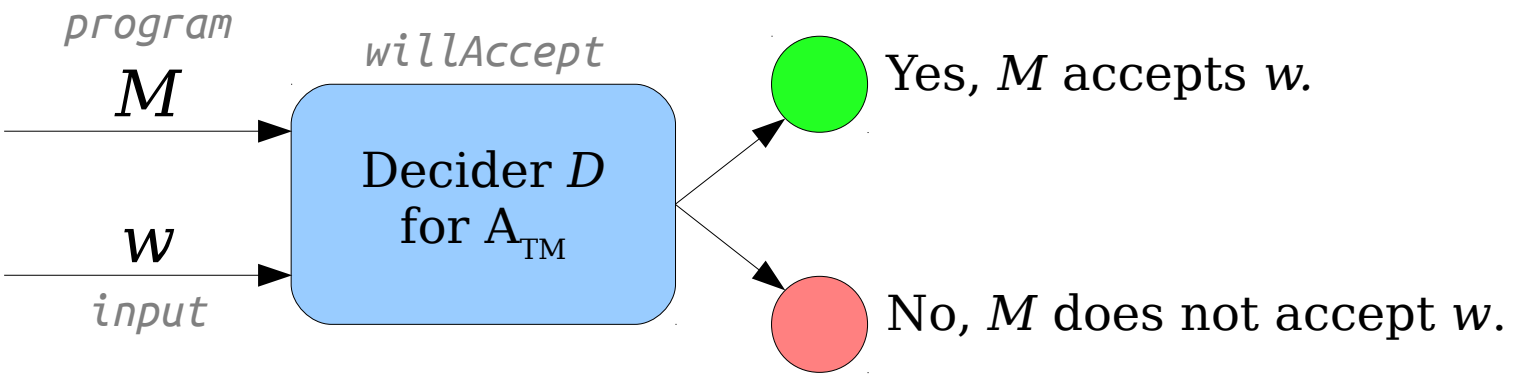


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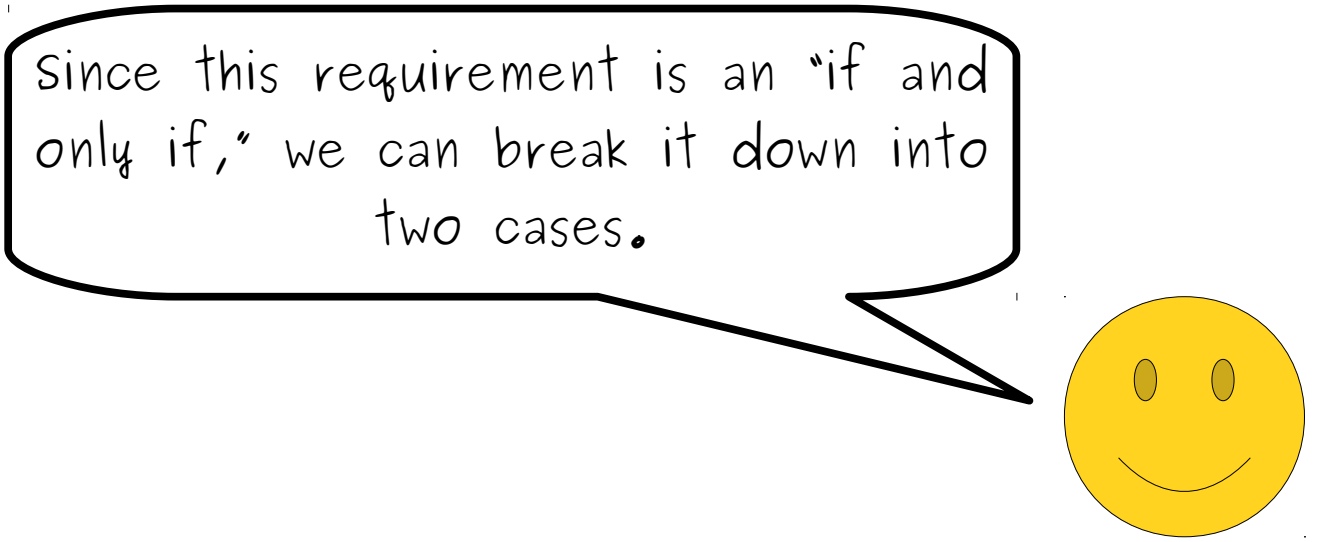

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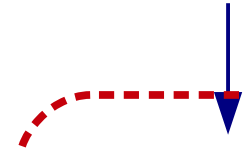

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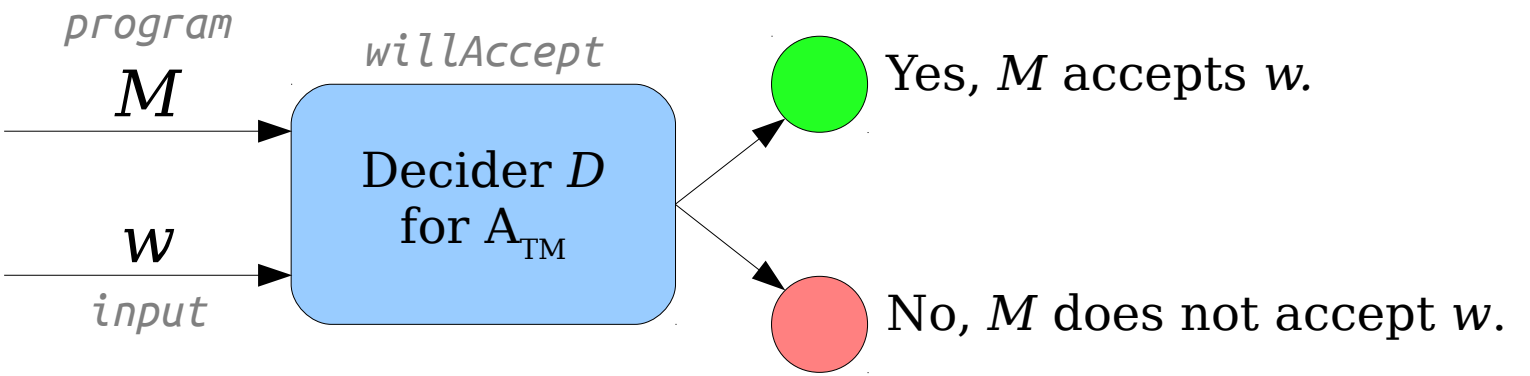


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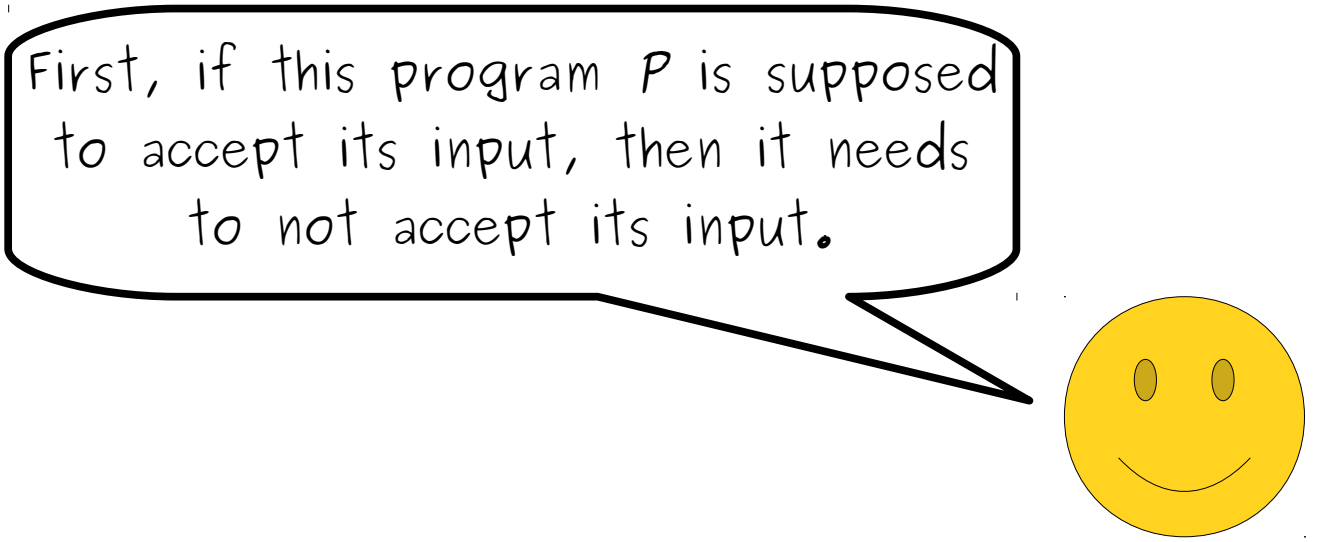

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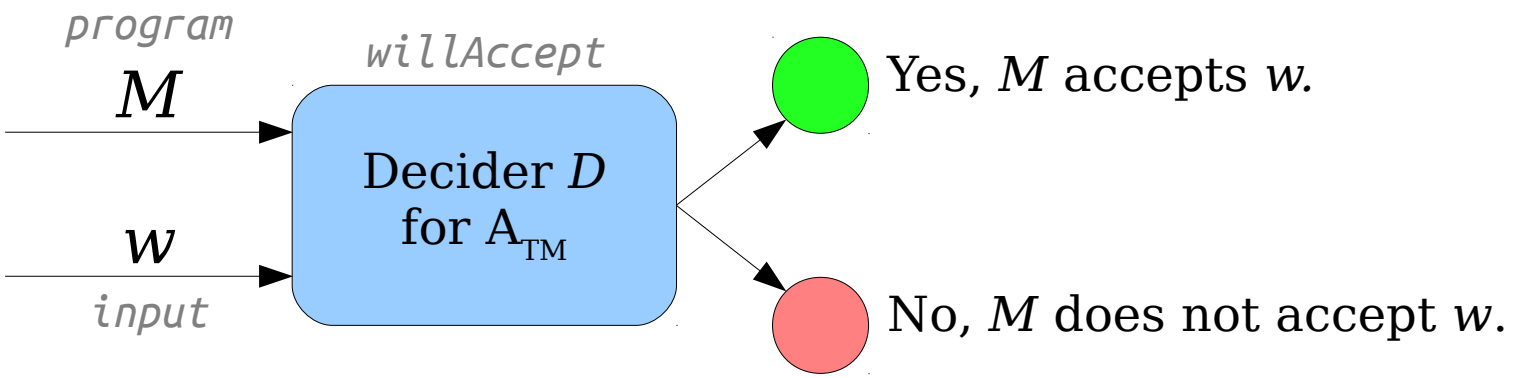


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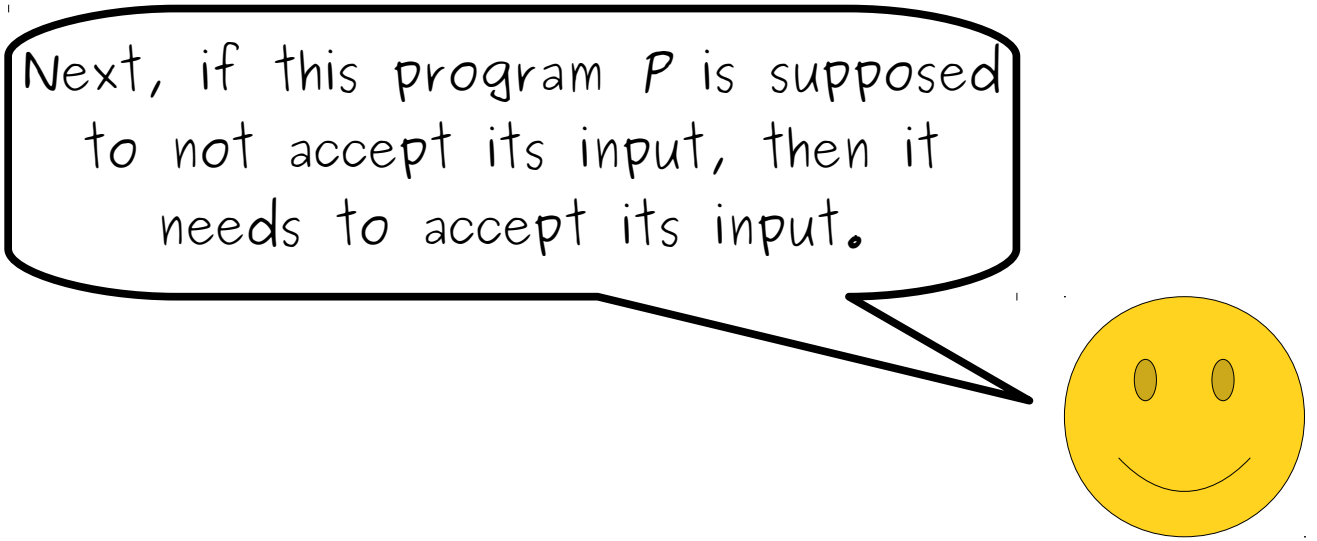

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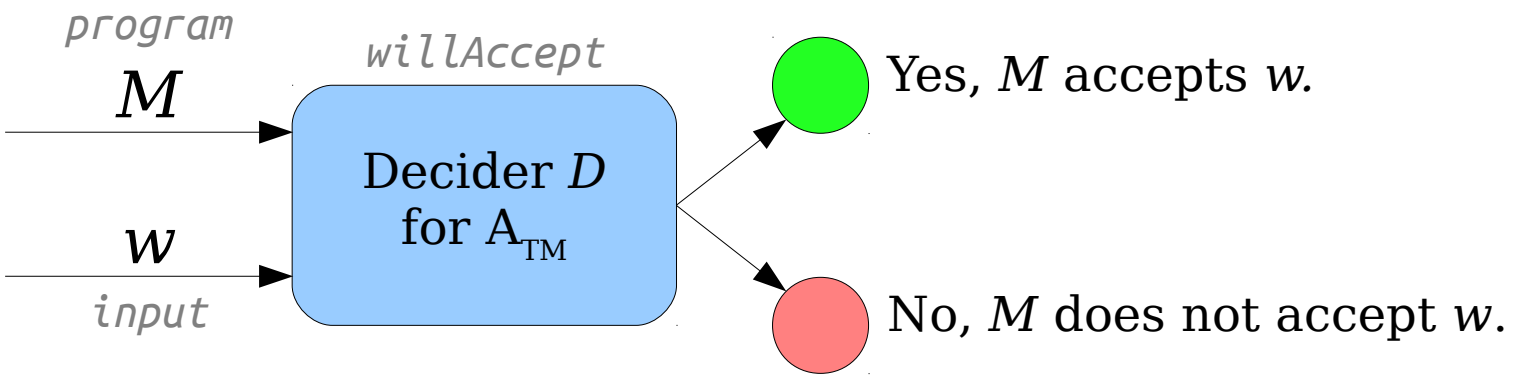


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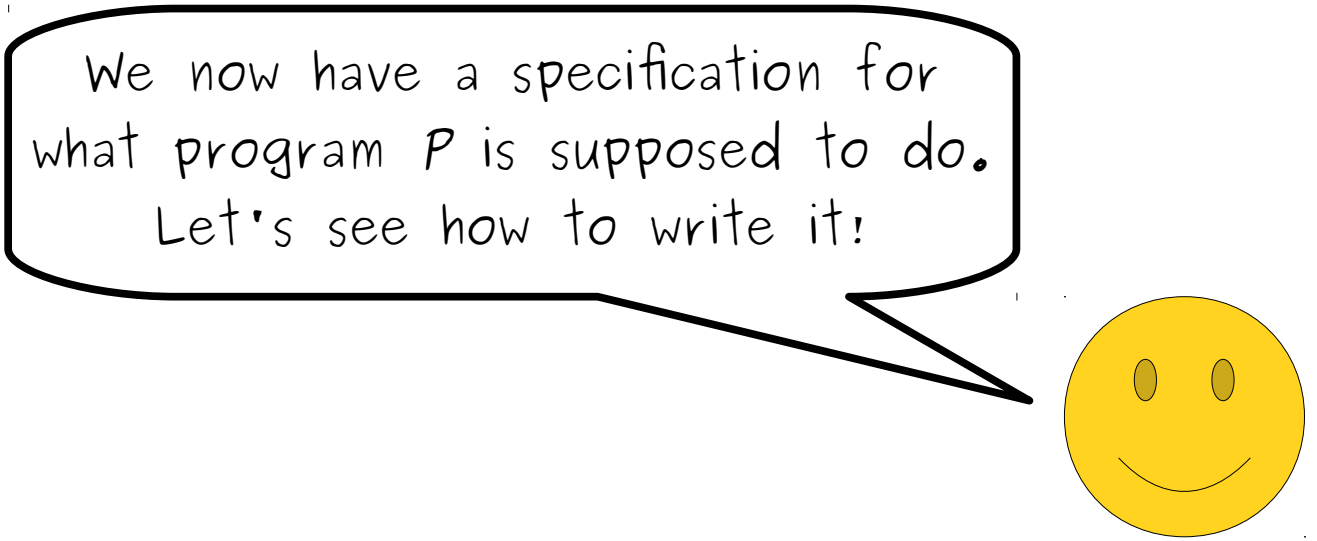

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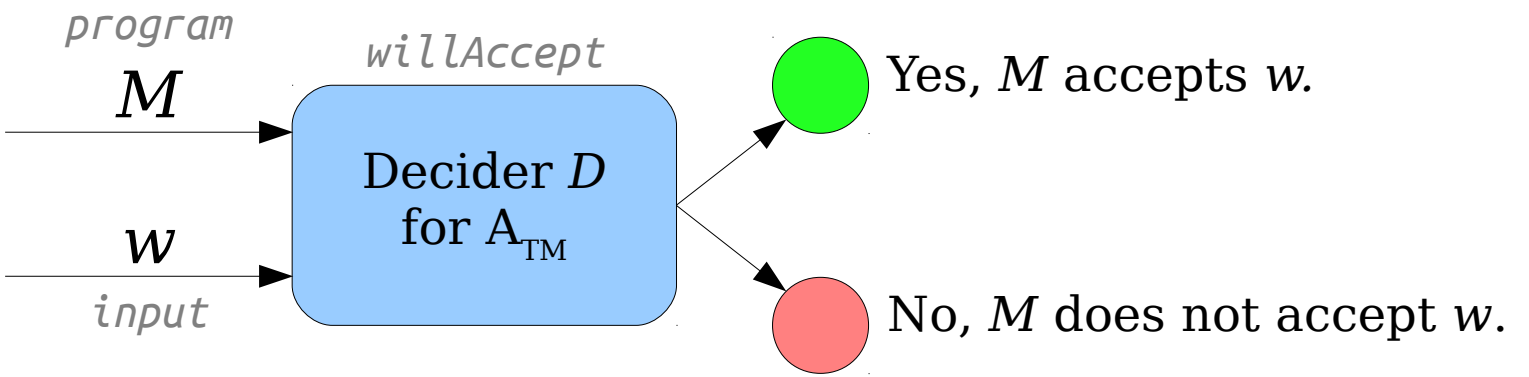


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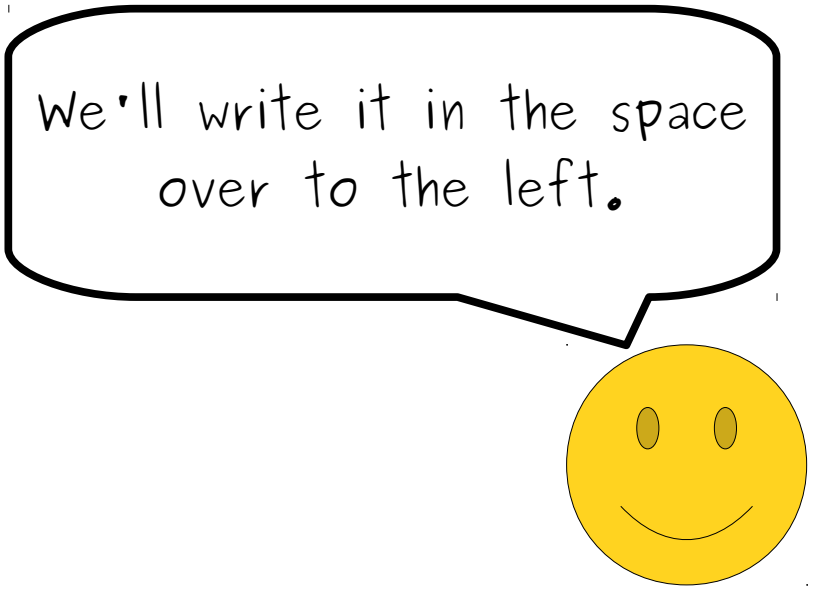

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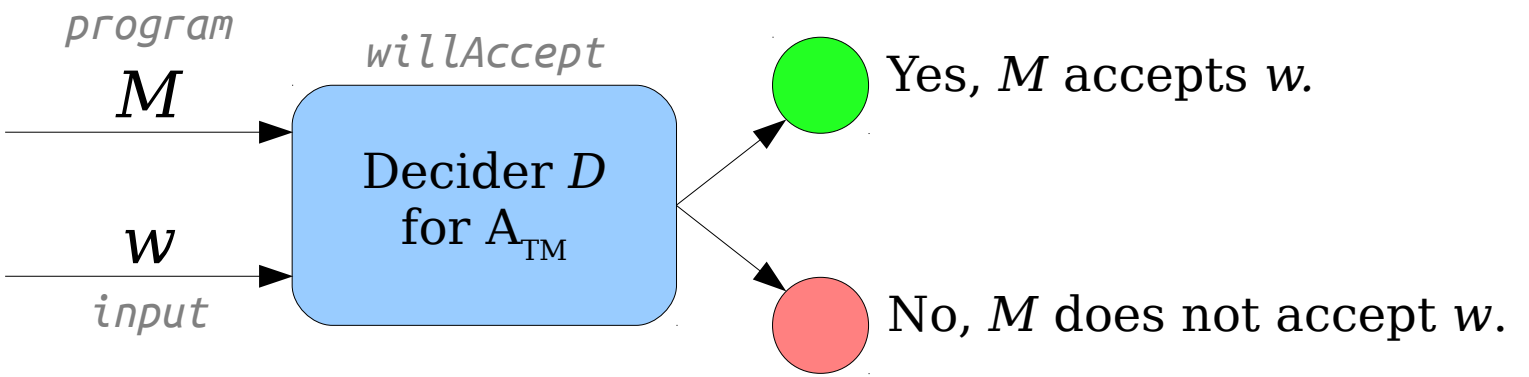


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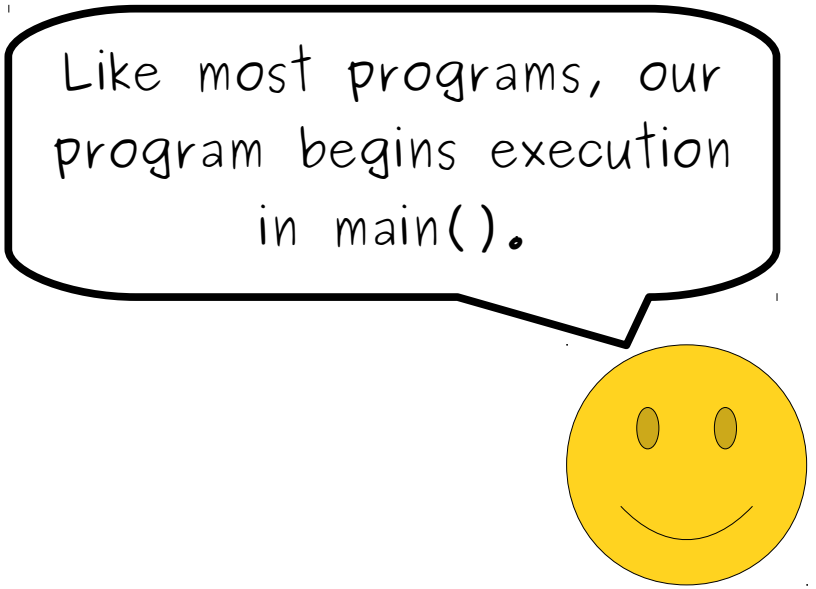

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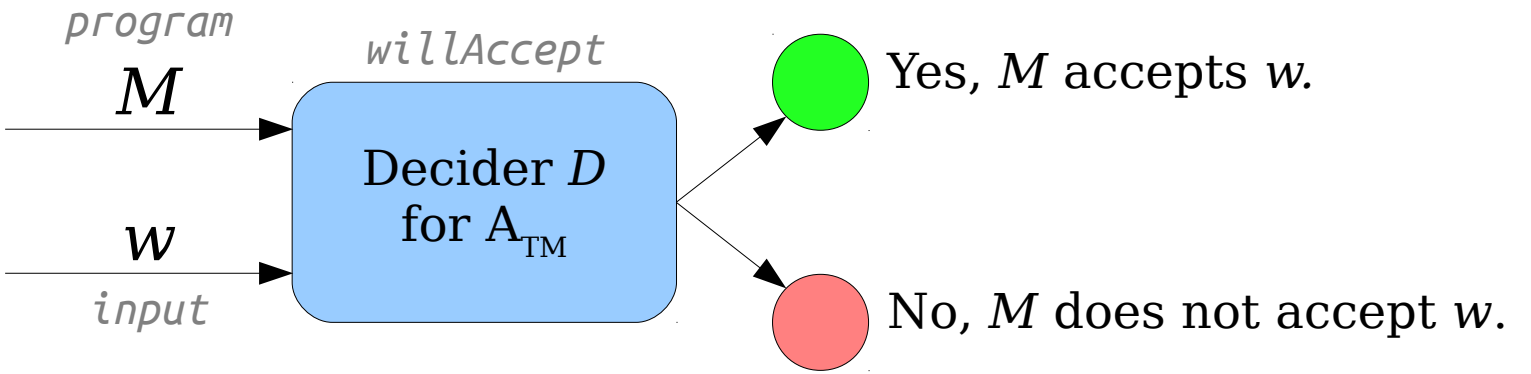


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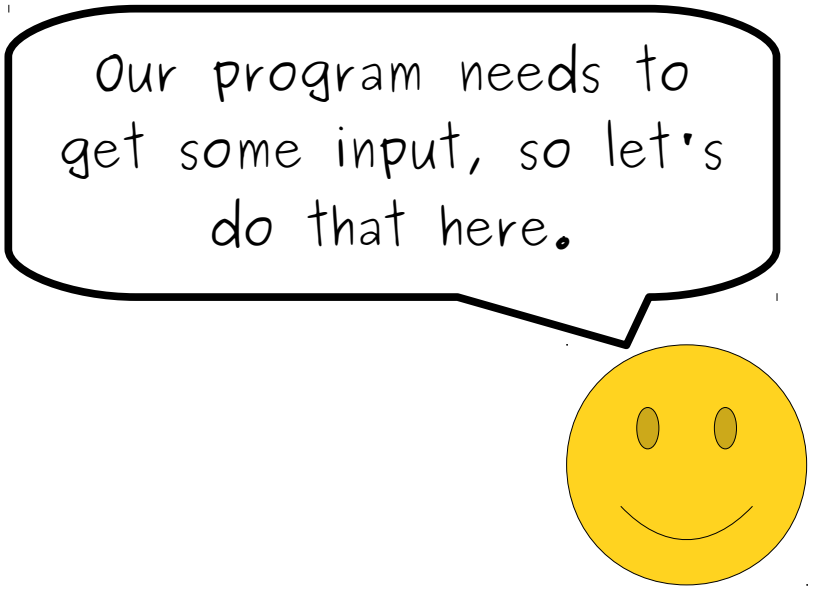

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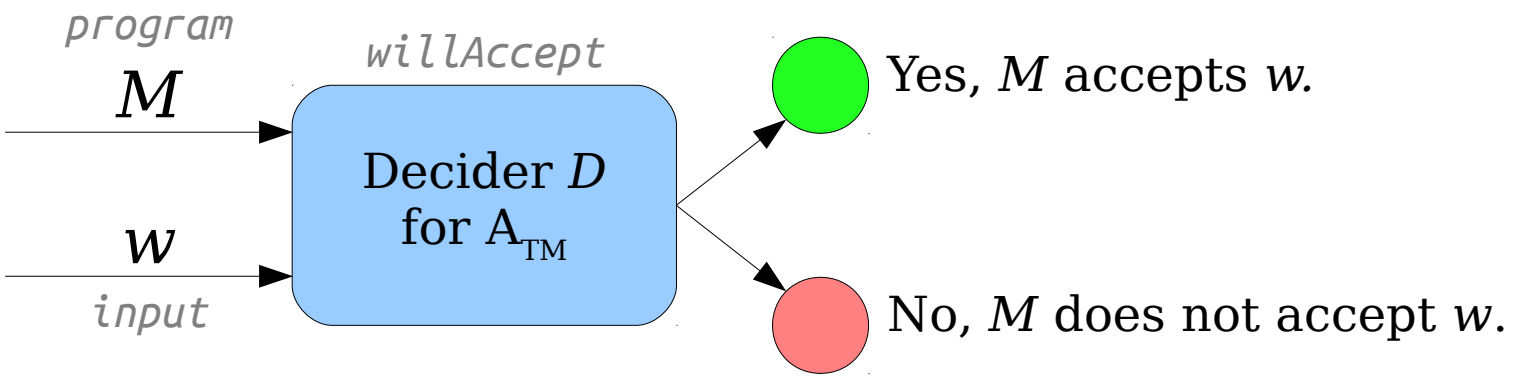


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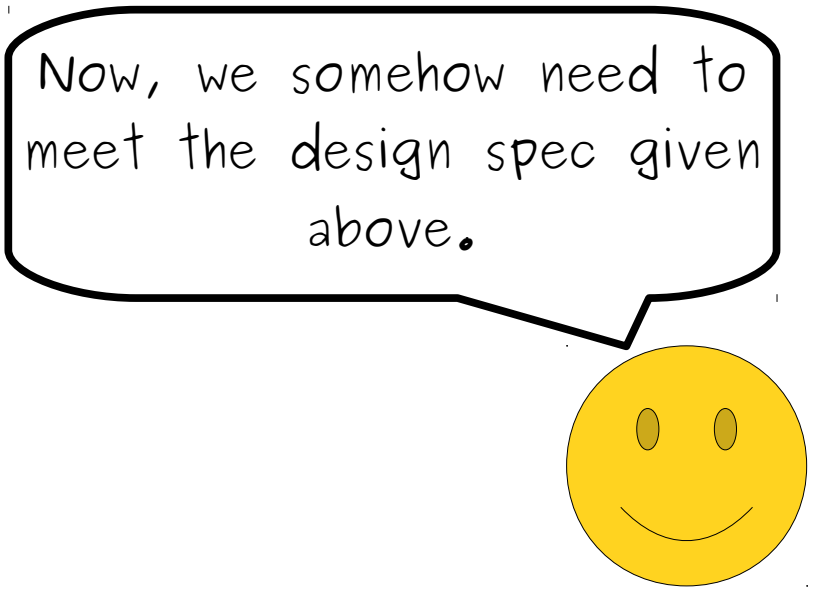

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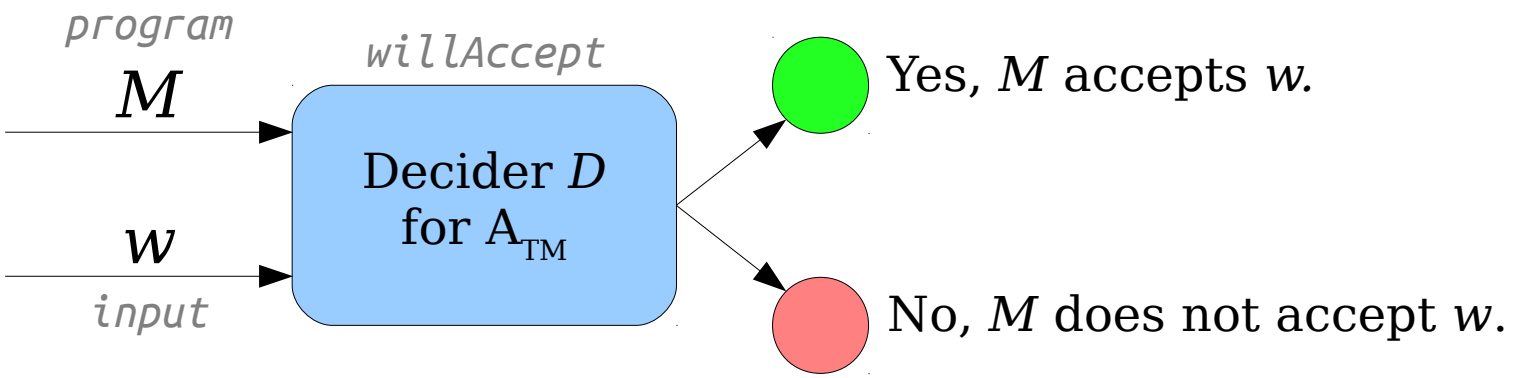


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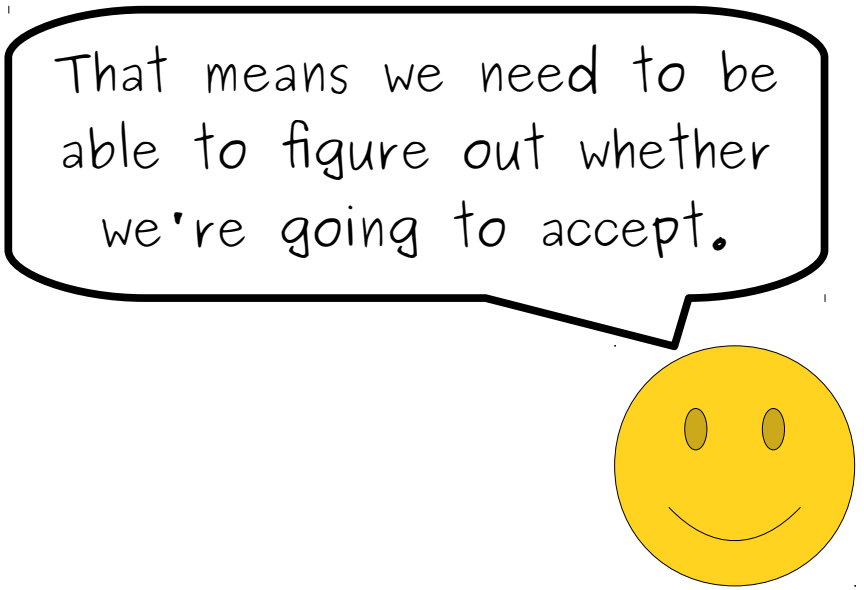

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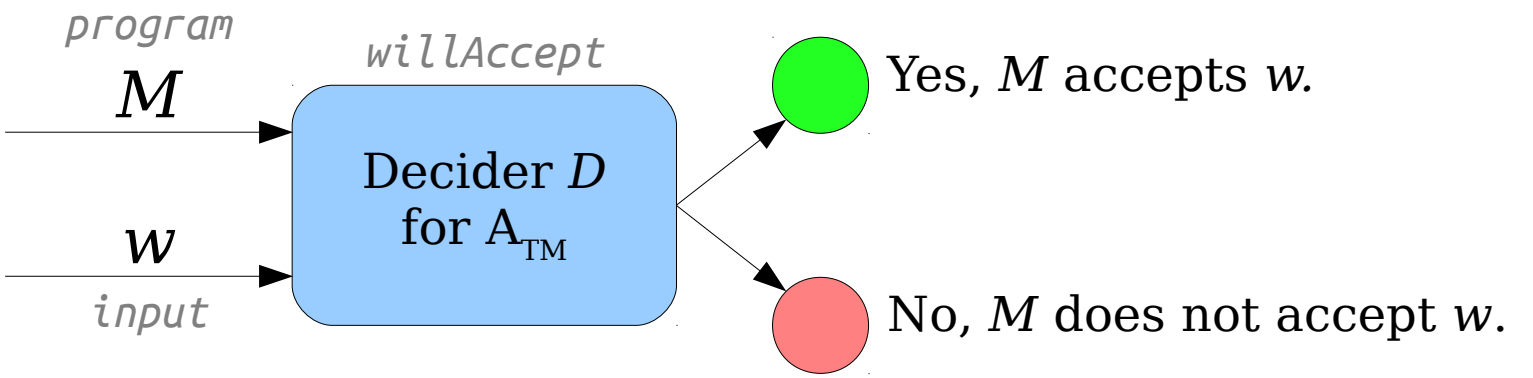


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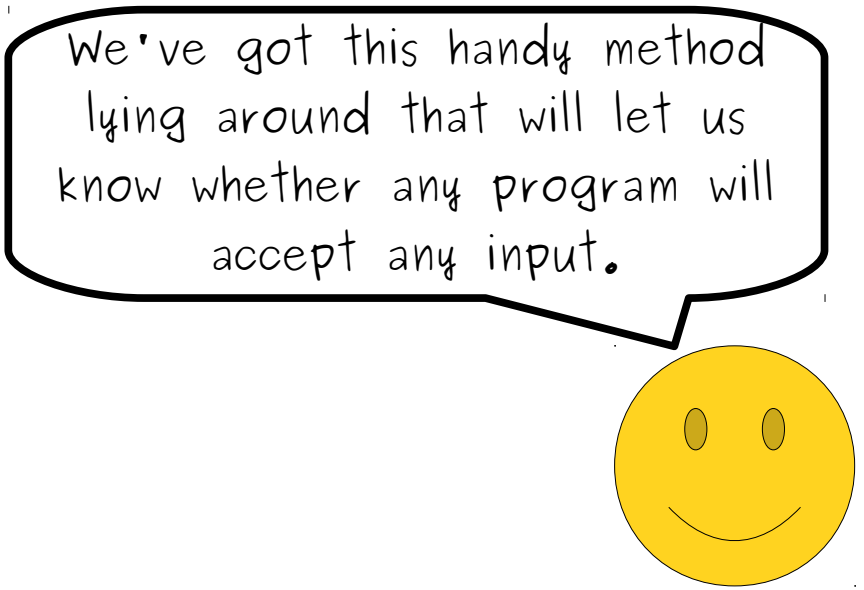

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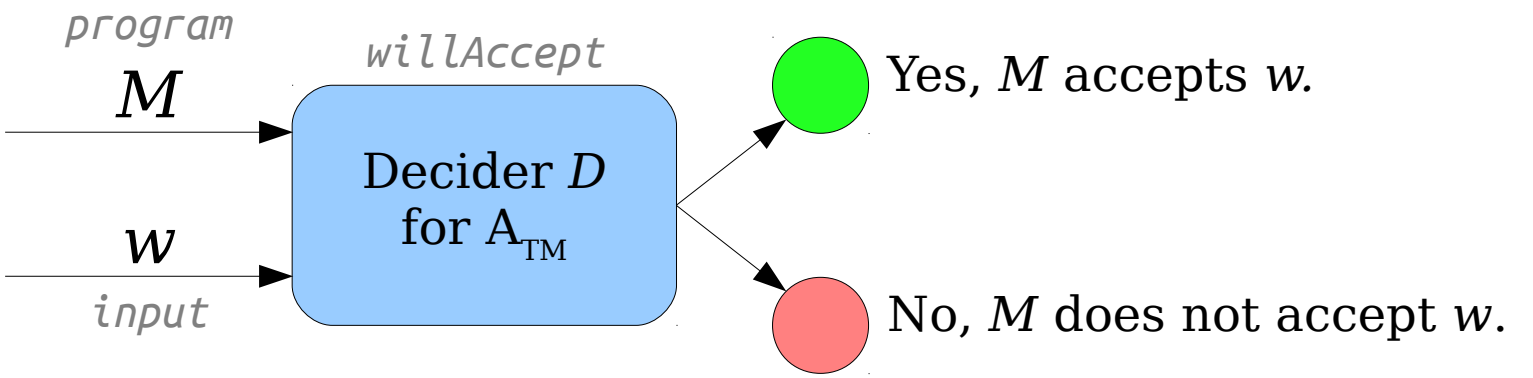


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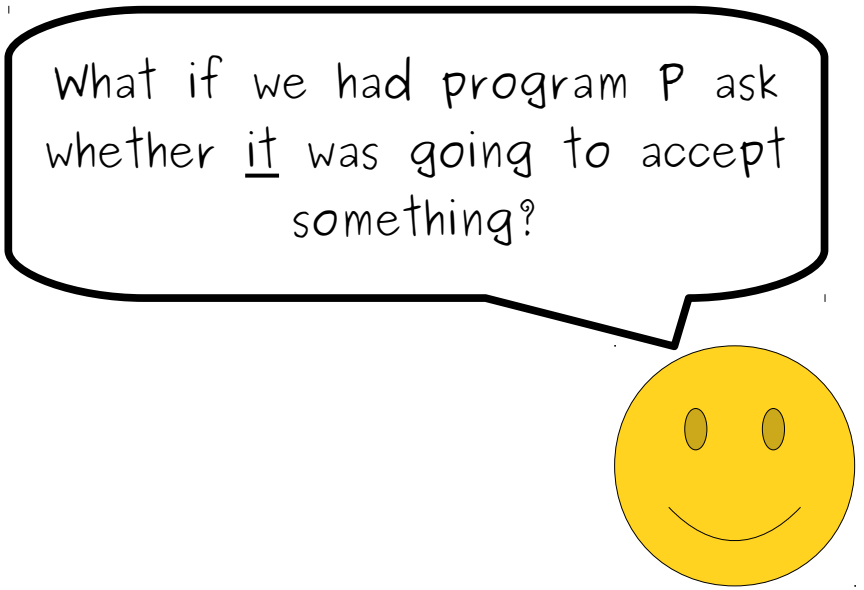

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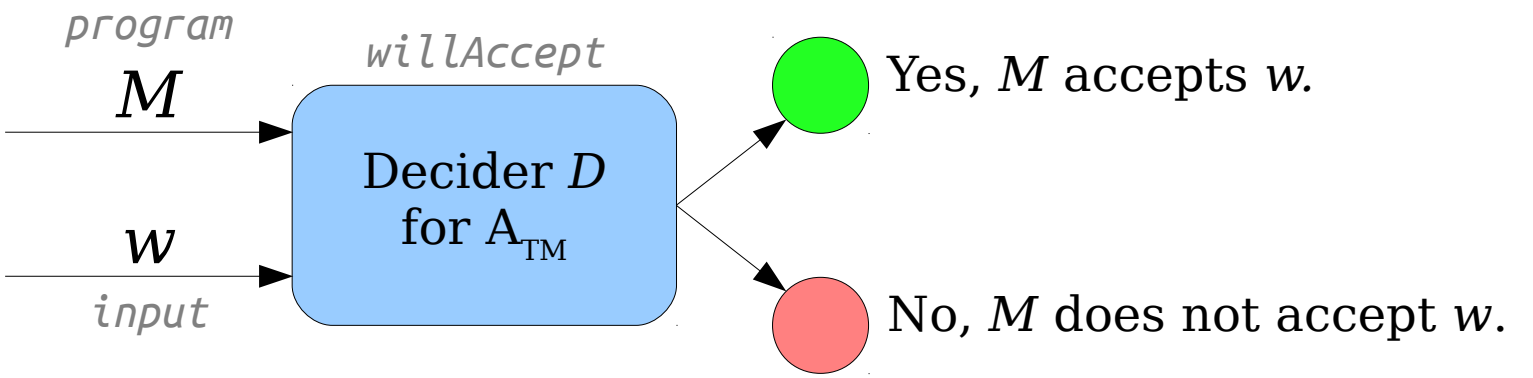


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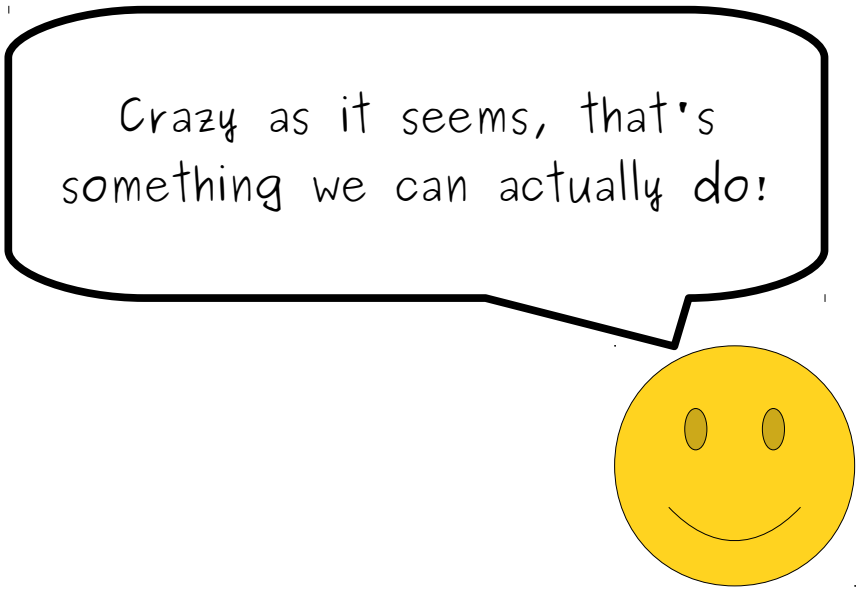

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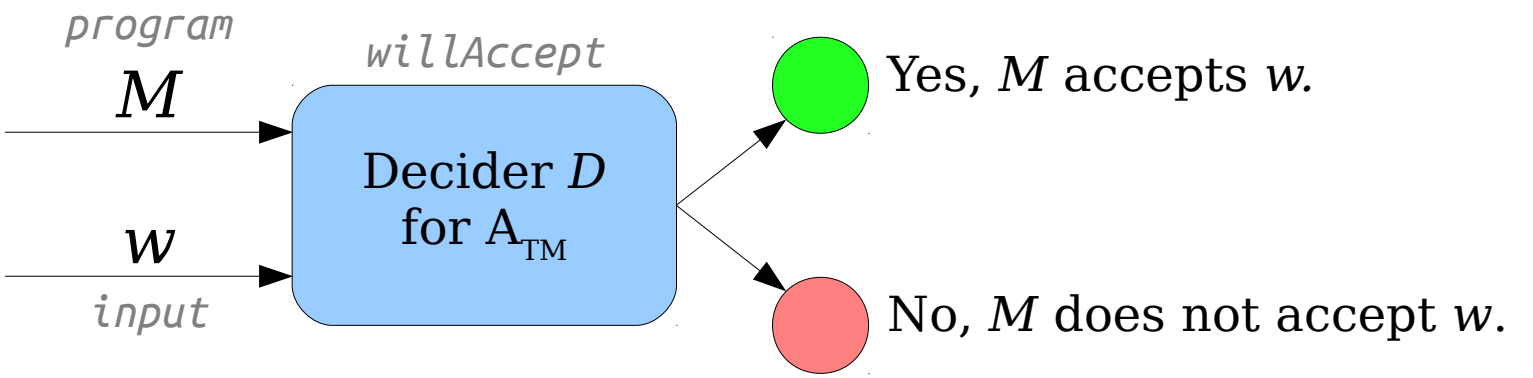


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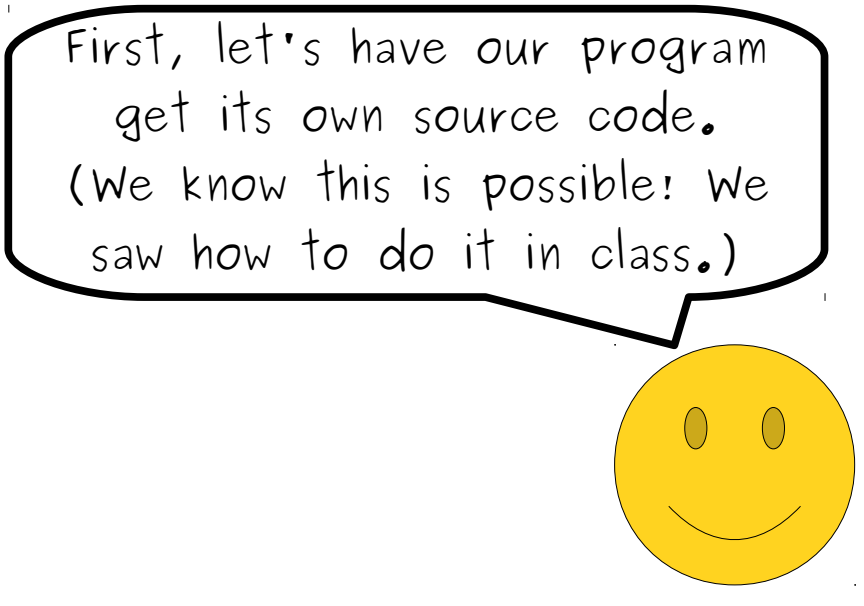

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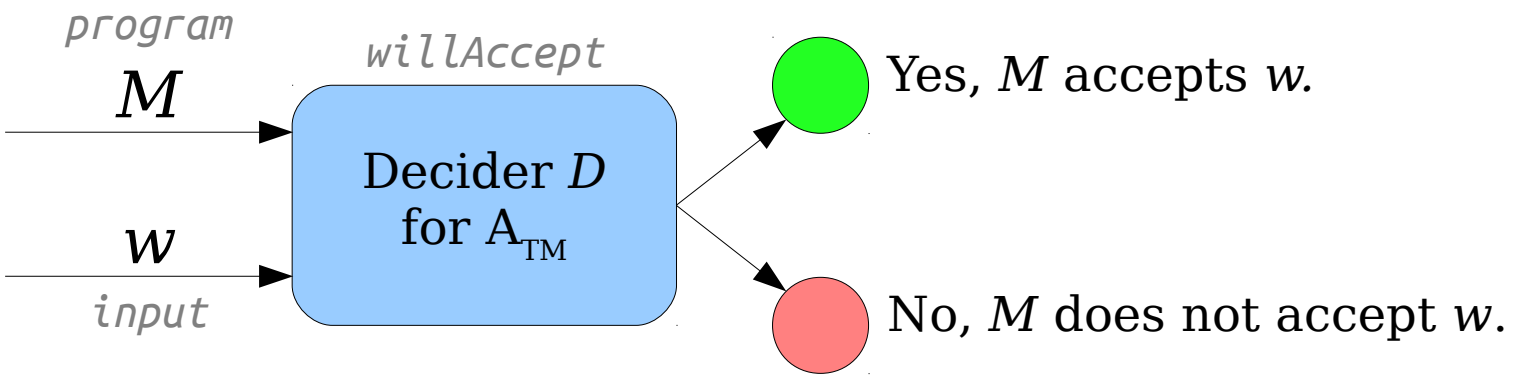


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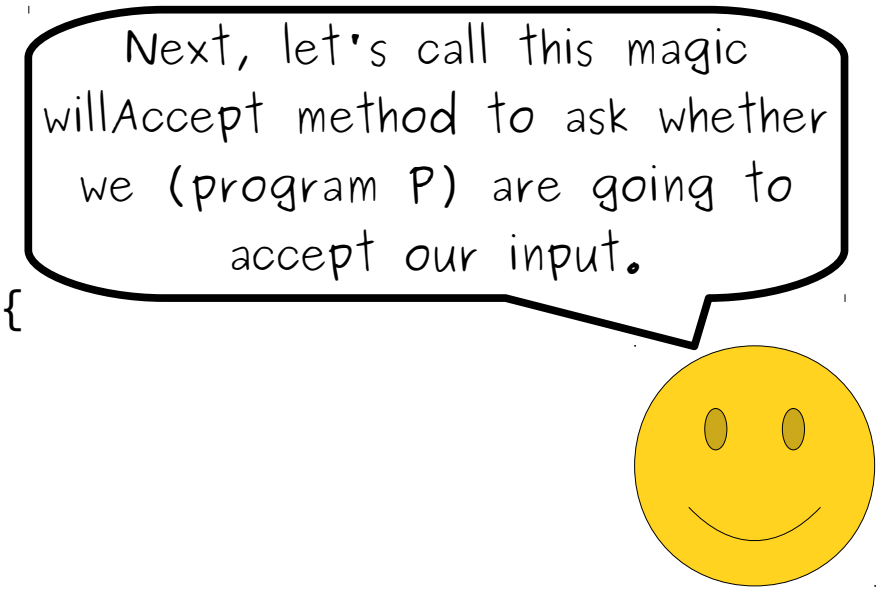

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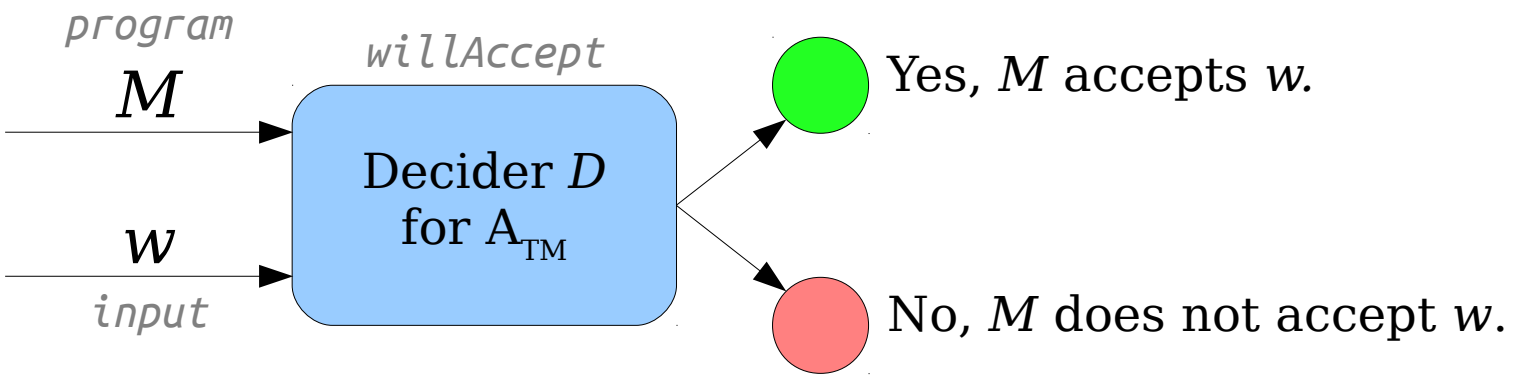


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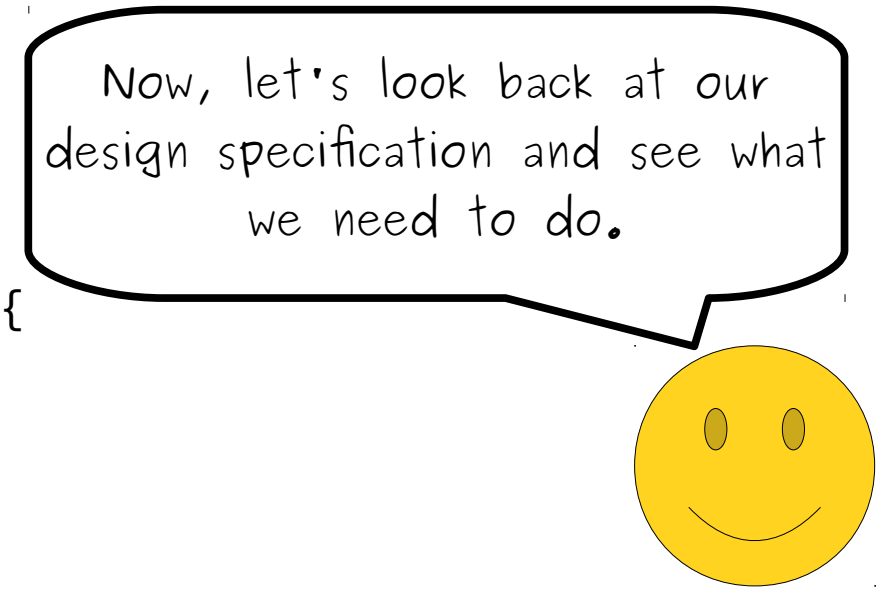

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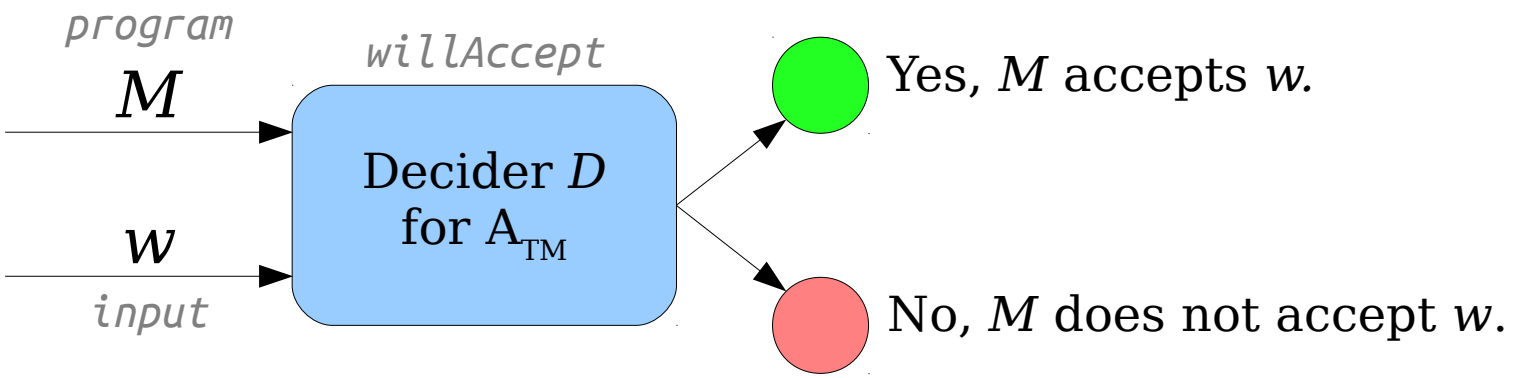


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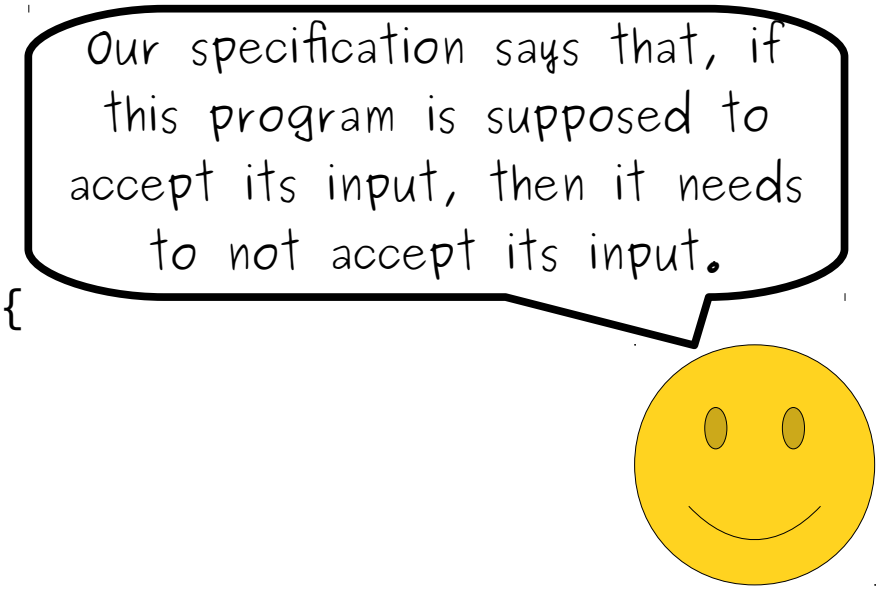

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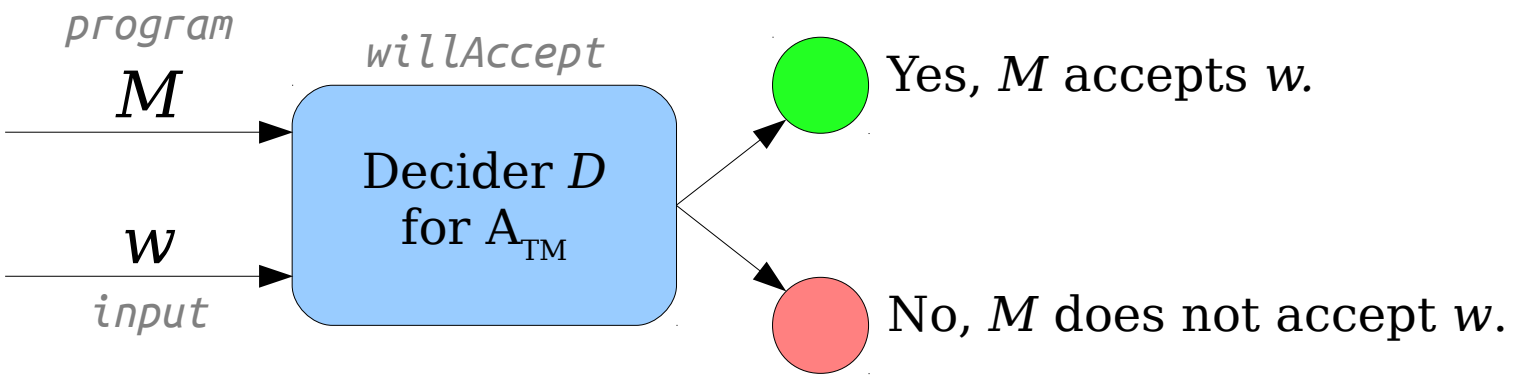


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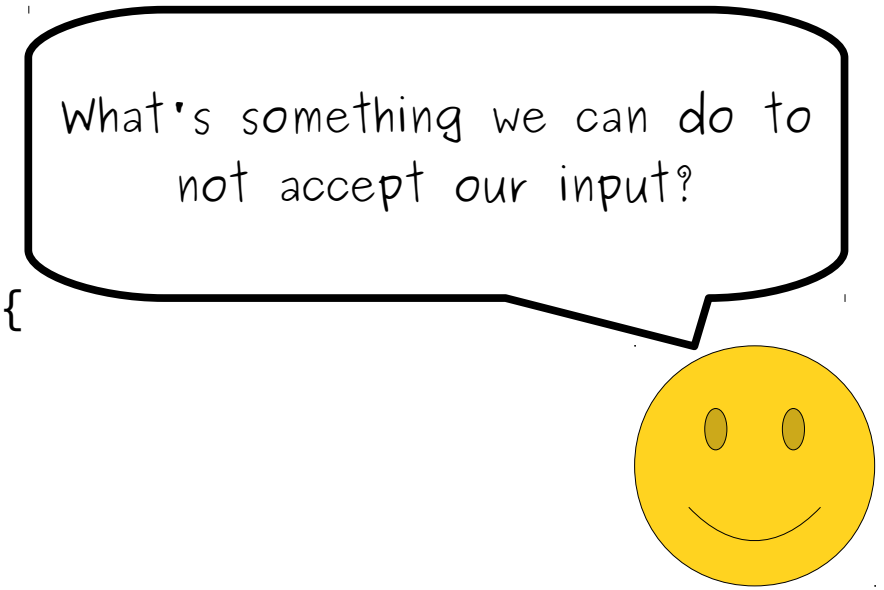

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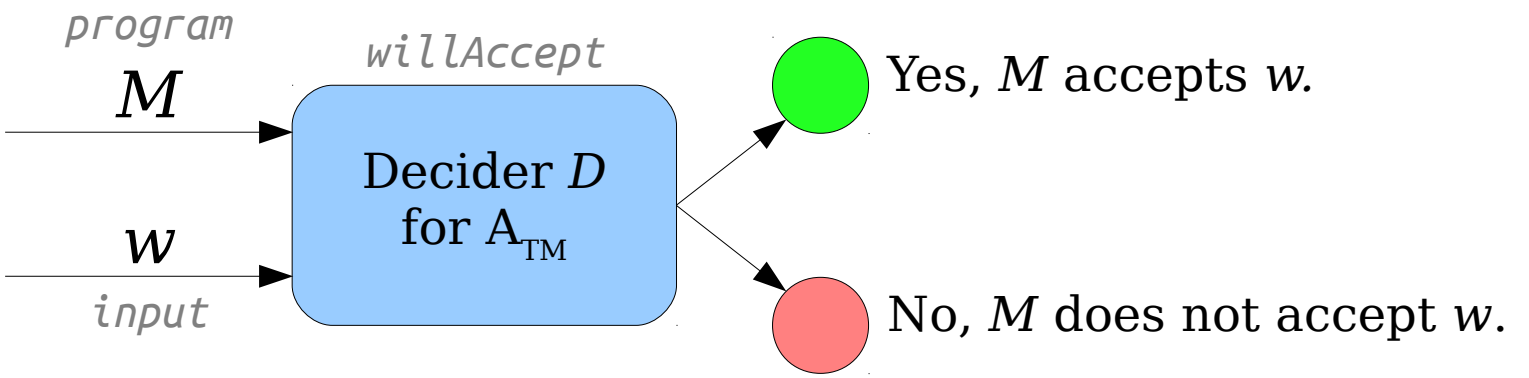


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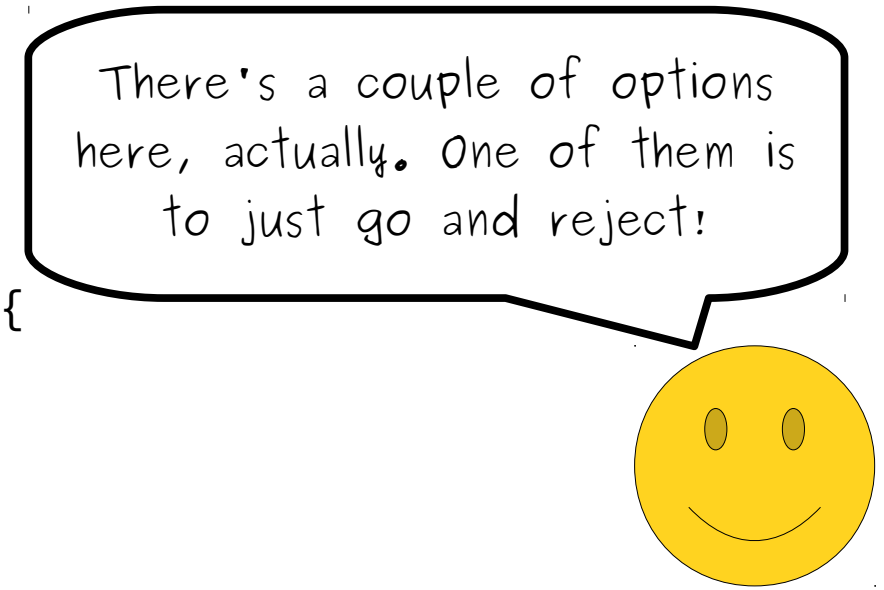

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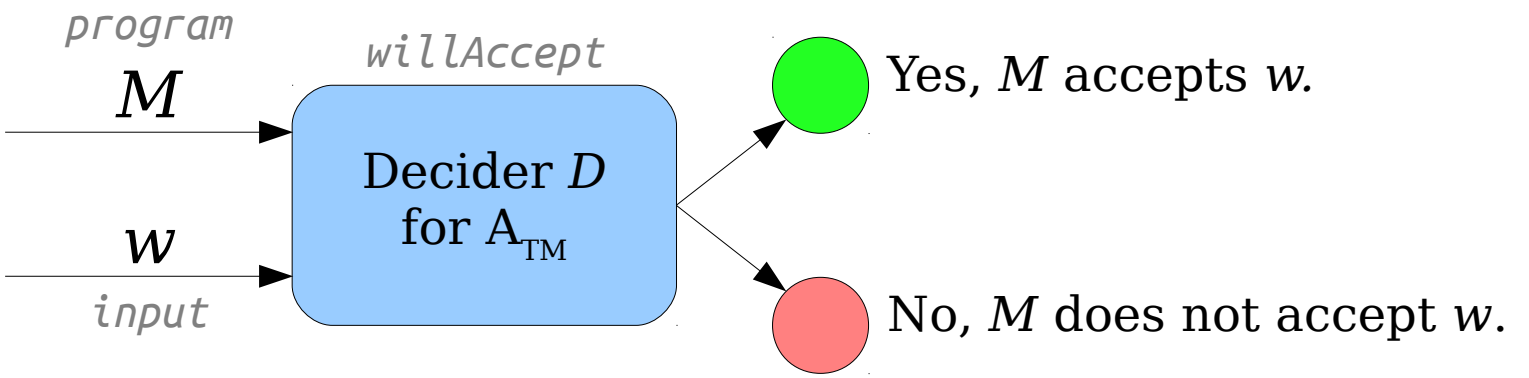


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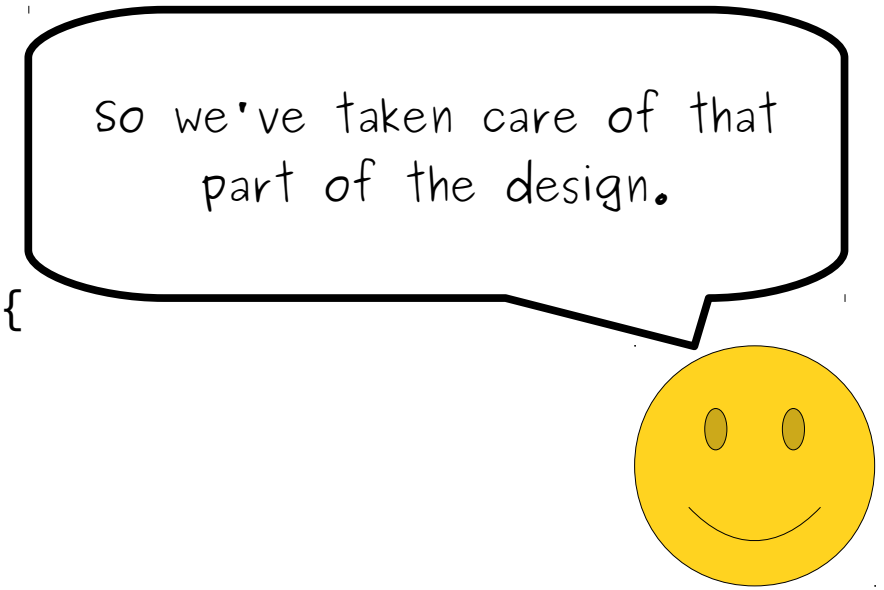

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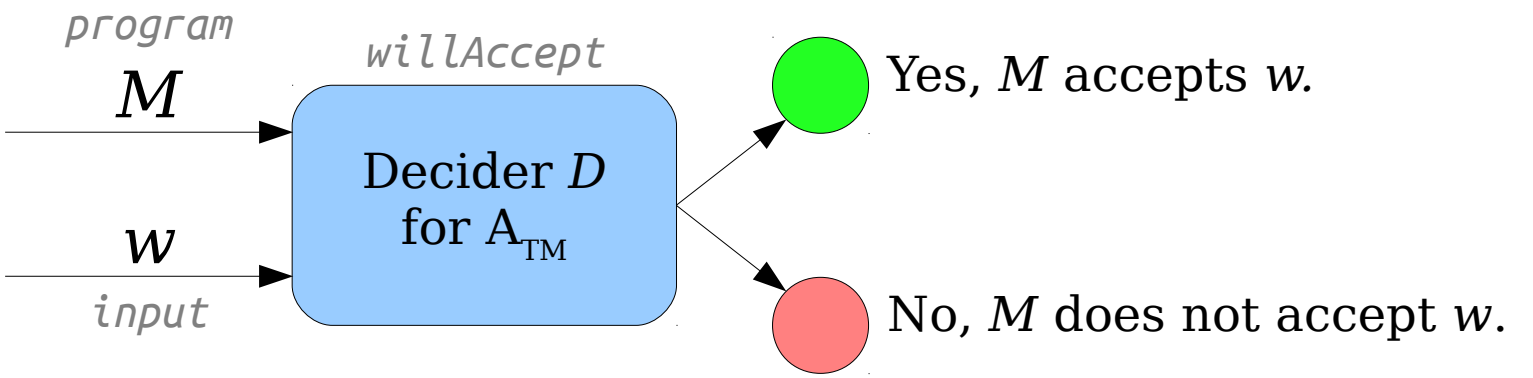


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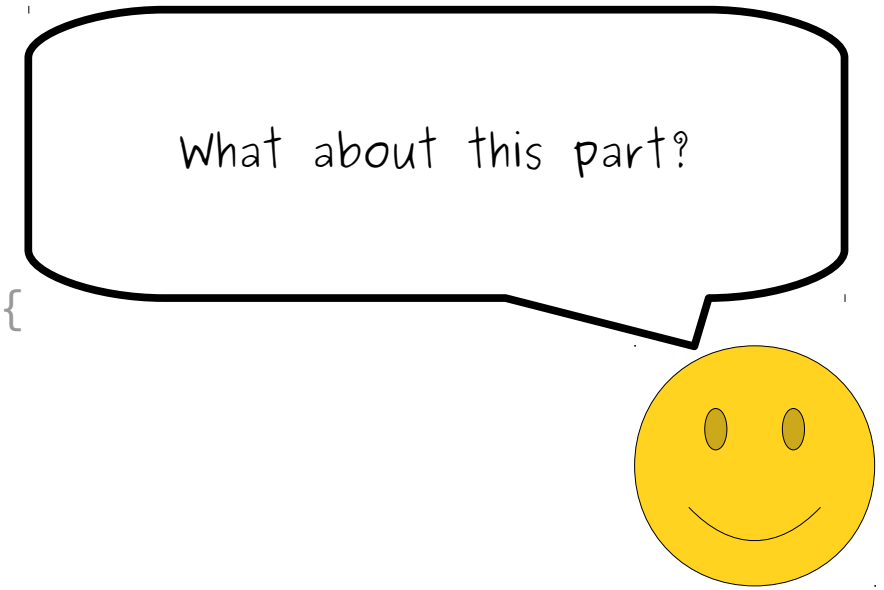

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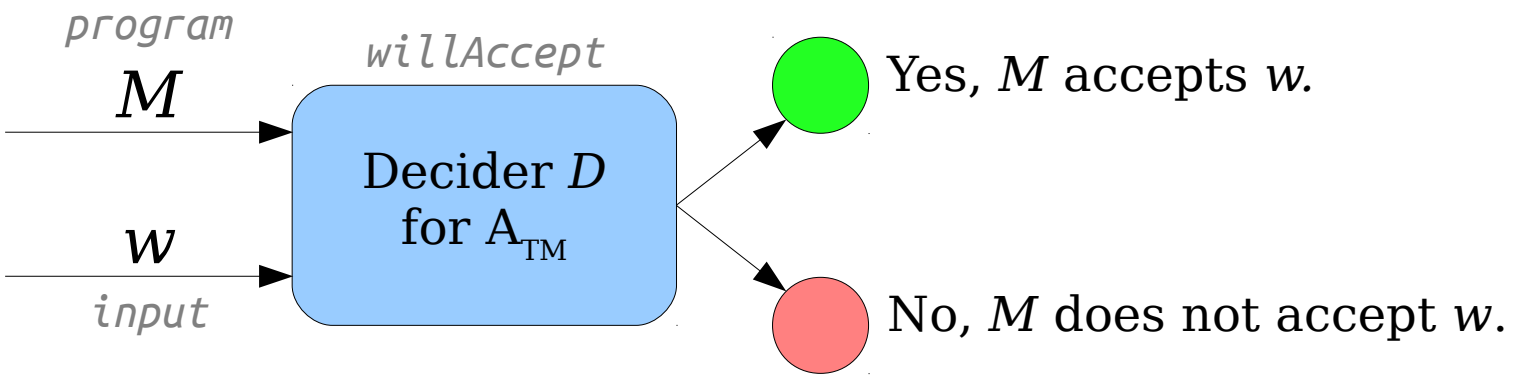


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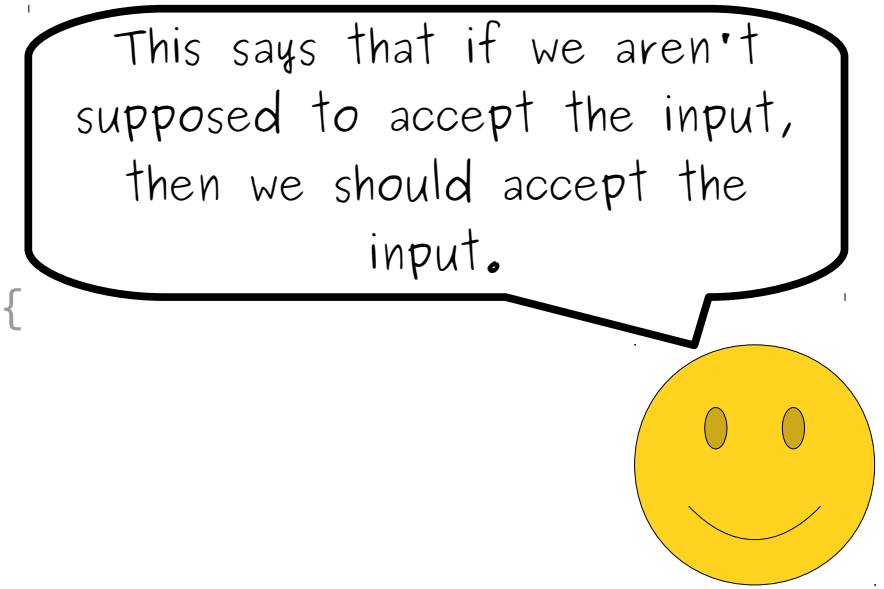

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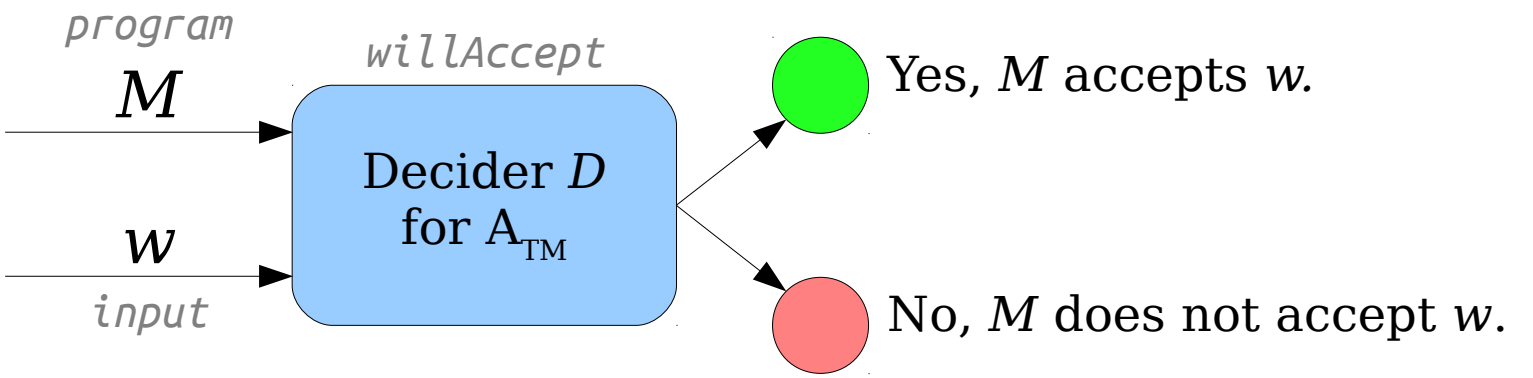


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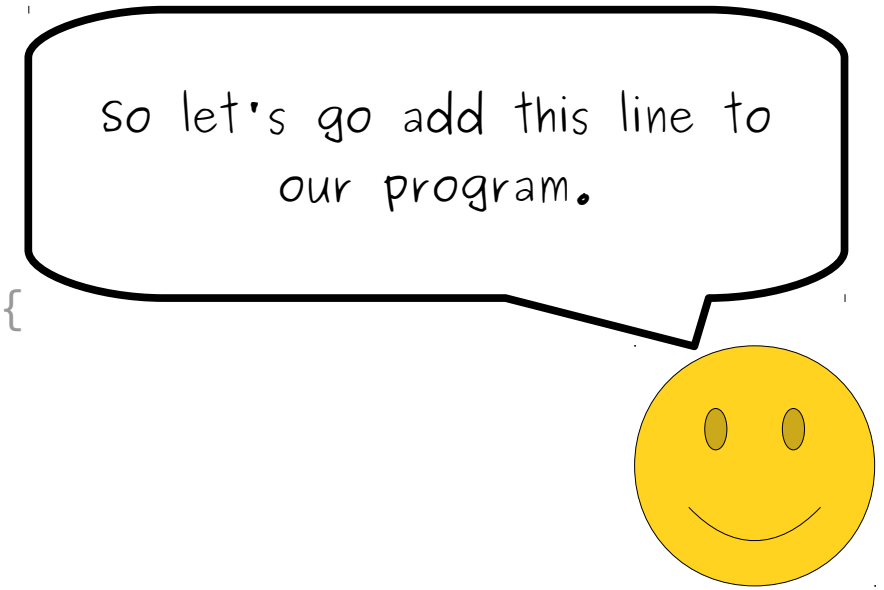

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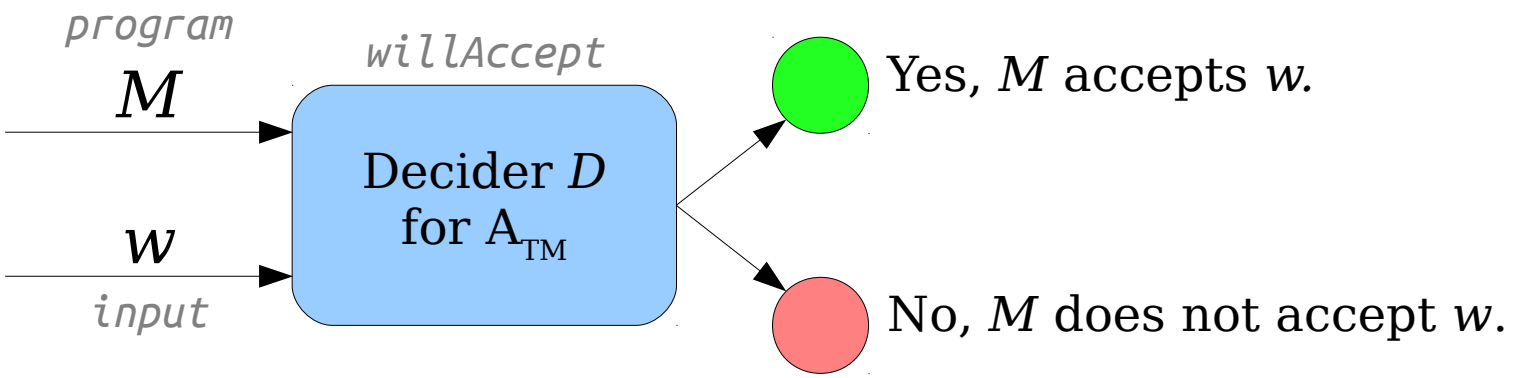


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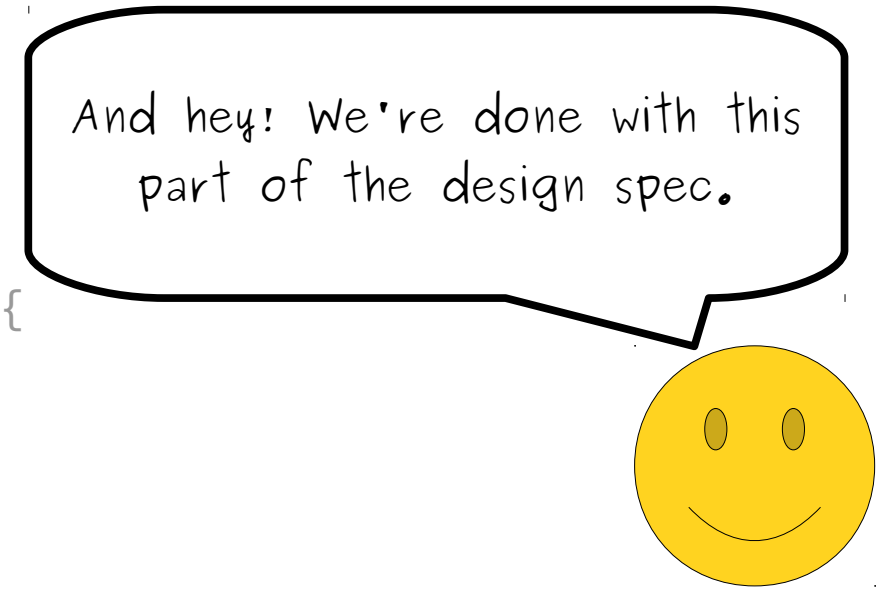

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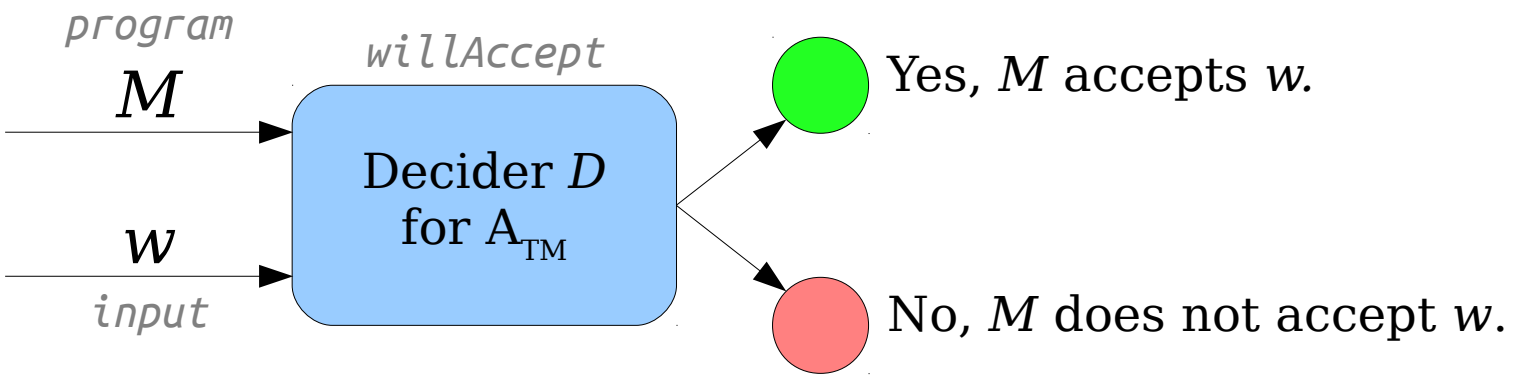


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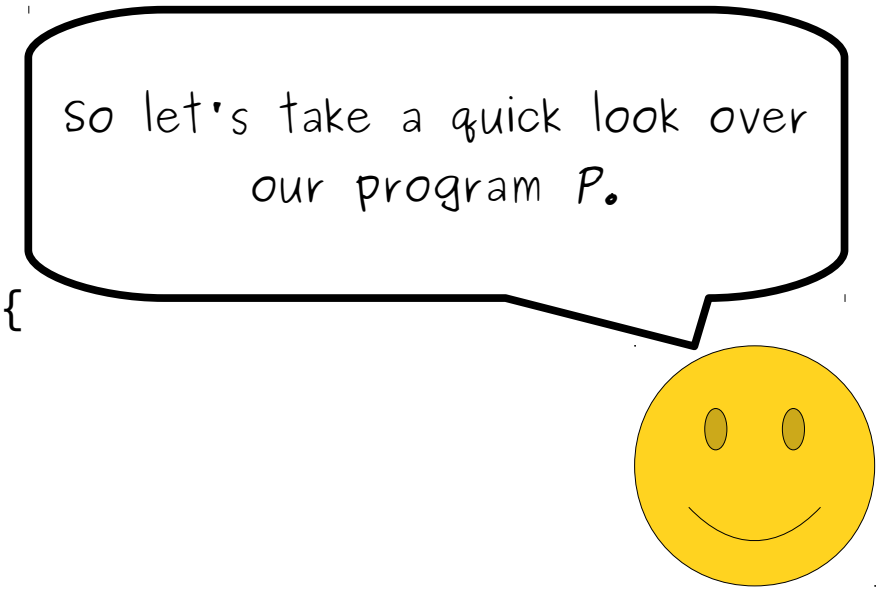

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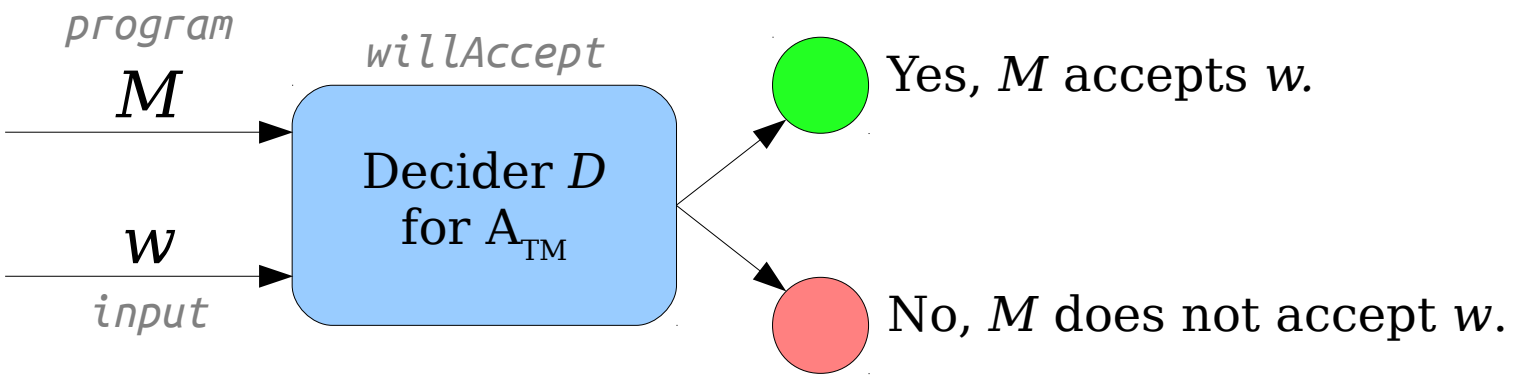


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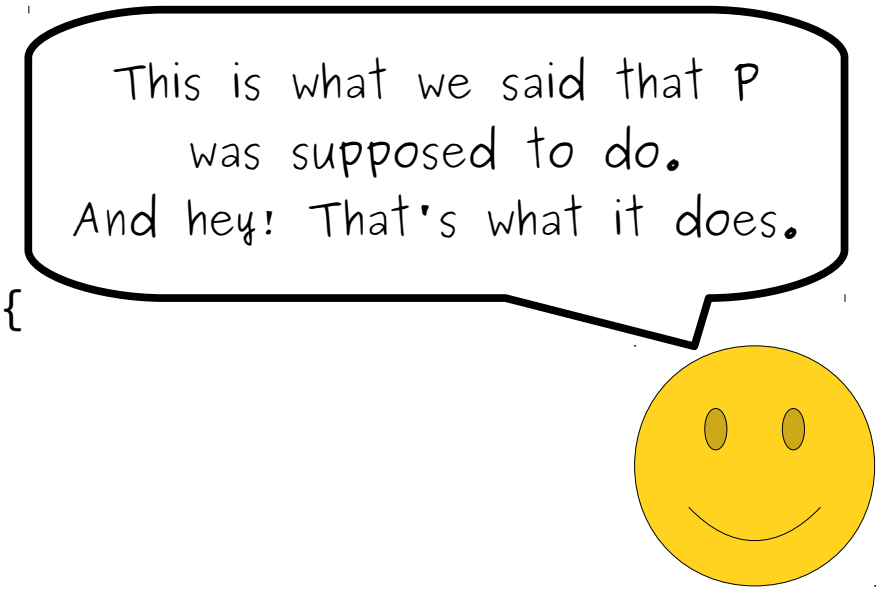

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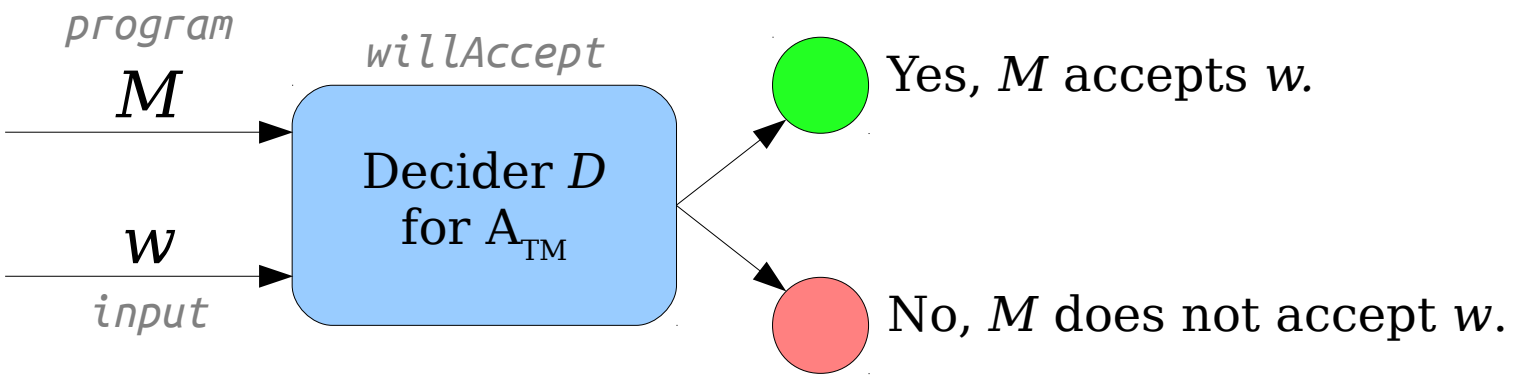


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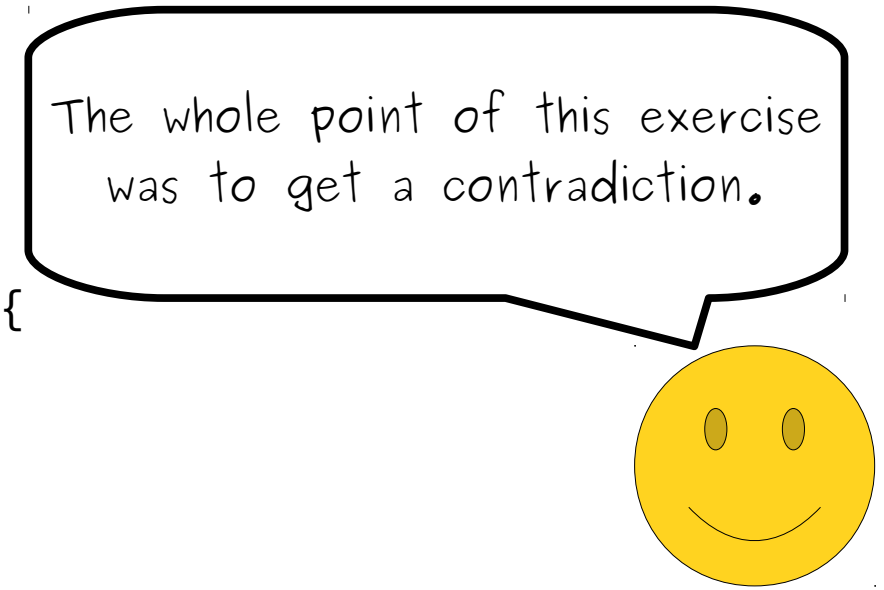

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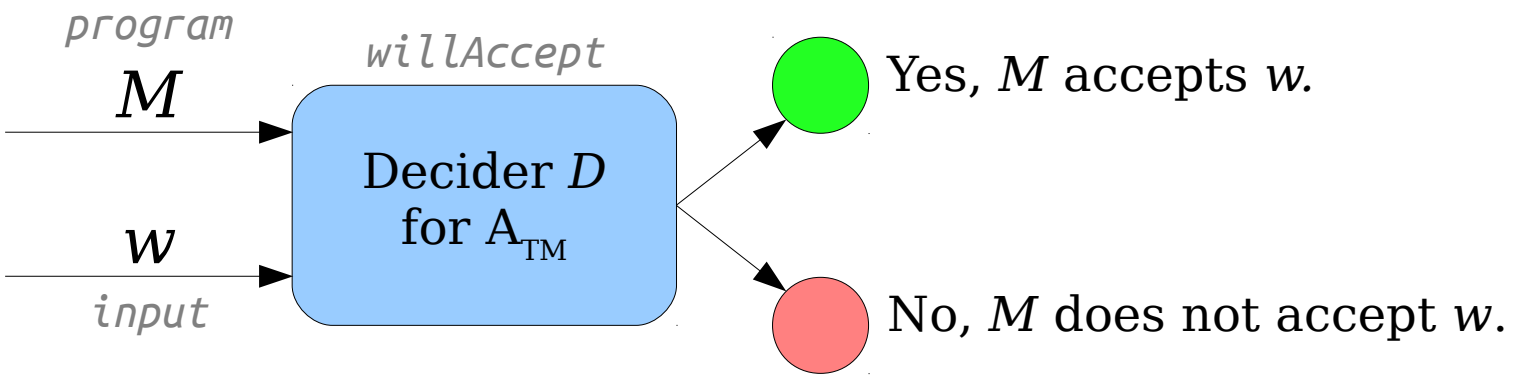


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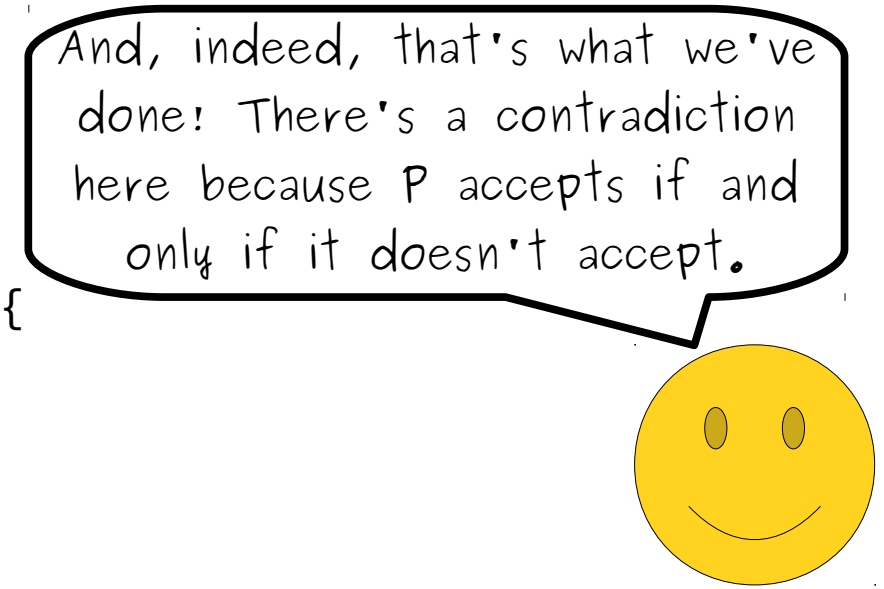

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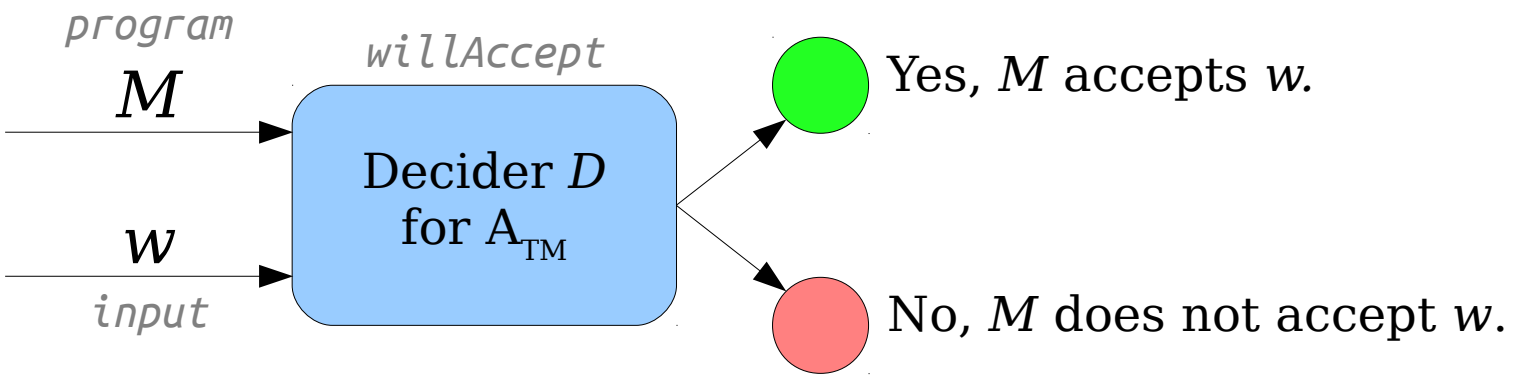


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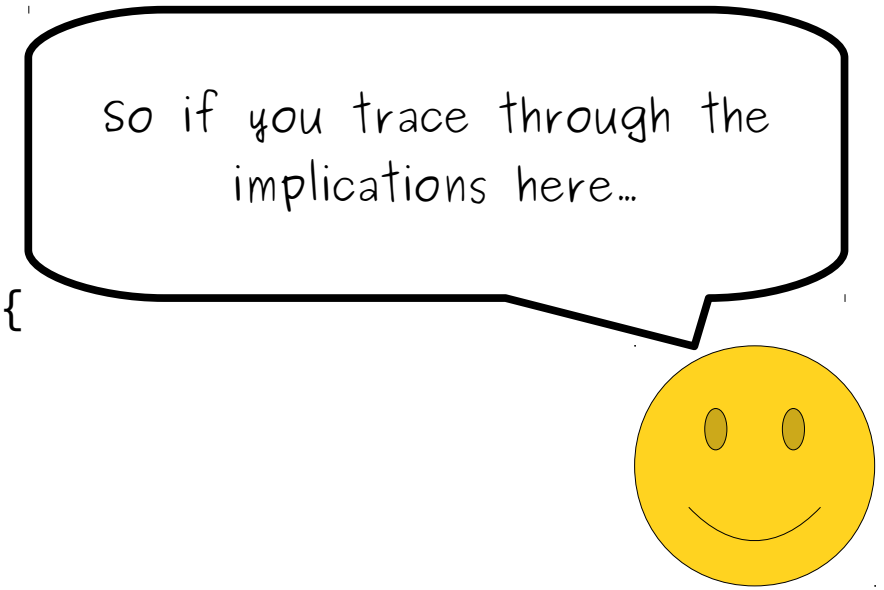

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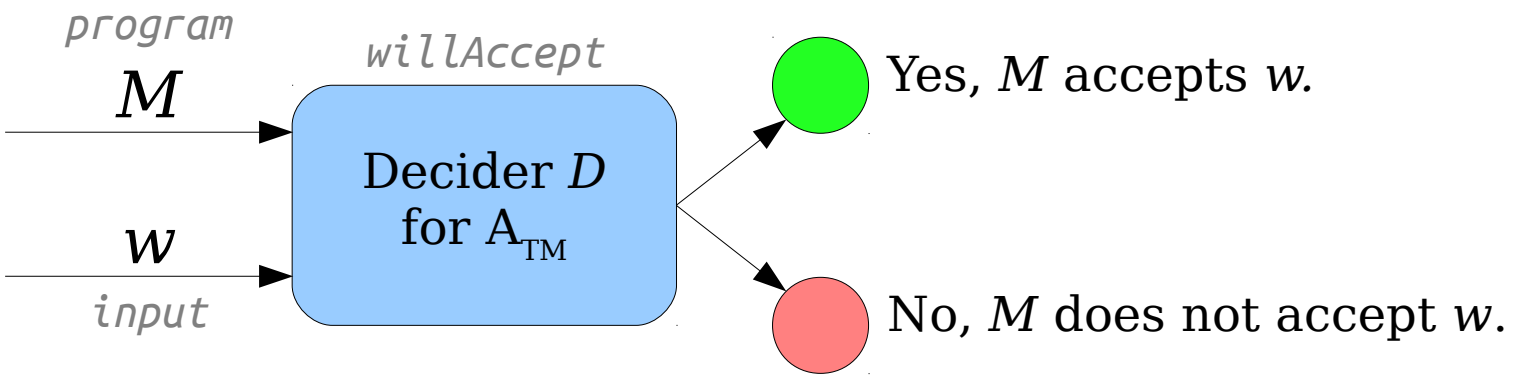


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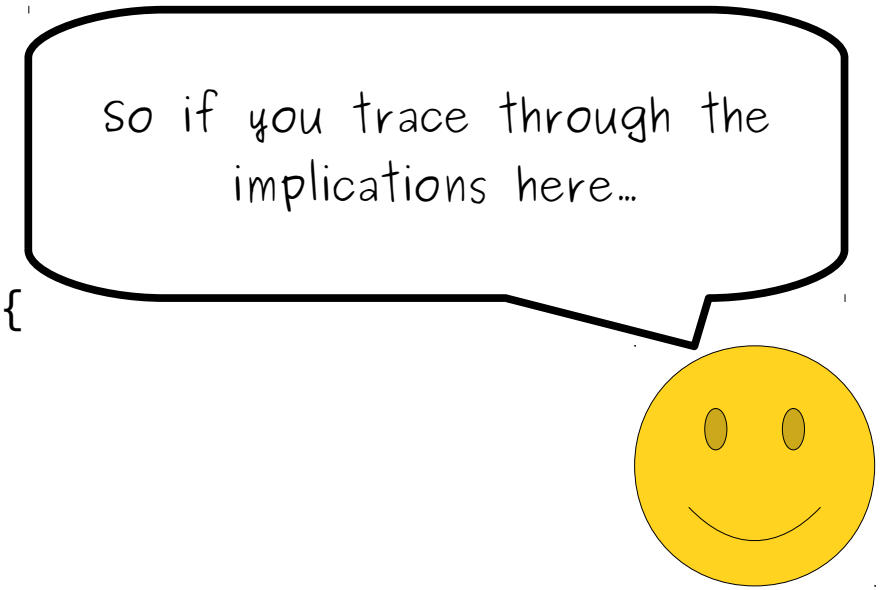

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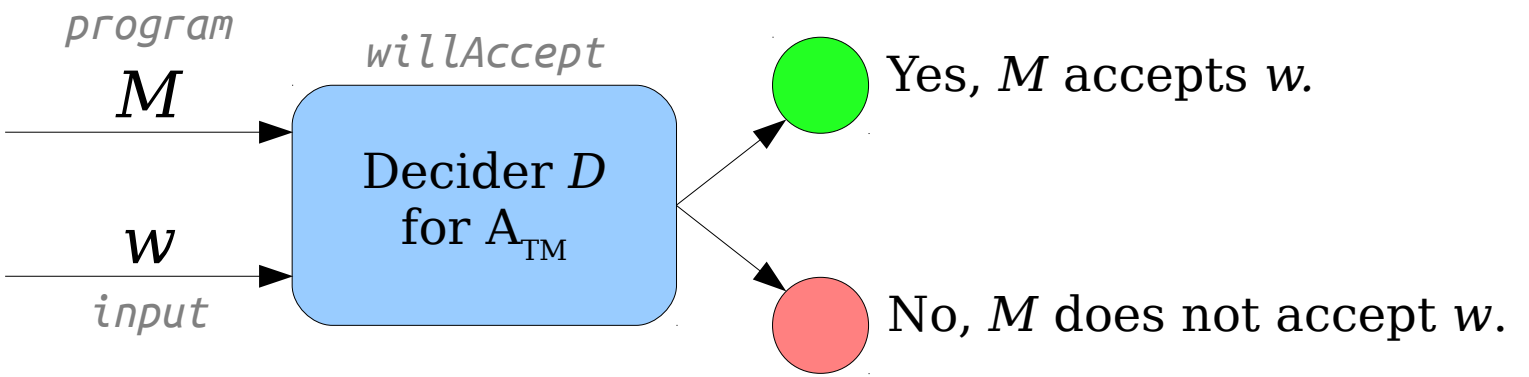


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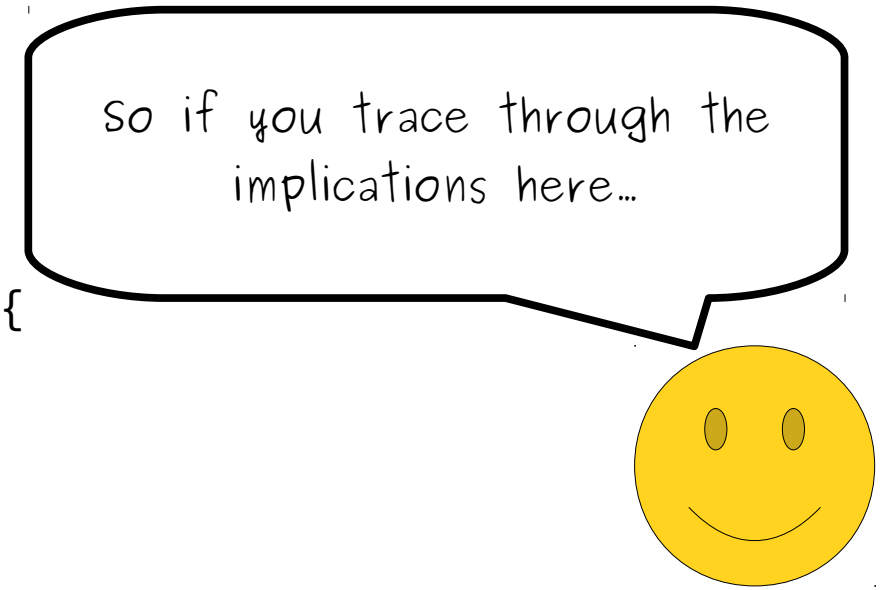

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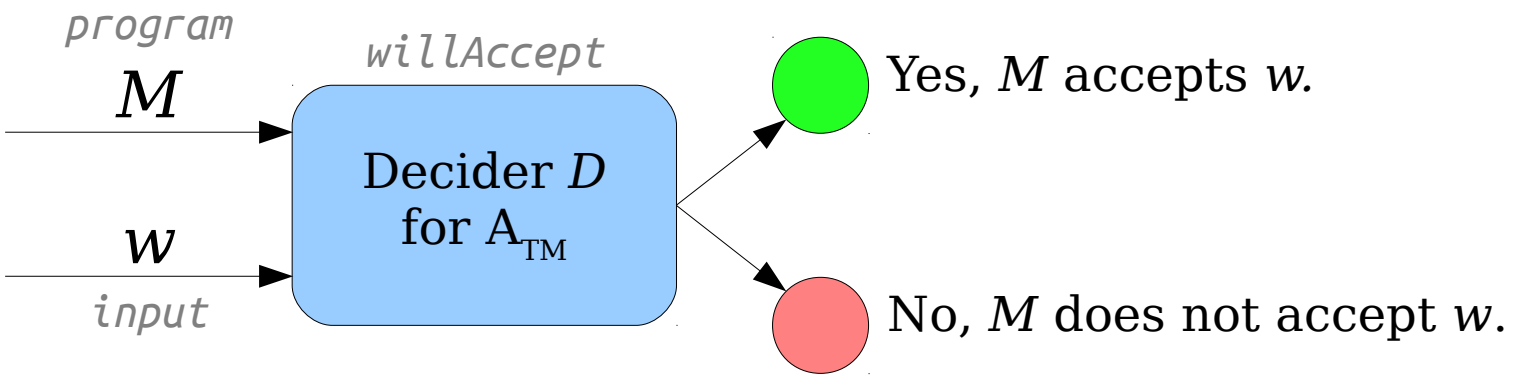


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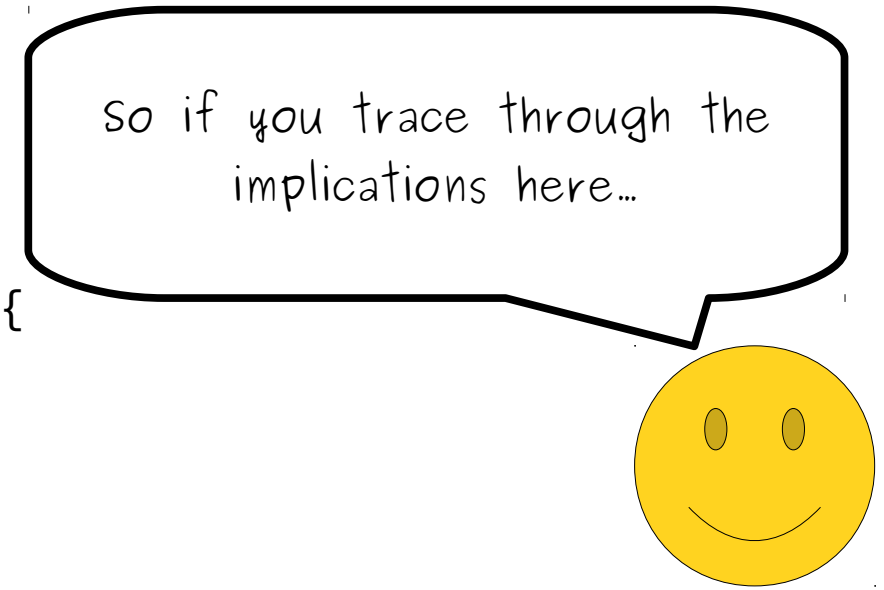

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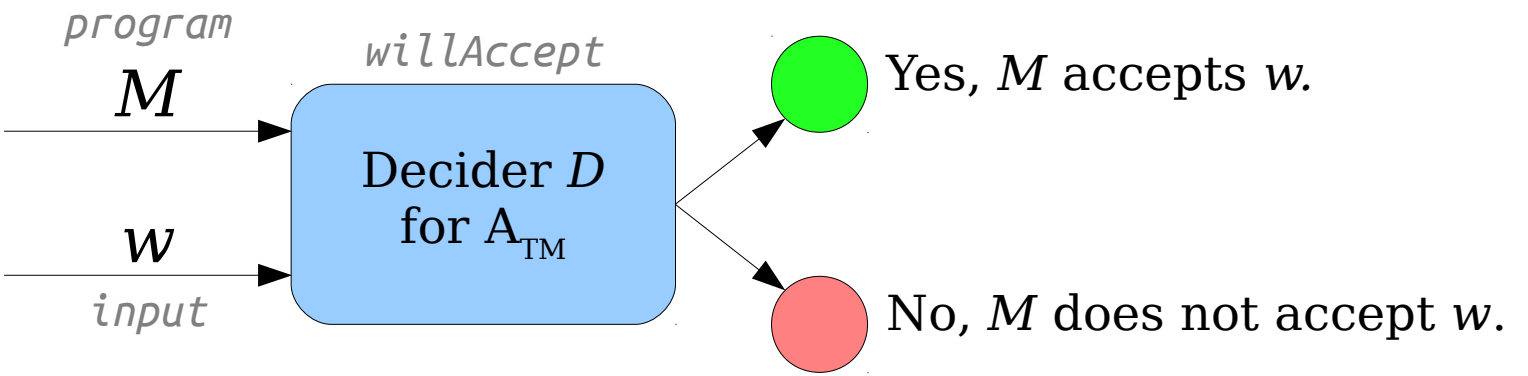


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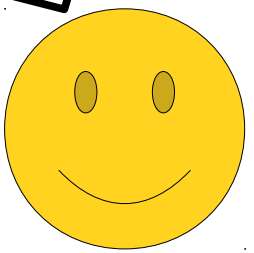
You can see that the starting assumption that A_{TM} is decidable leads to a contradiction – we're done!



What does this program do?

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   /* … some implementation … */
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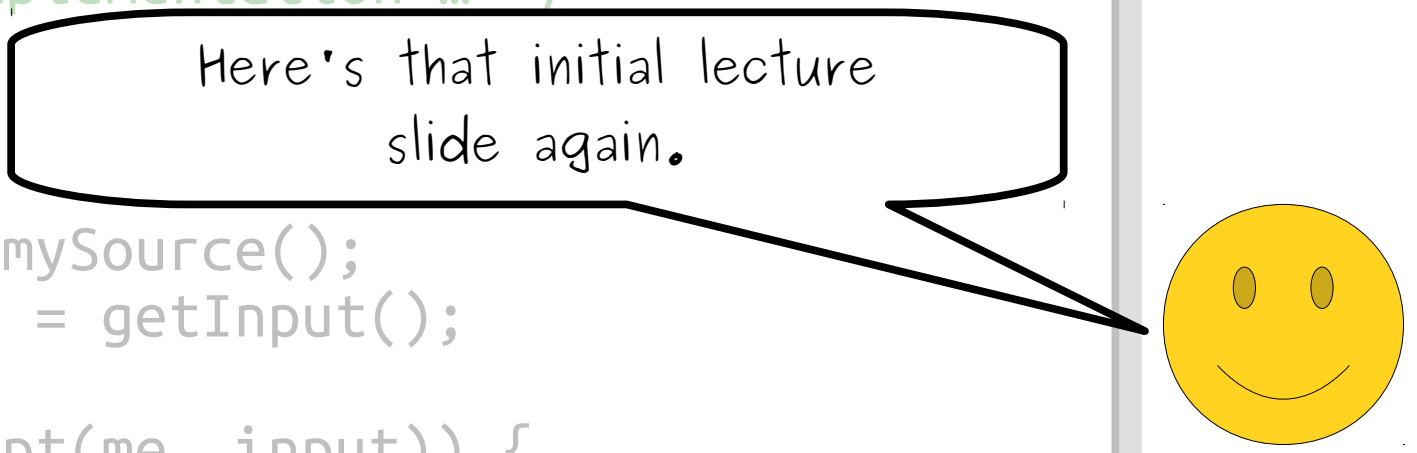

Try running this program on any input. What happens if

… this program accepts its input?
It rejects the input!

… this program doesn't accept its input?
It accepts the input!

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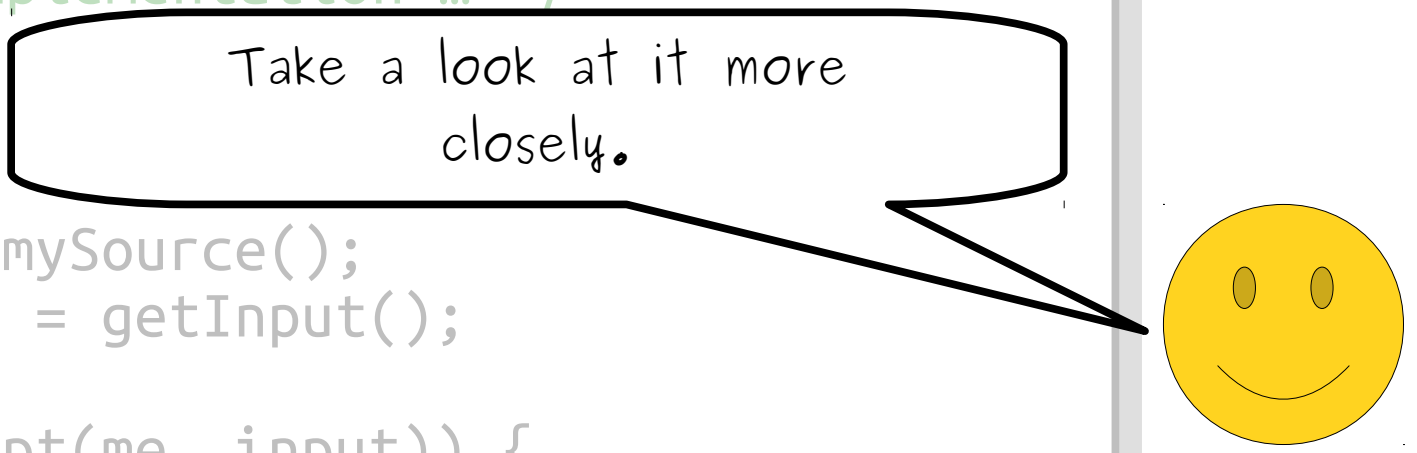

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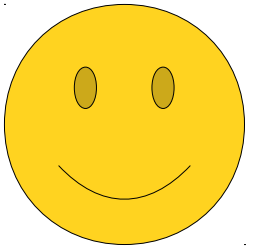
Recognize this code? Now you know where it comes from!



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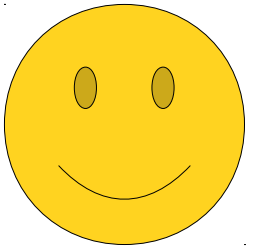
We created it to get these contradictions.



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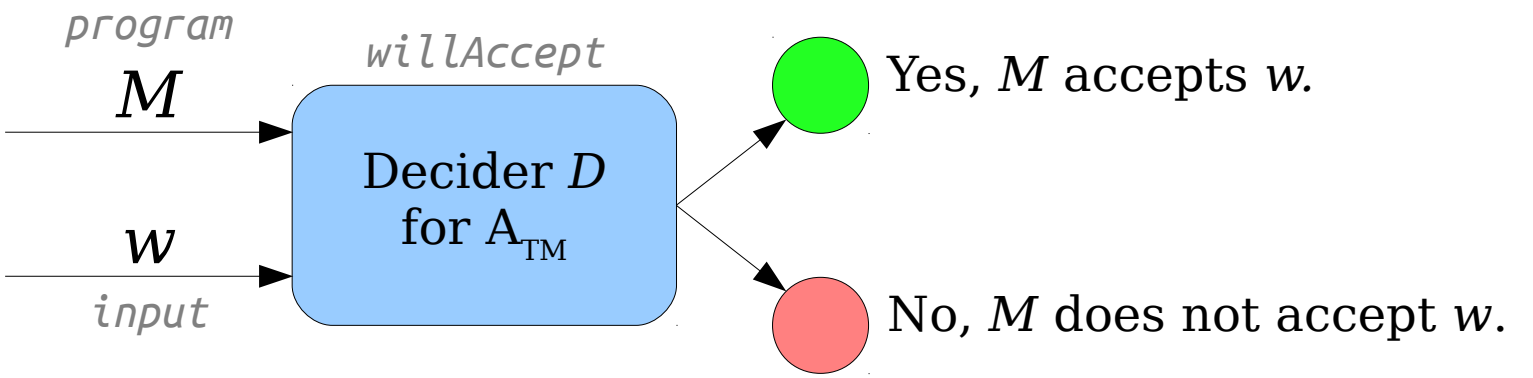


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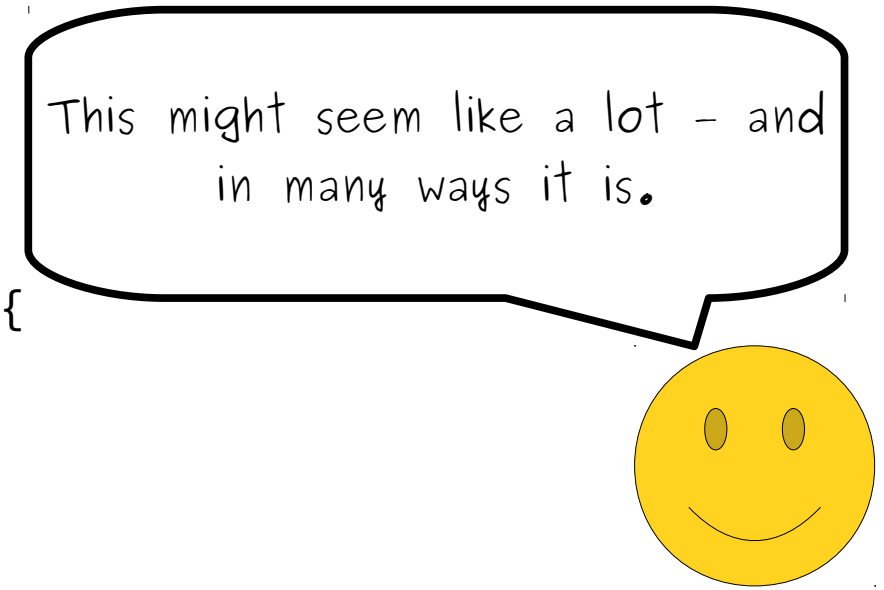

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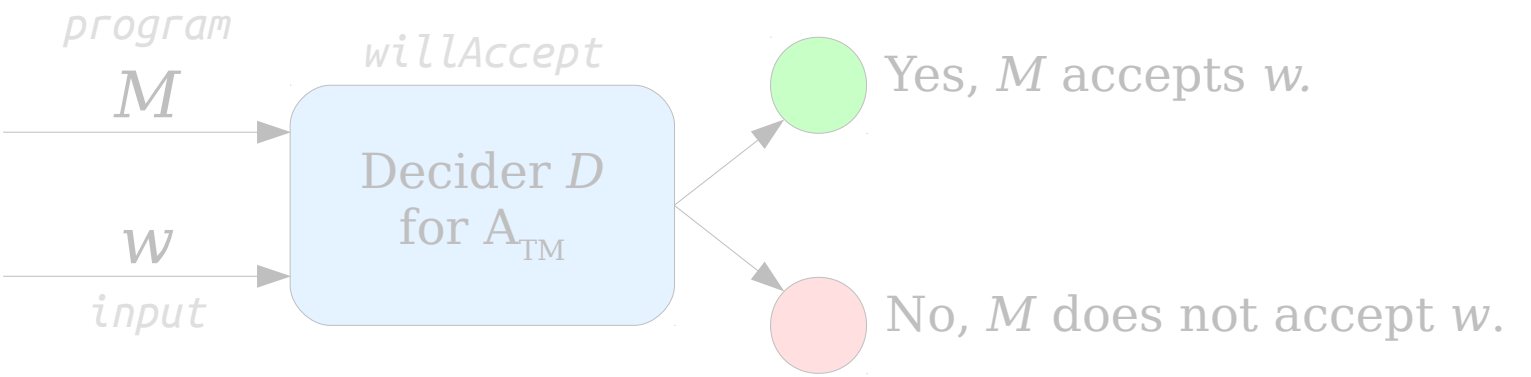


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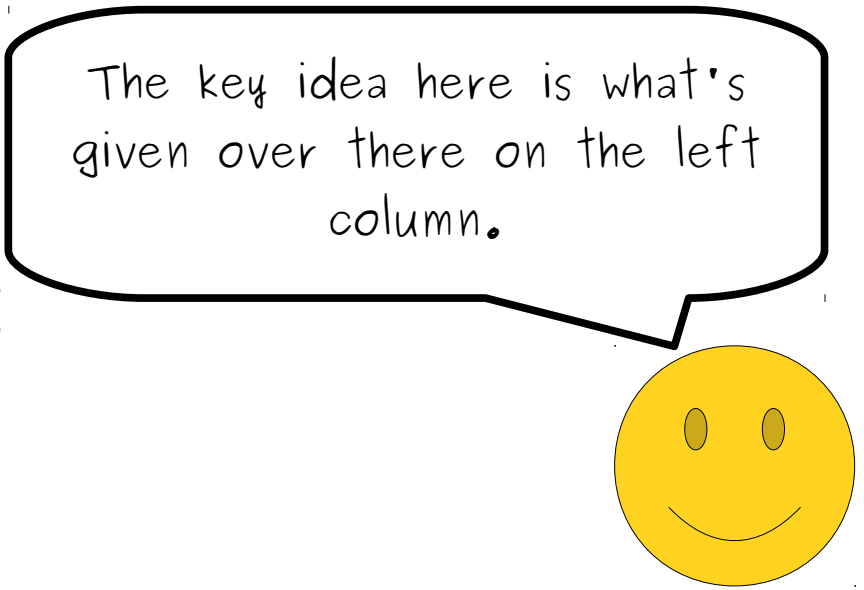

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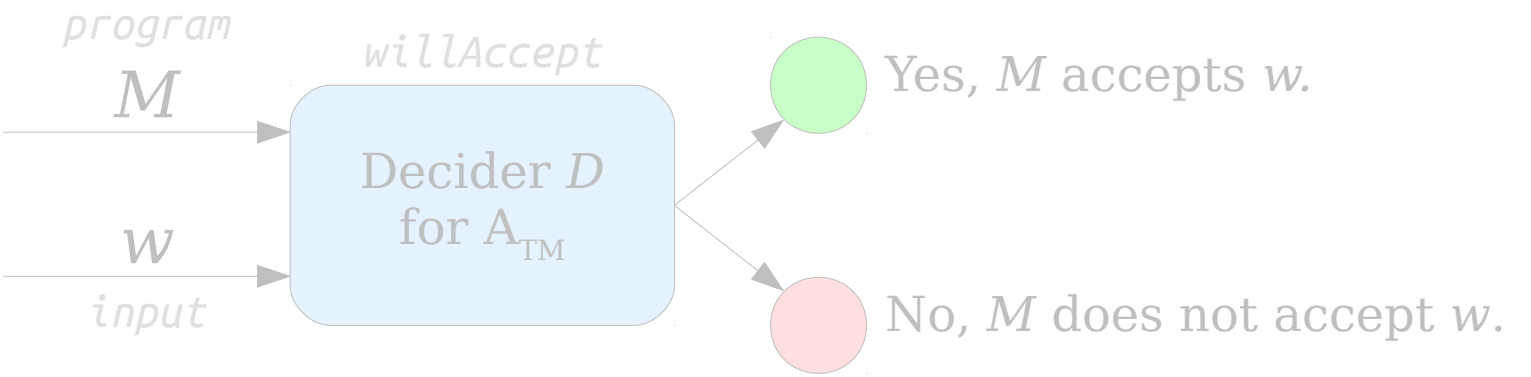


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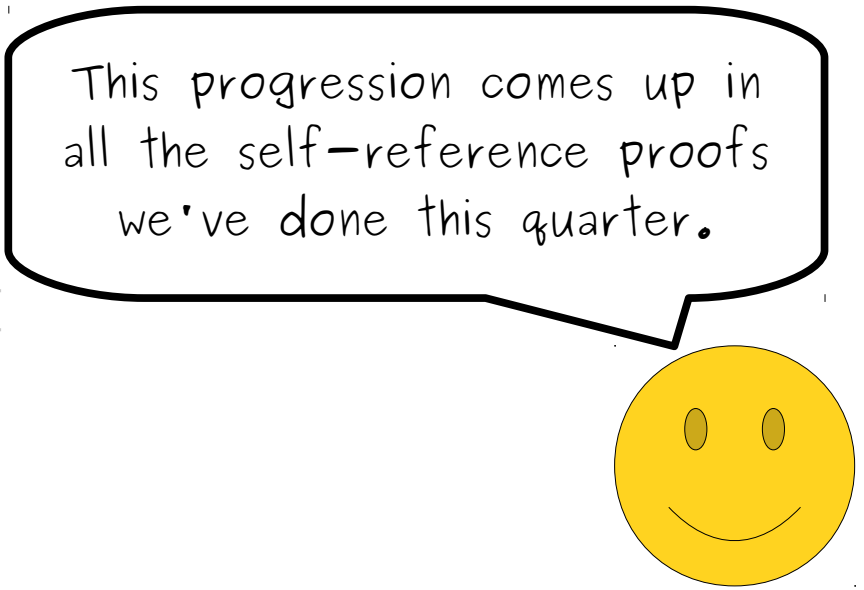

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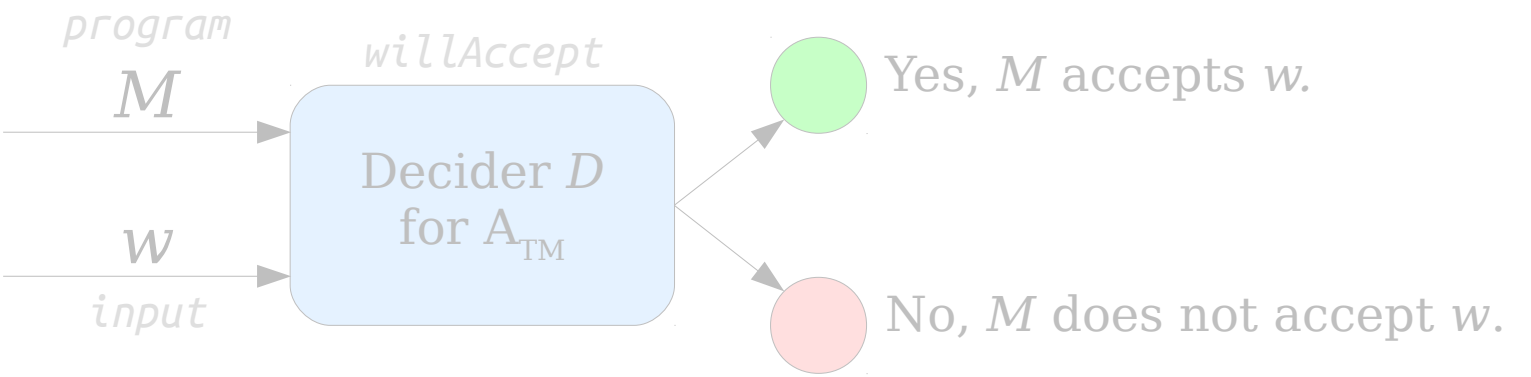


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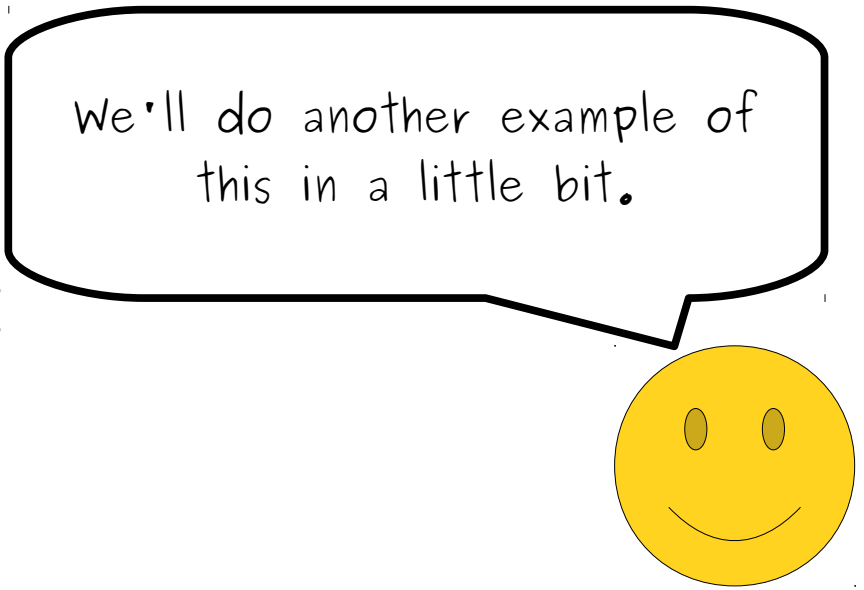

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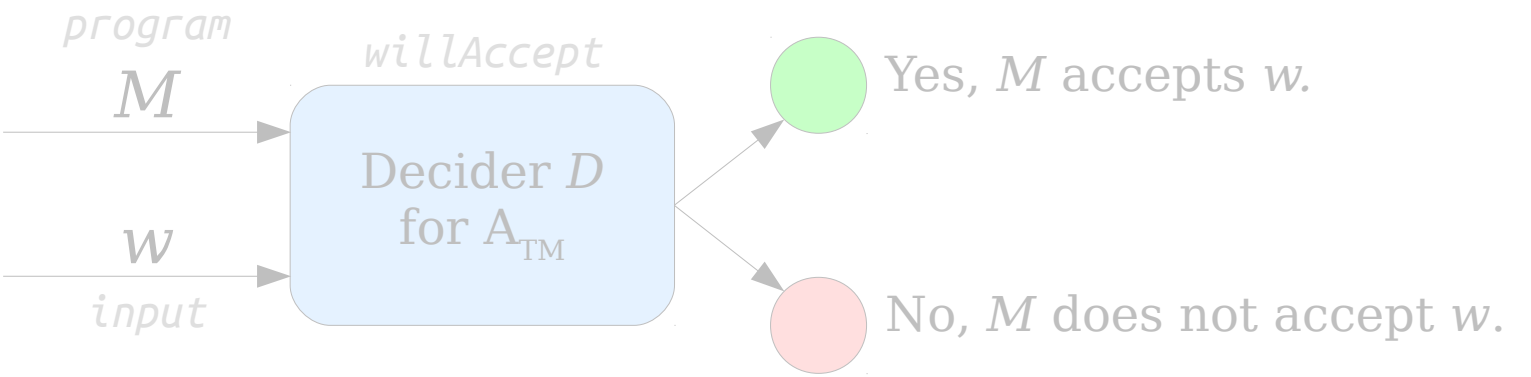


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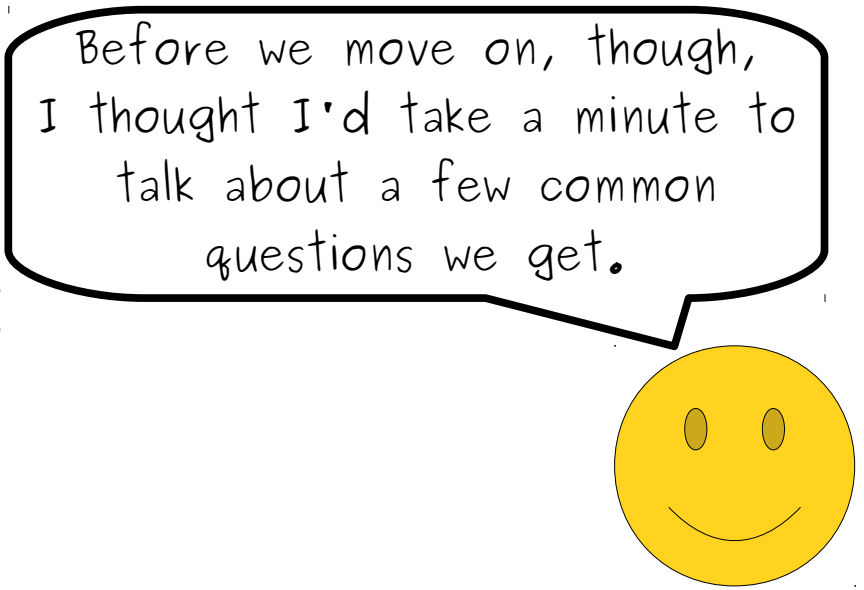

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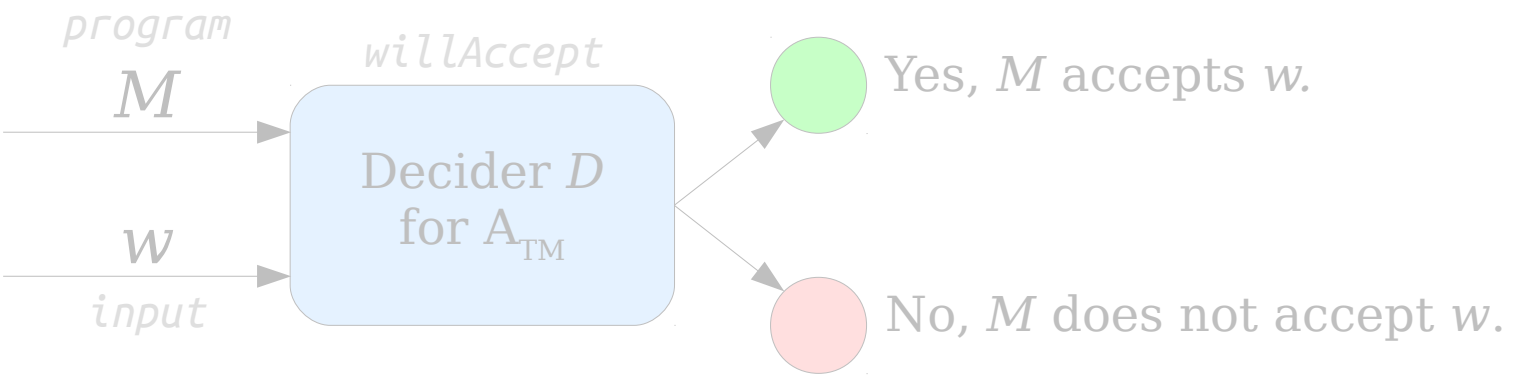


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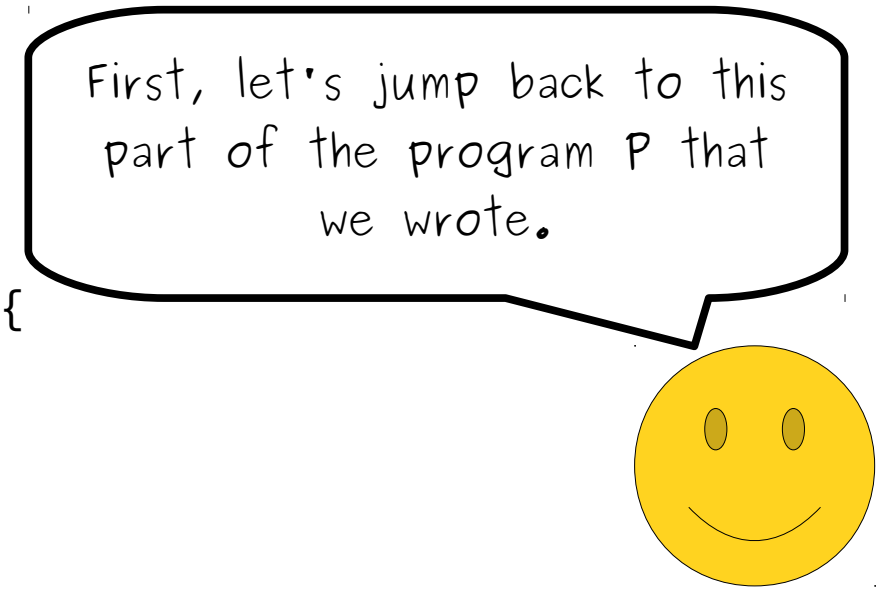

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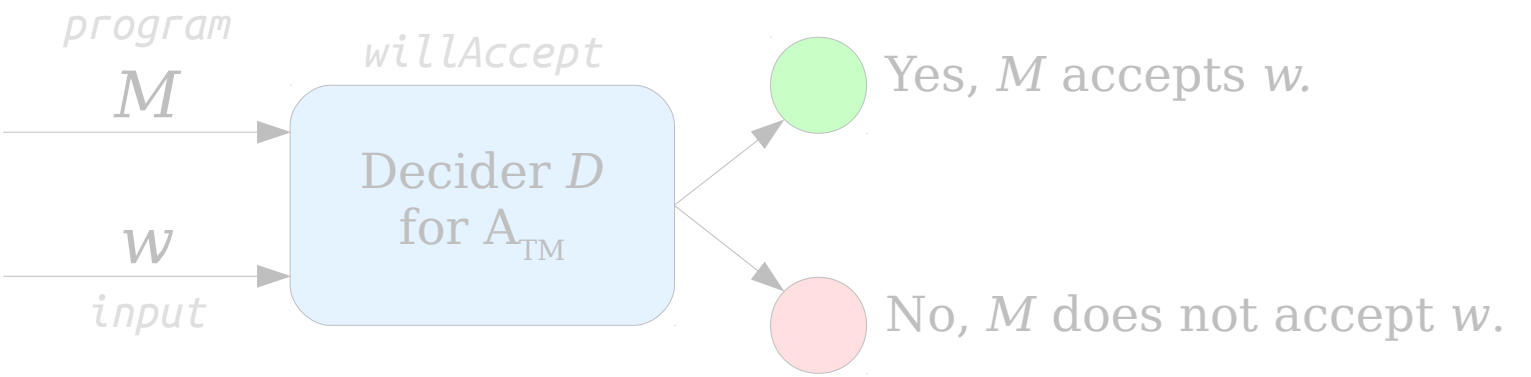


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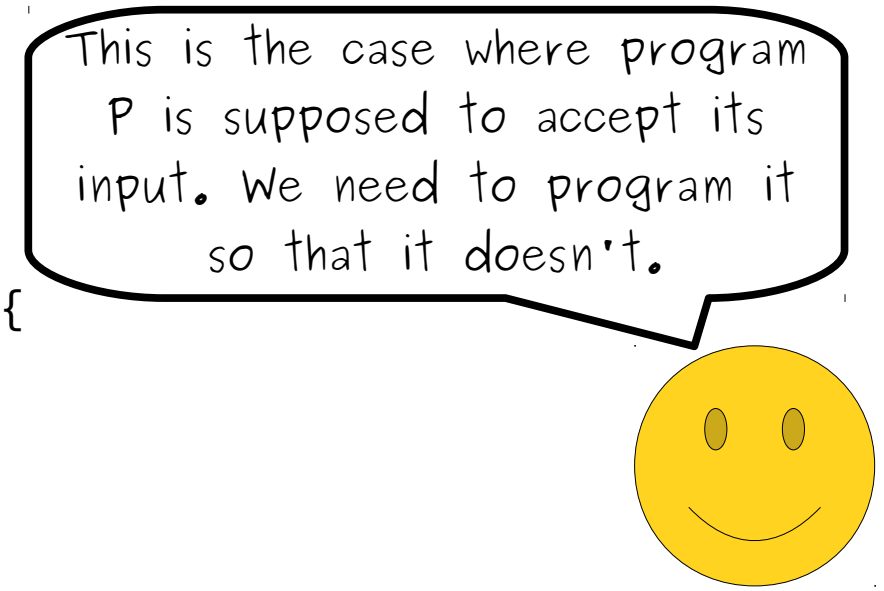

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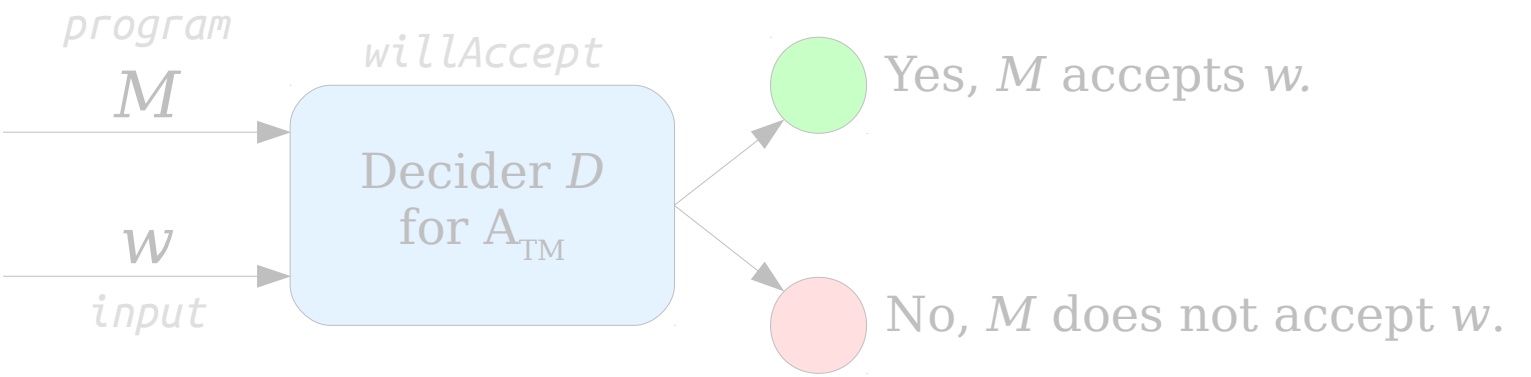


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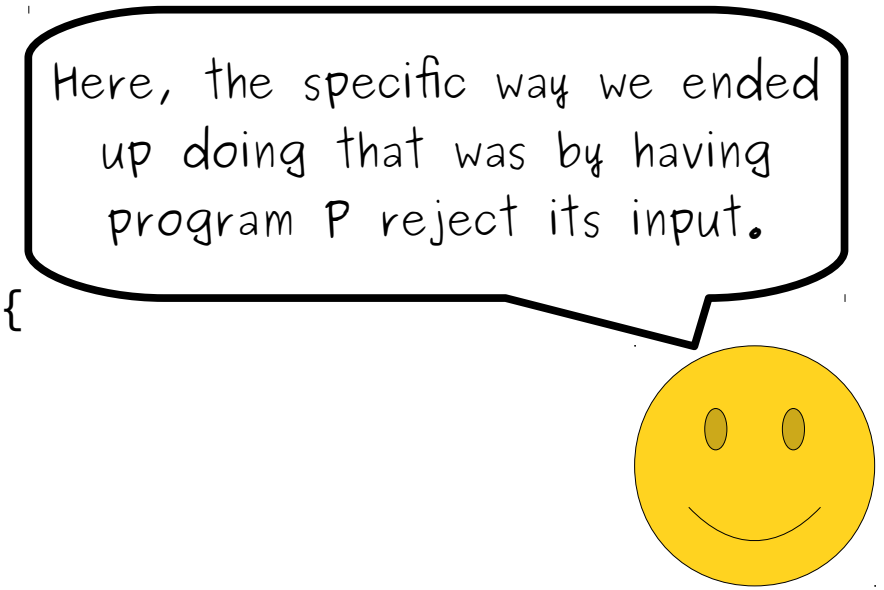

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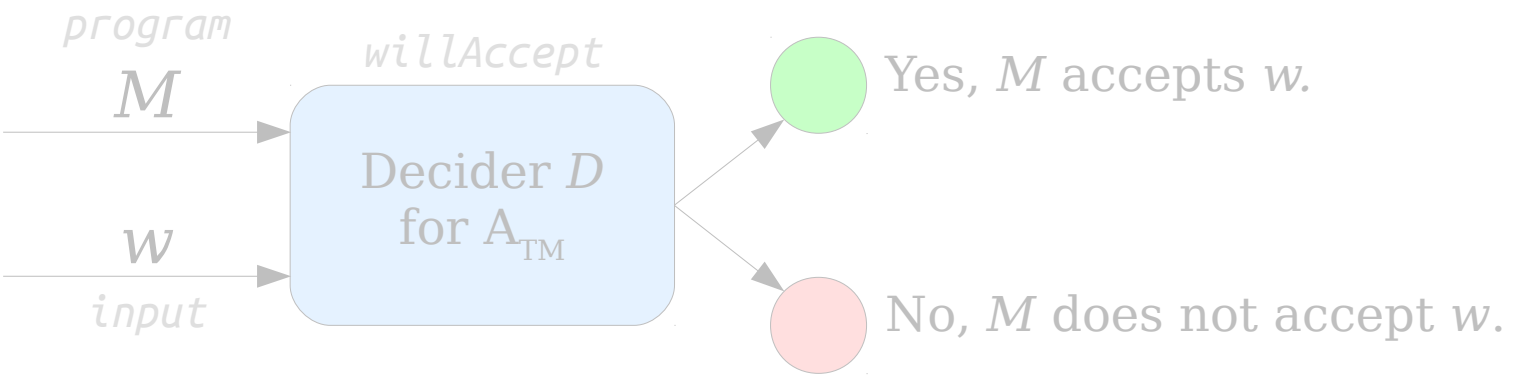


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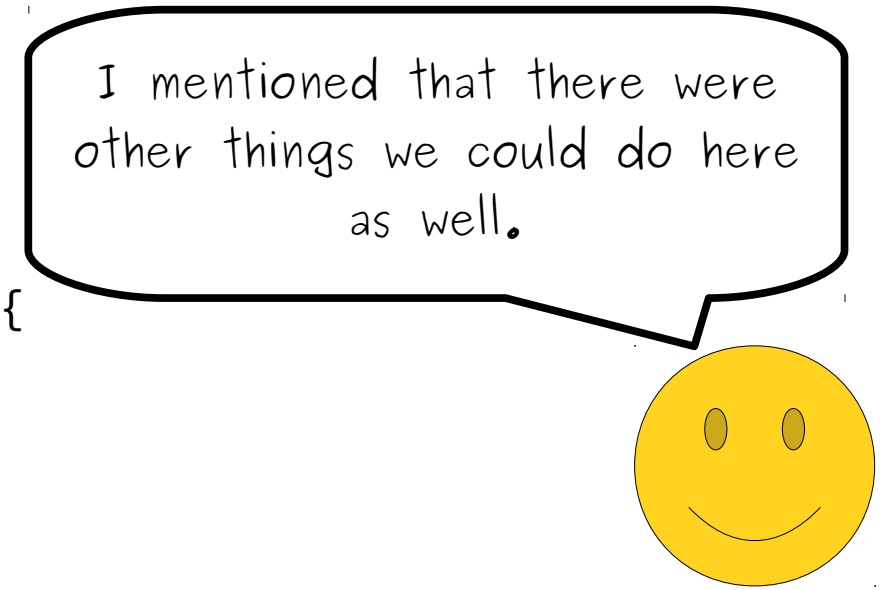

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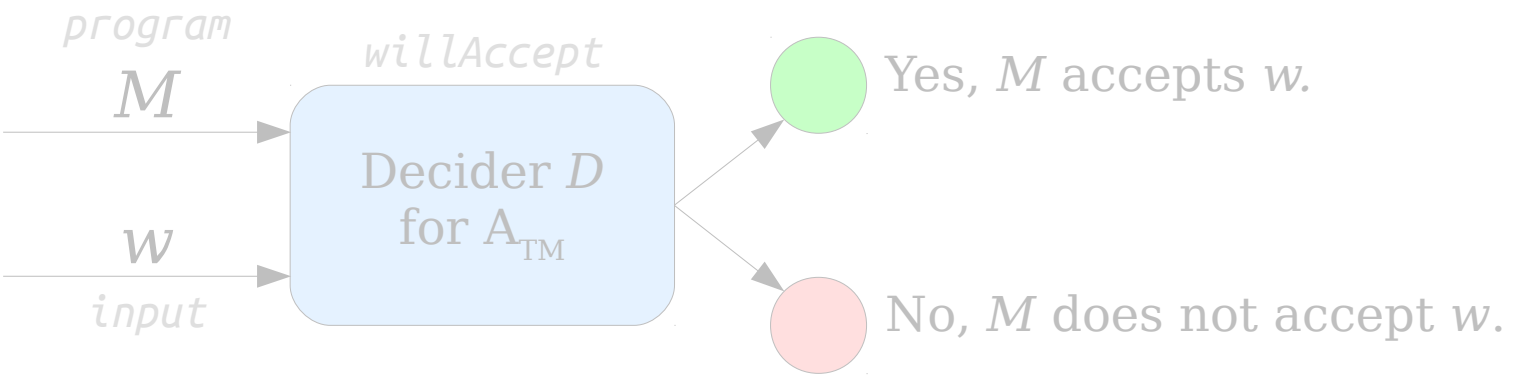


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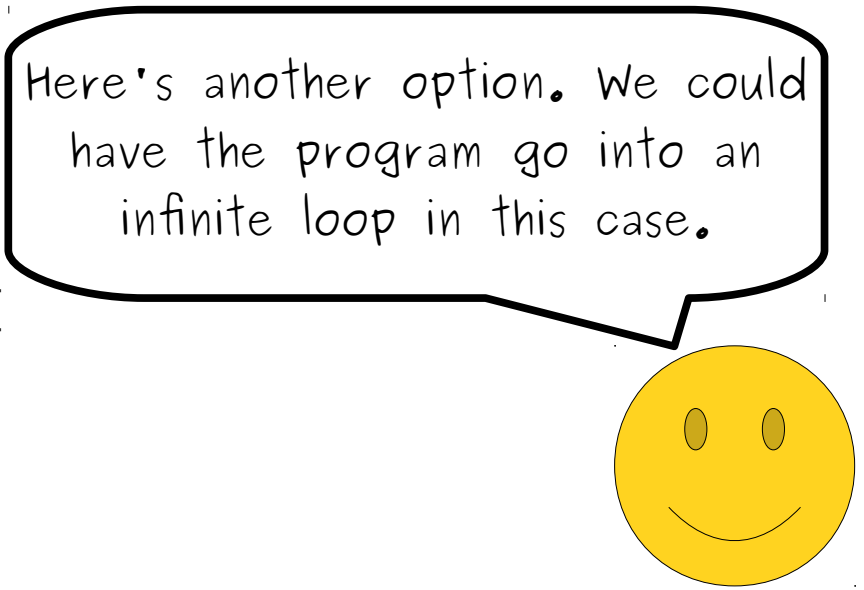

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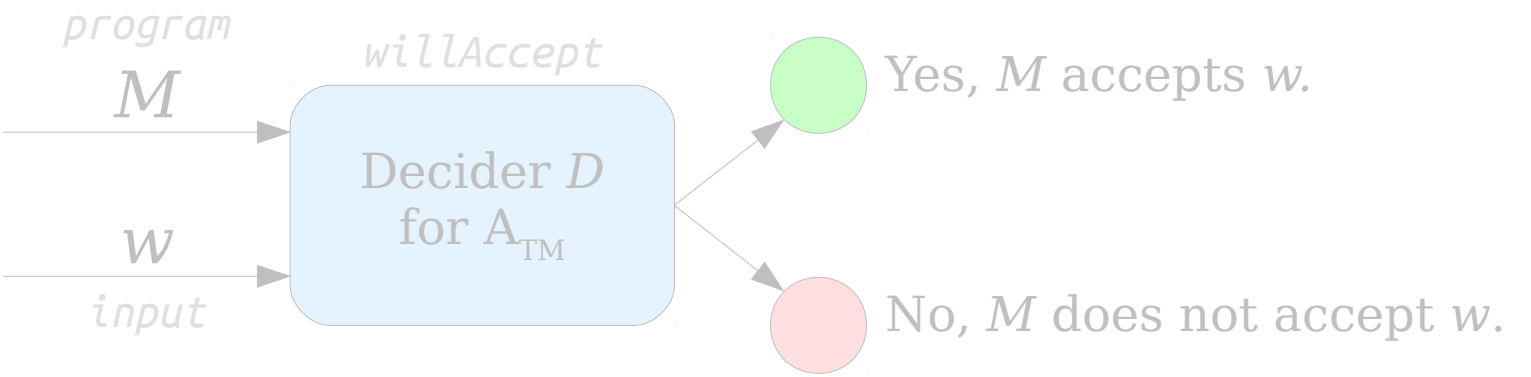


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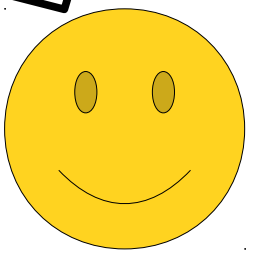
The design spec here says that P needs to not accept in this case, and indeed, that's what happens!



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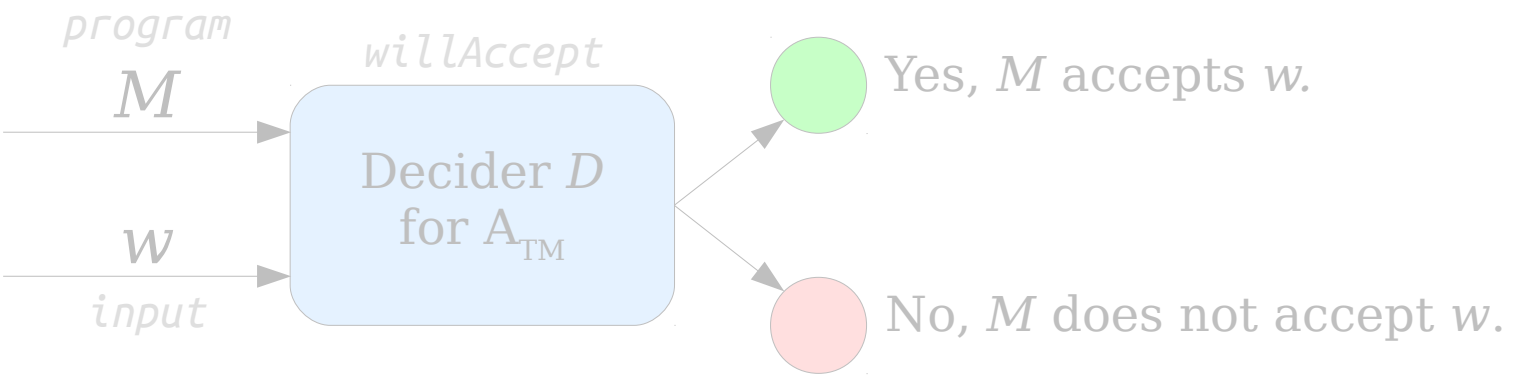


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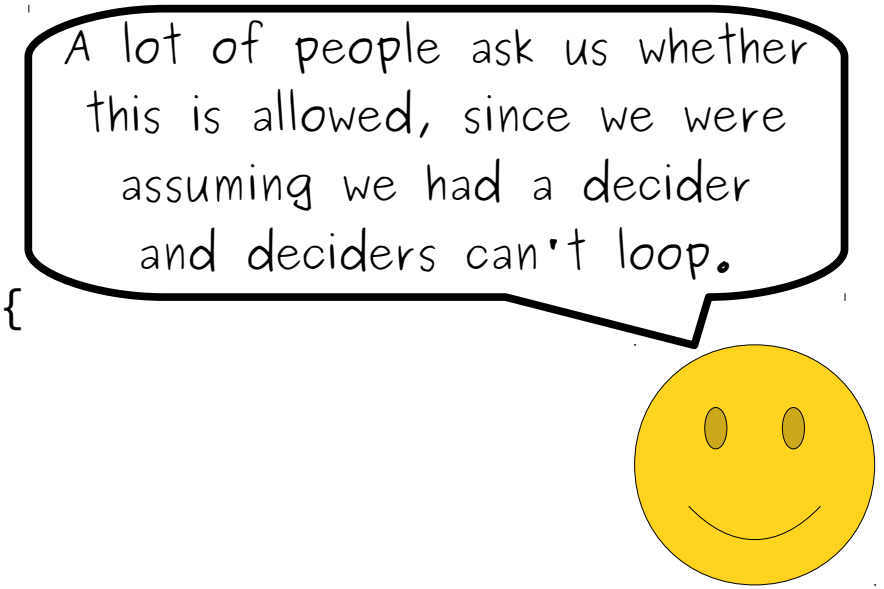

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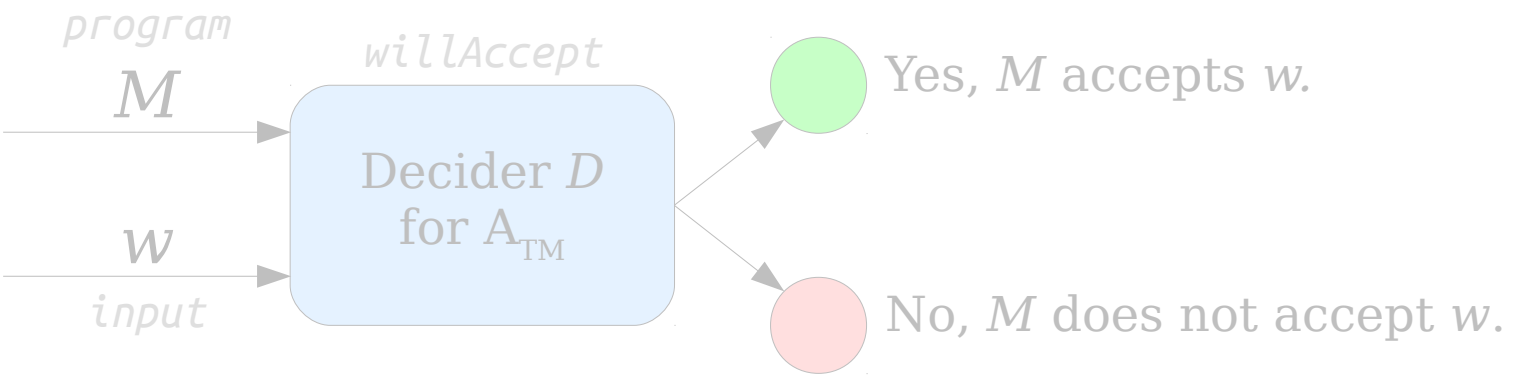


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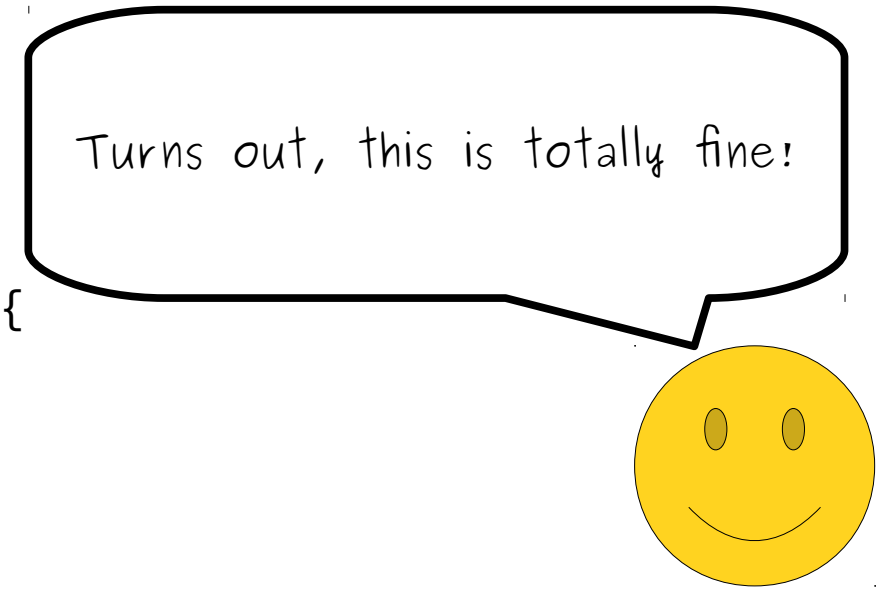

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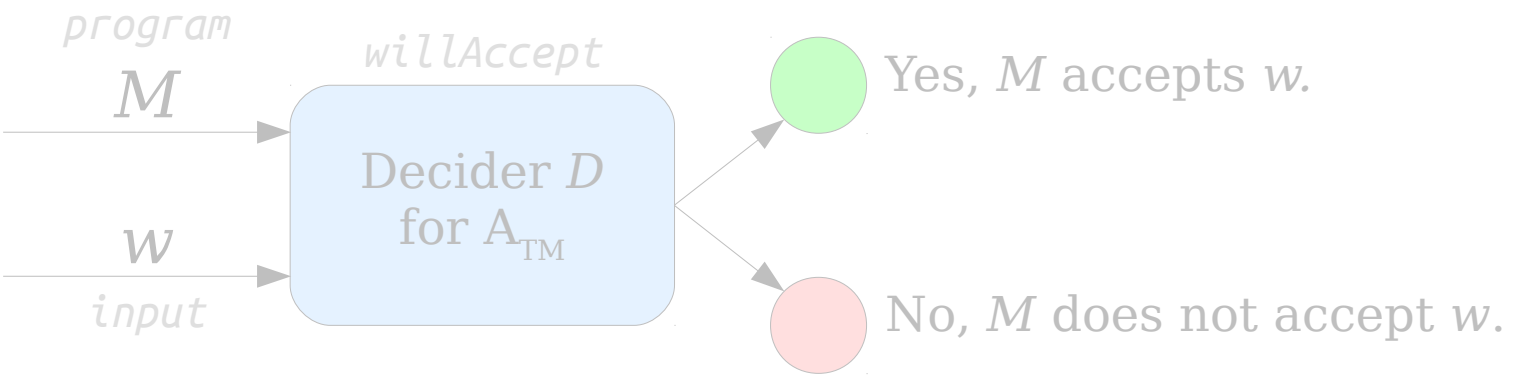


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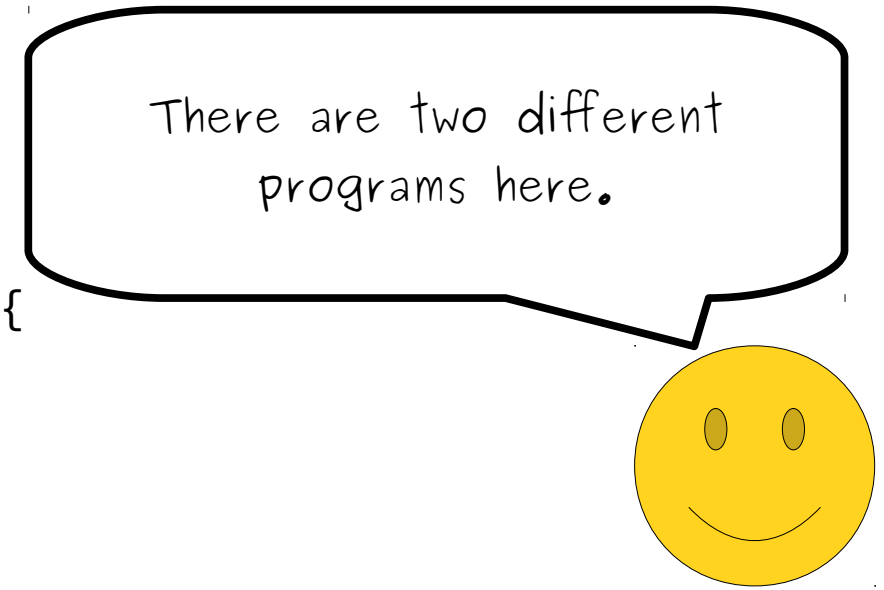

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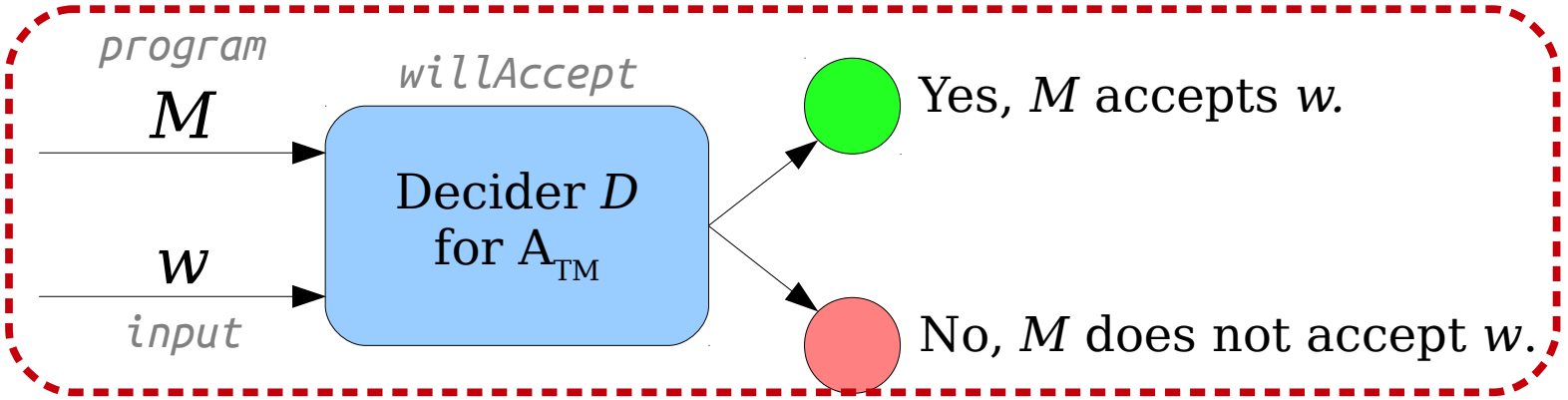


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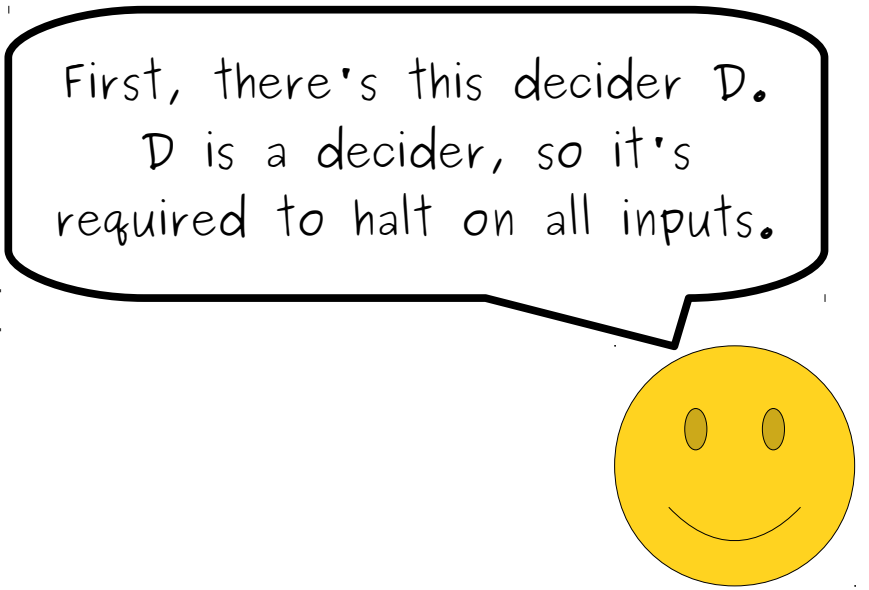

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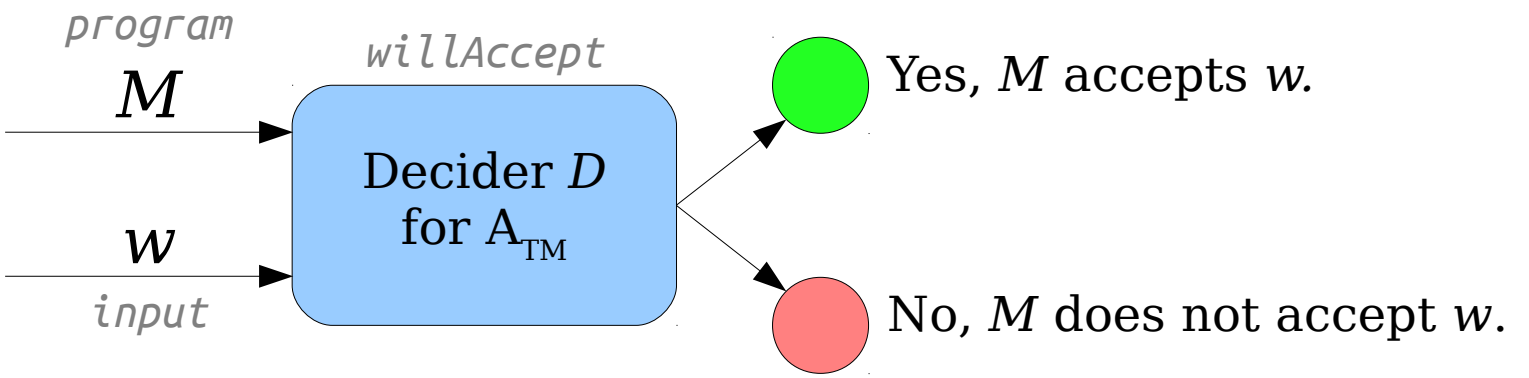


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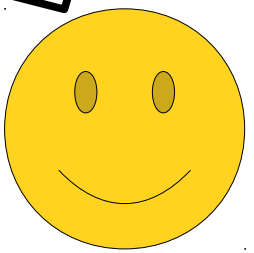
There's also this program P. Program P isn't the decider for A_{TM}, so it's not required to halt on all inputs.



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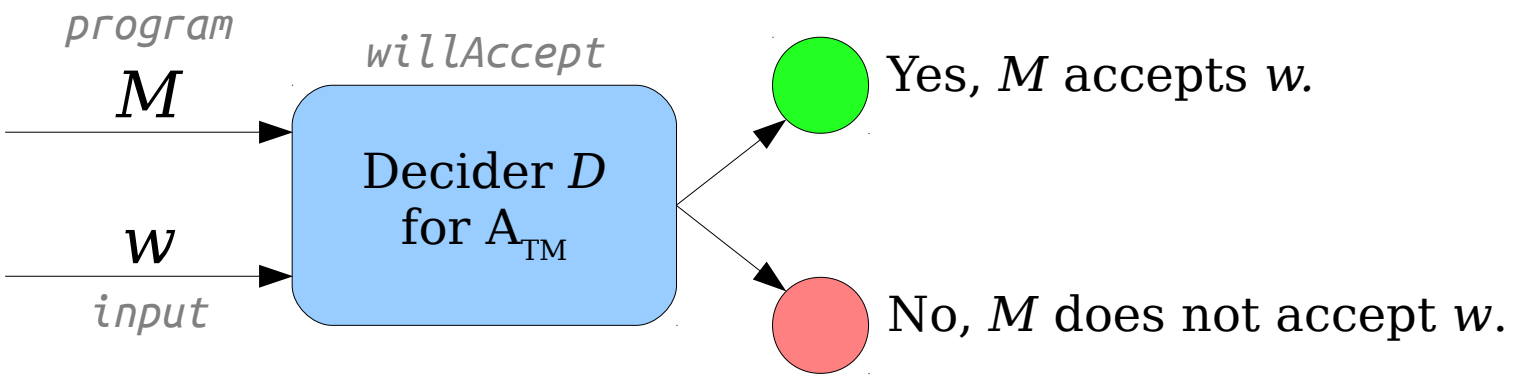


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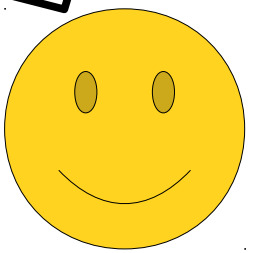
Going forward, remember that these proofs involve two different programs: the decider for the language, and the self-referential program.



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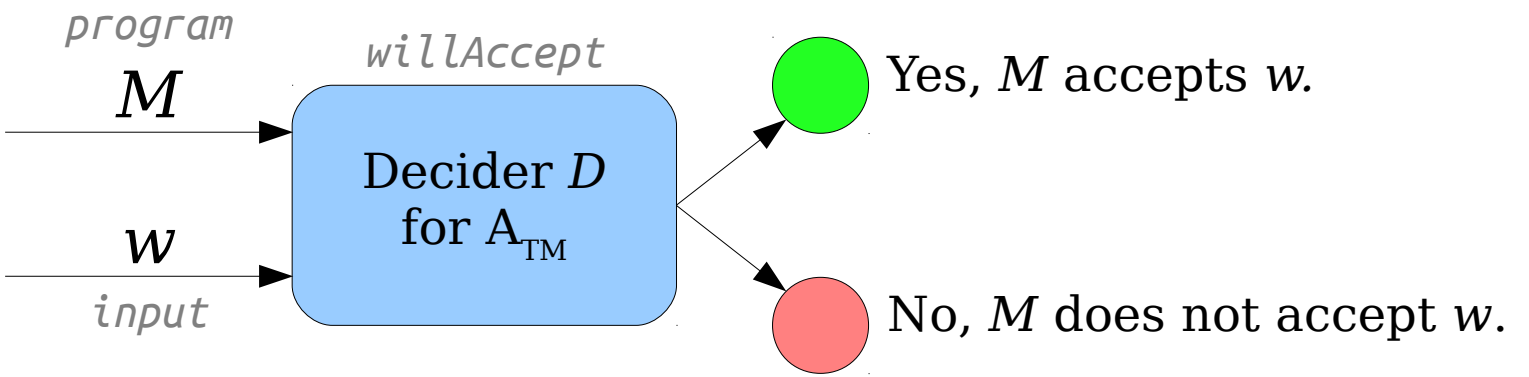


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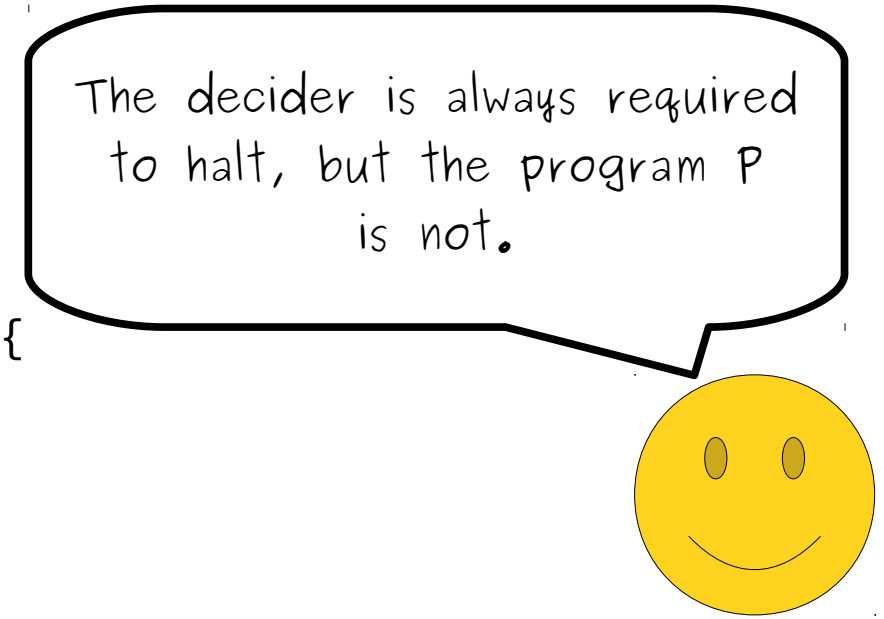

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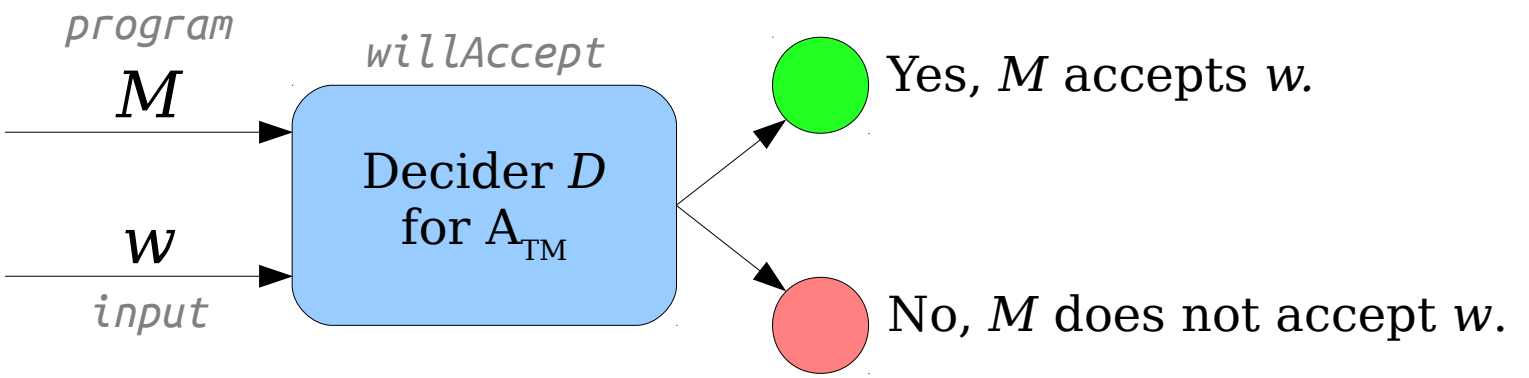


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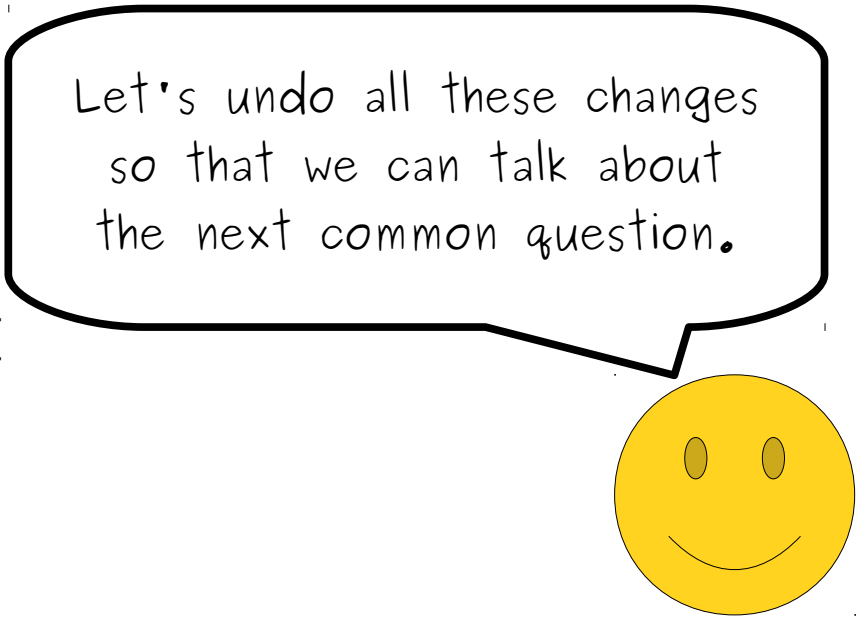

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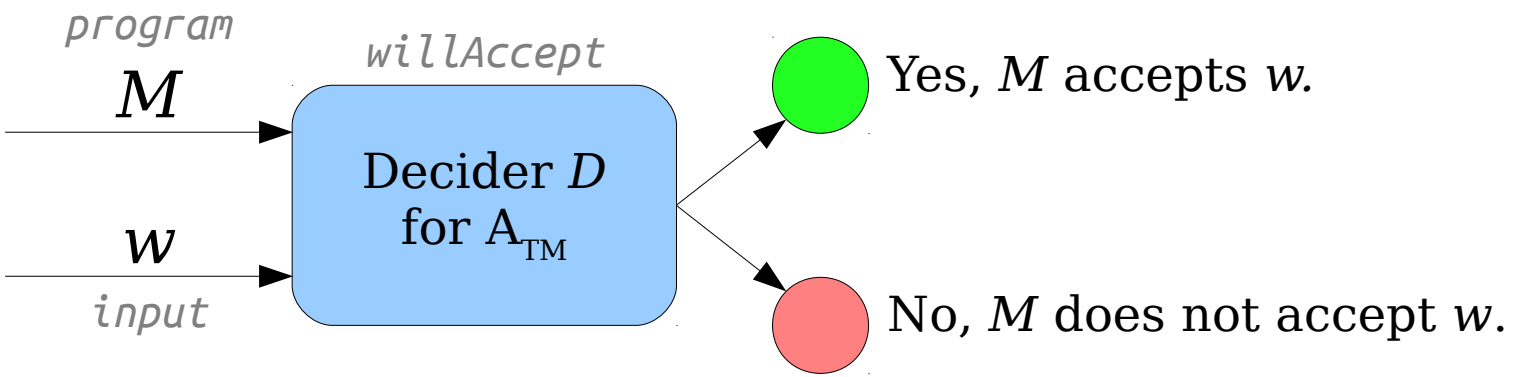


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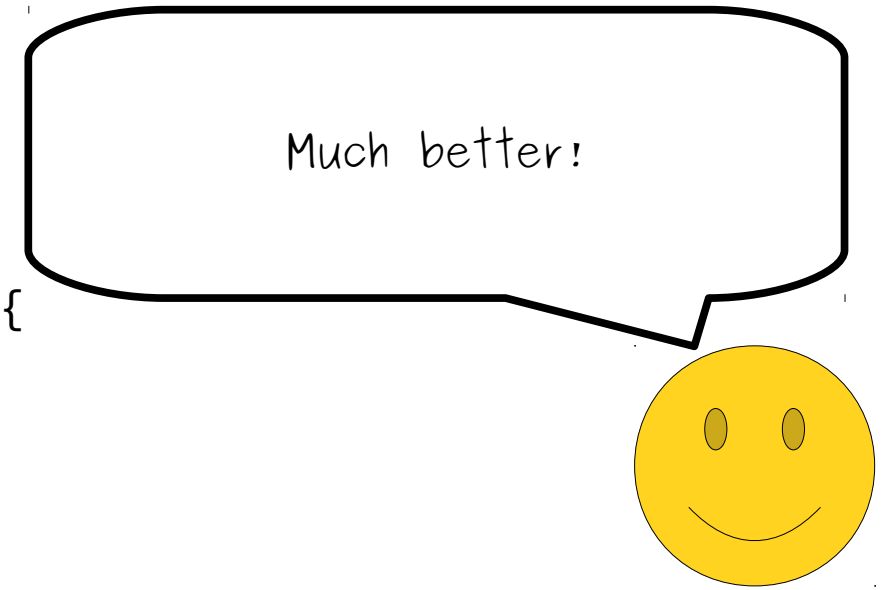

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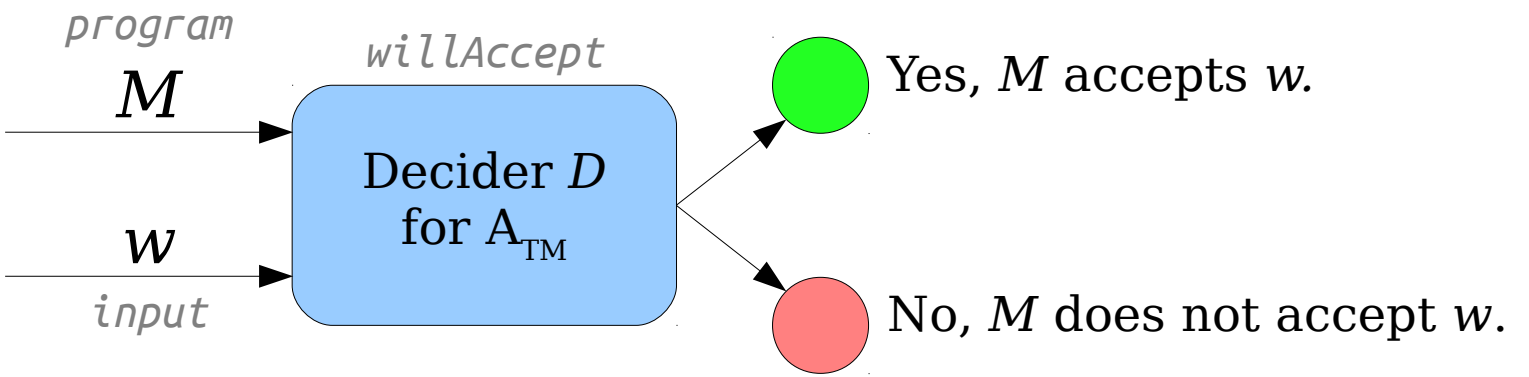


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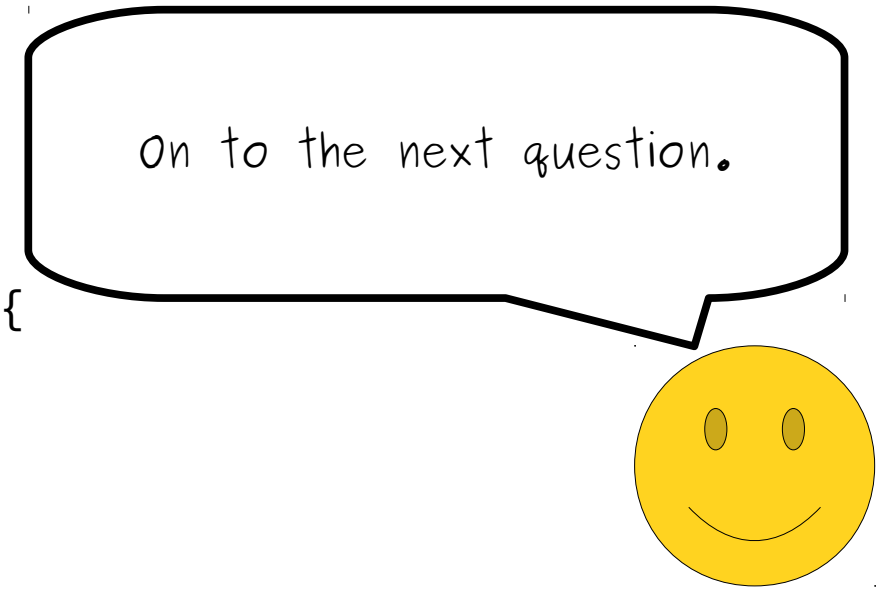

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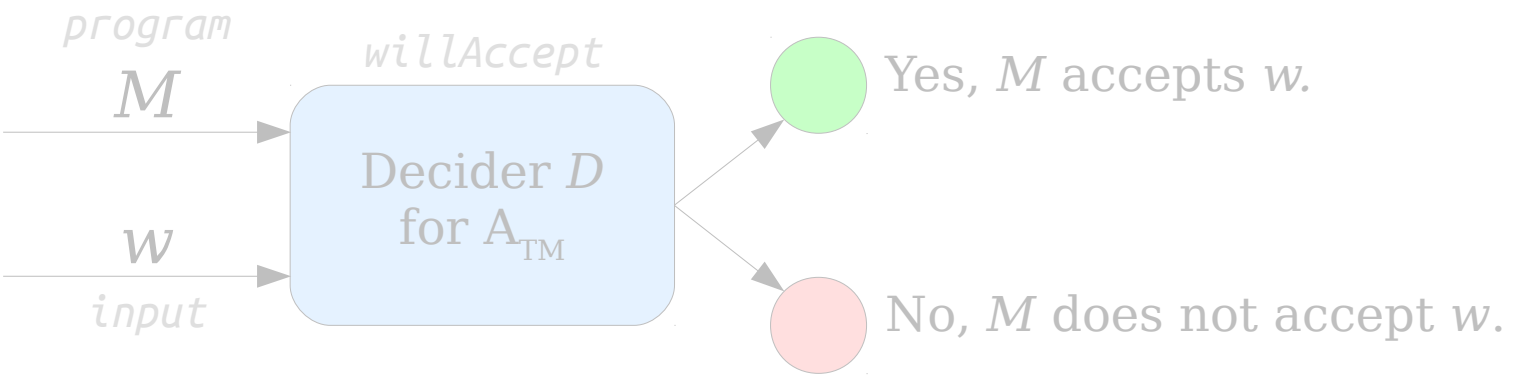


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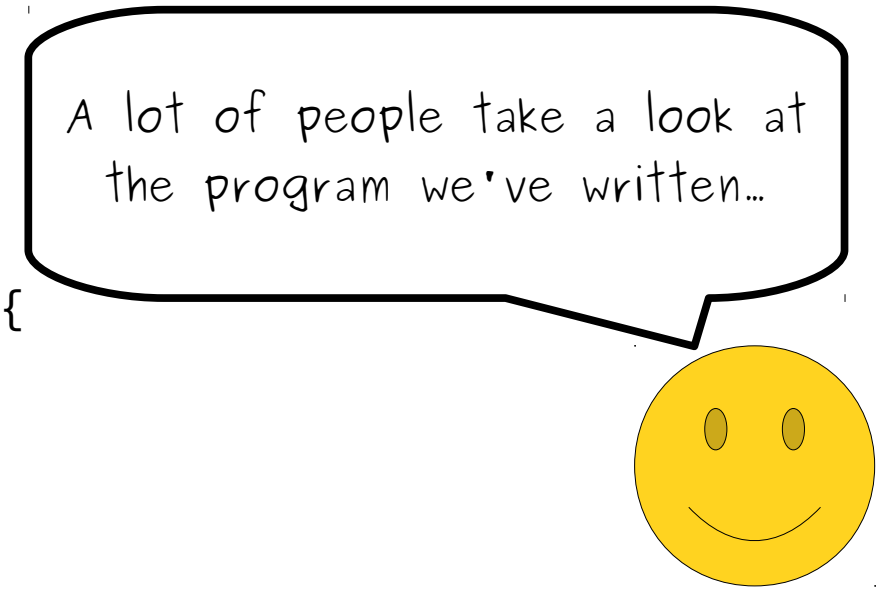

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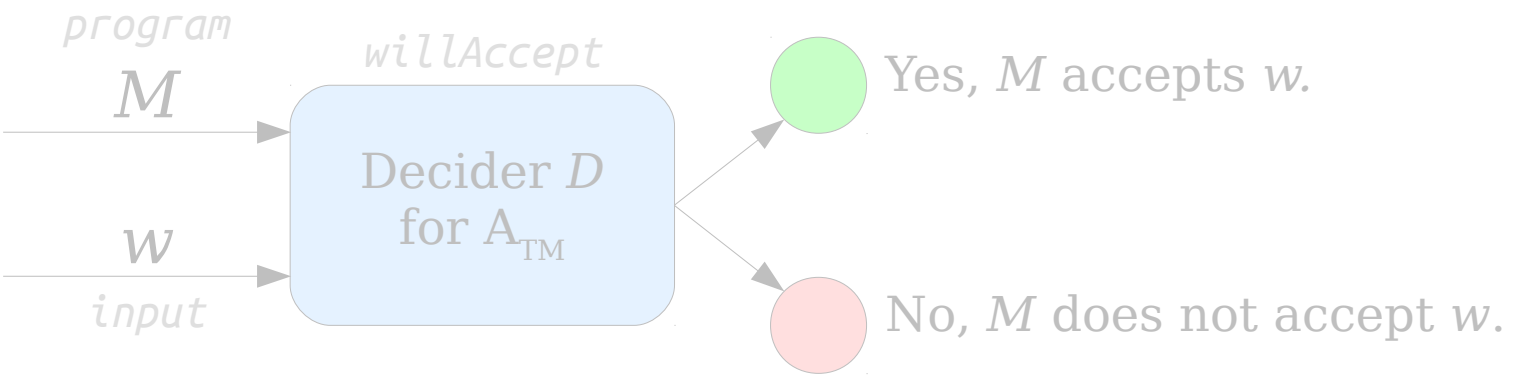


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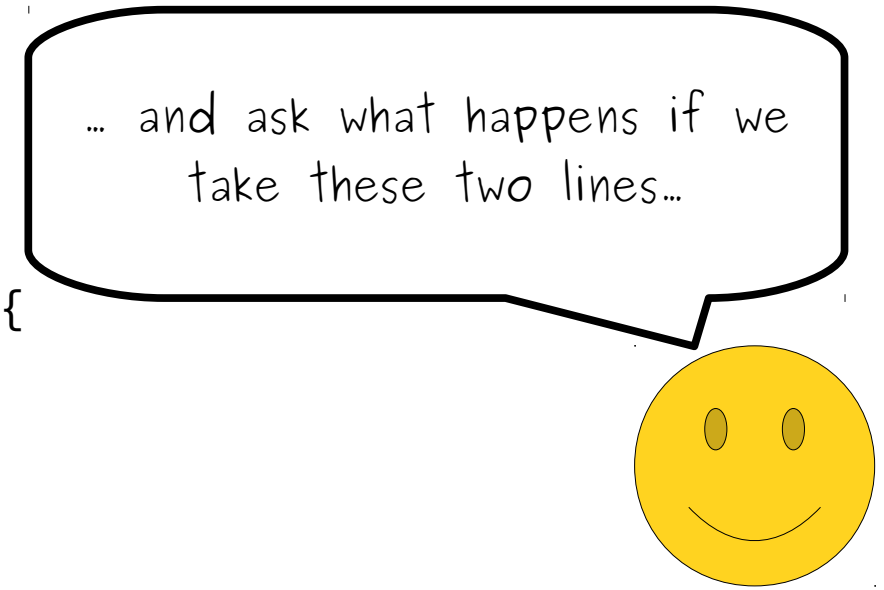

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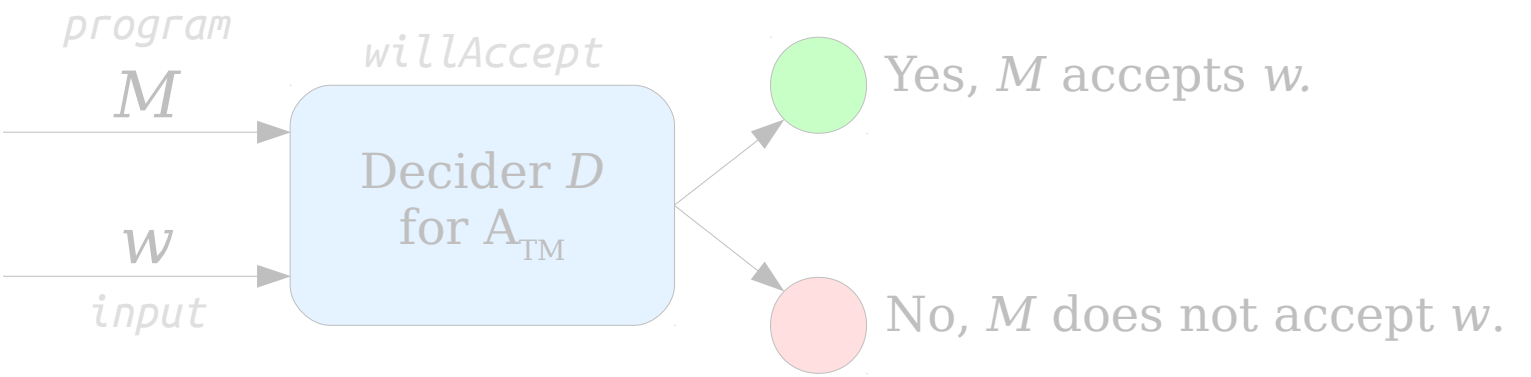


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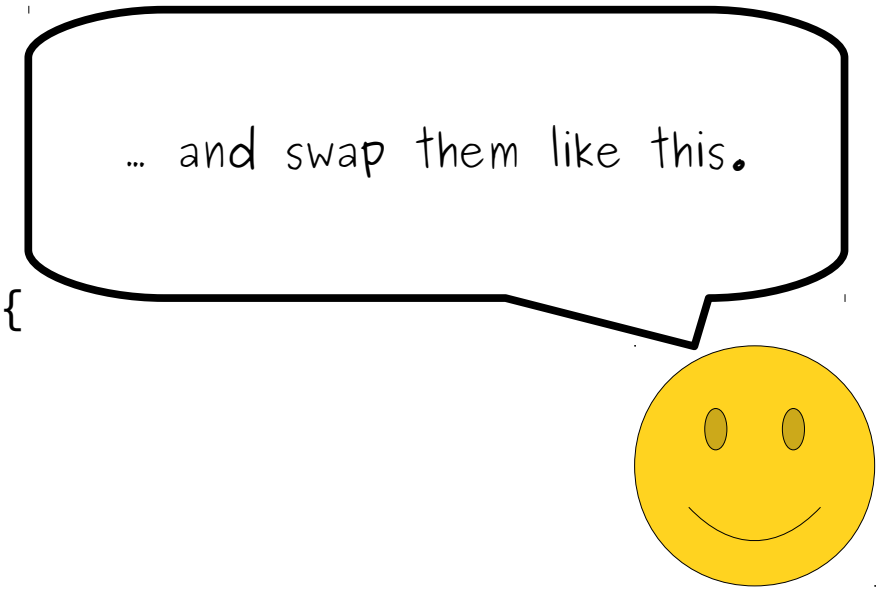

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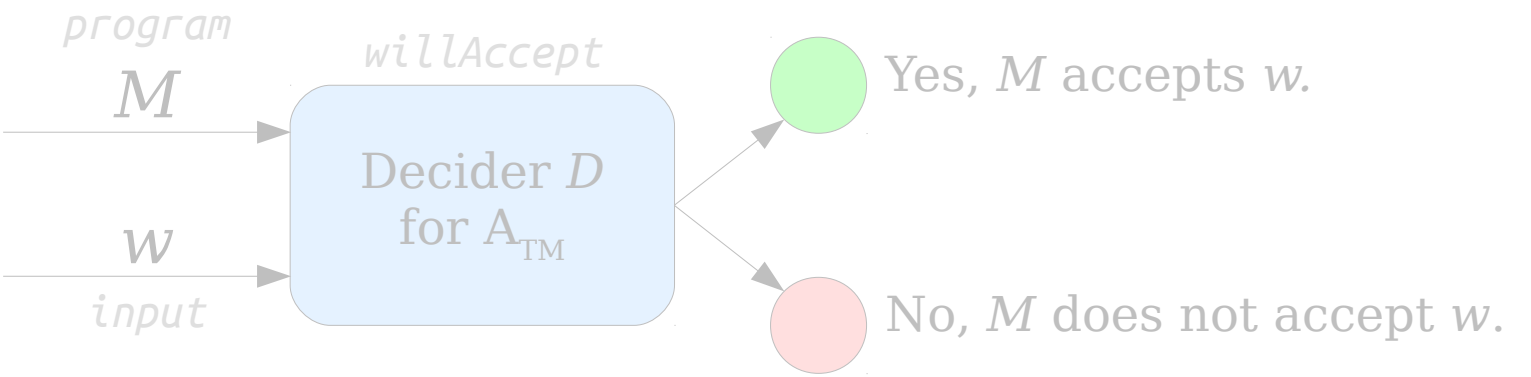


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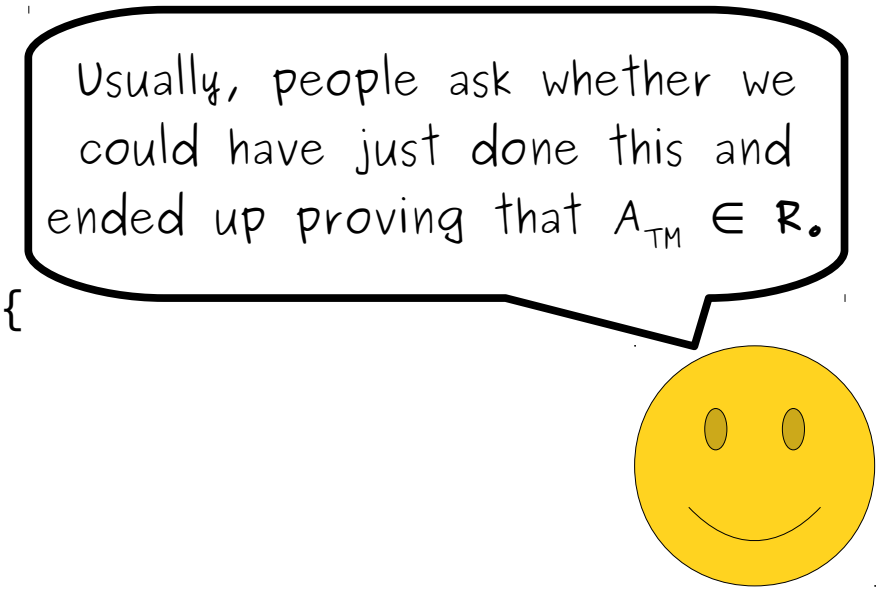

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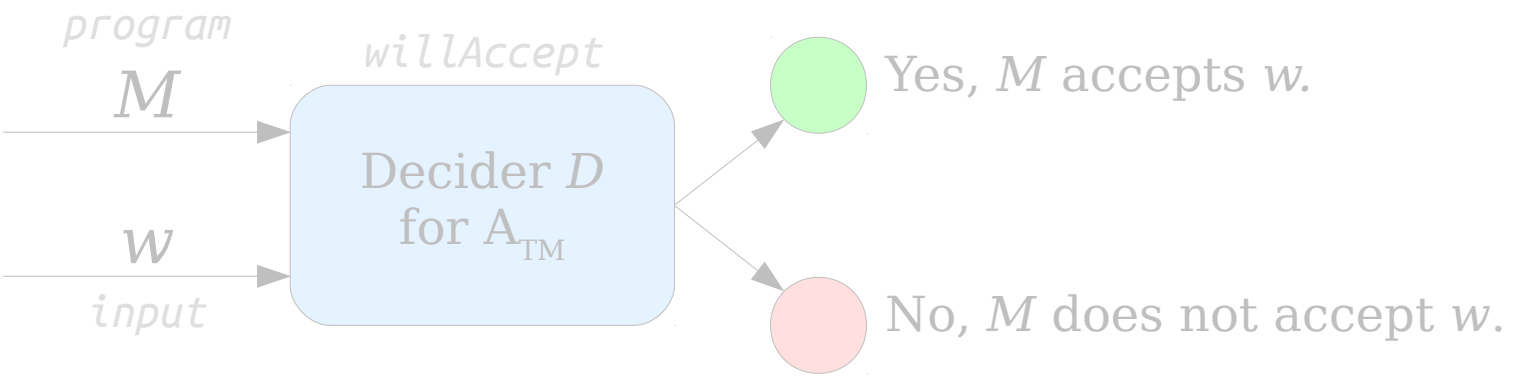


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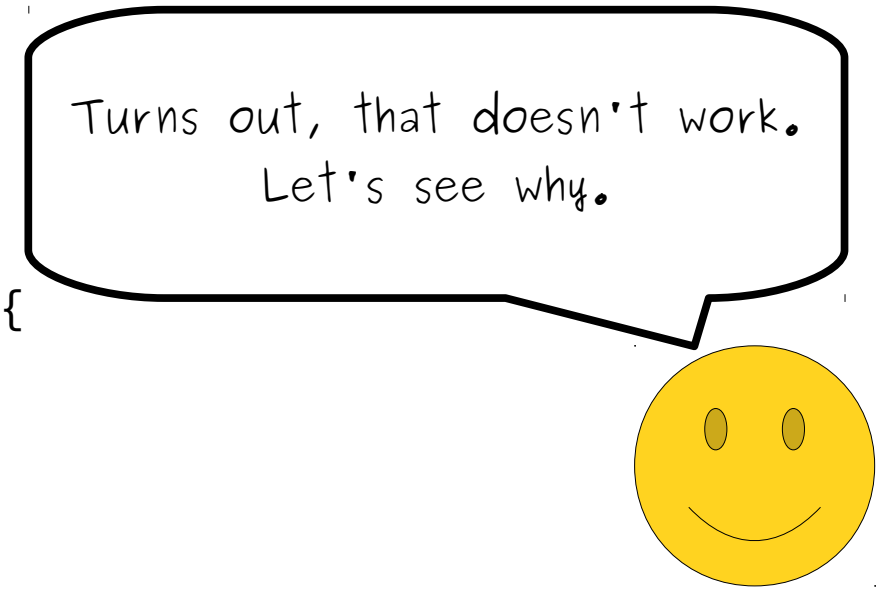

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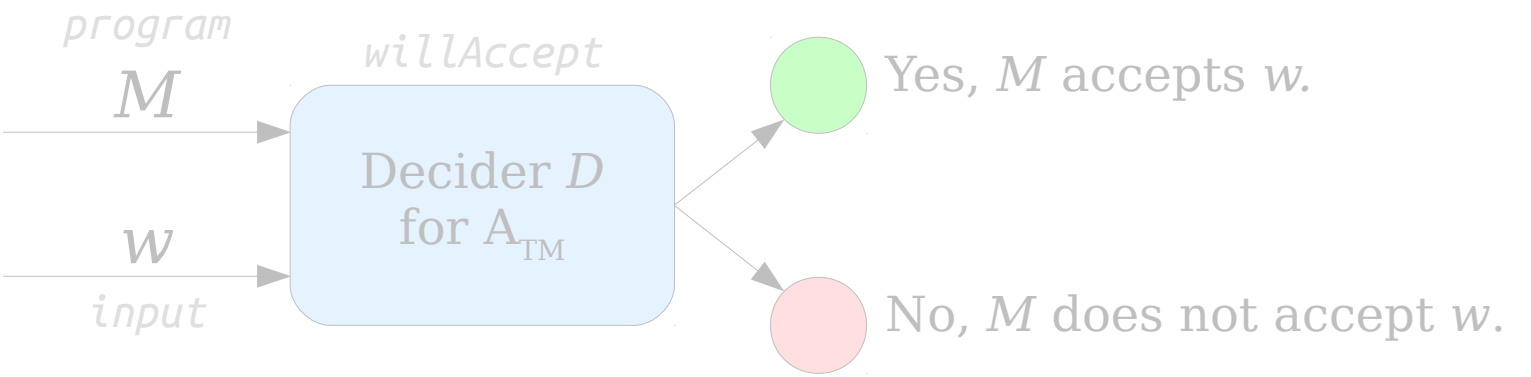


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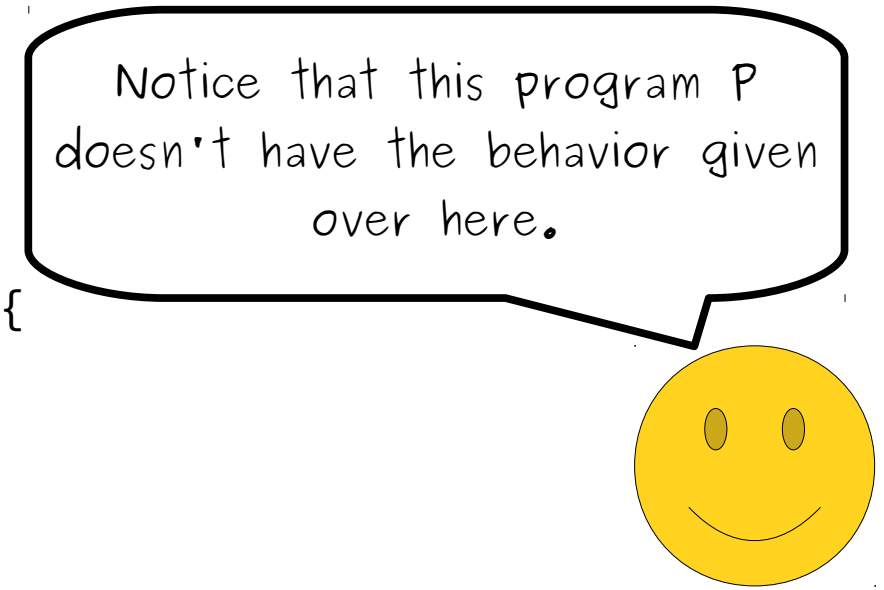

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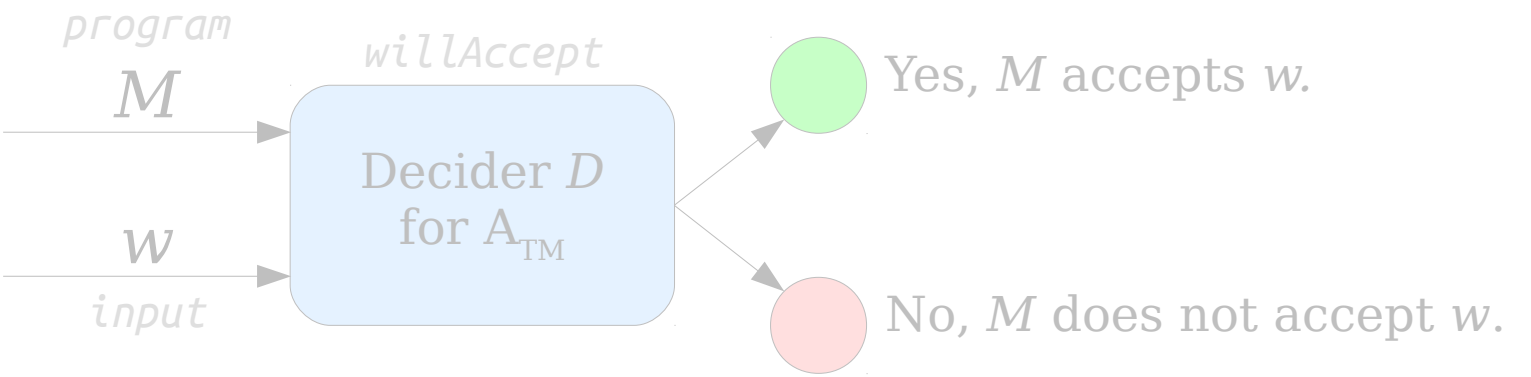


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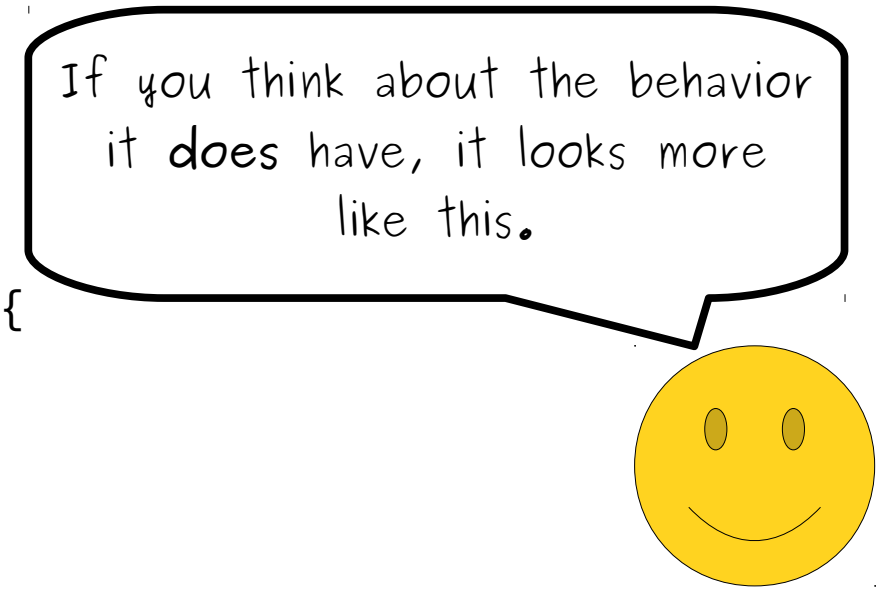

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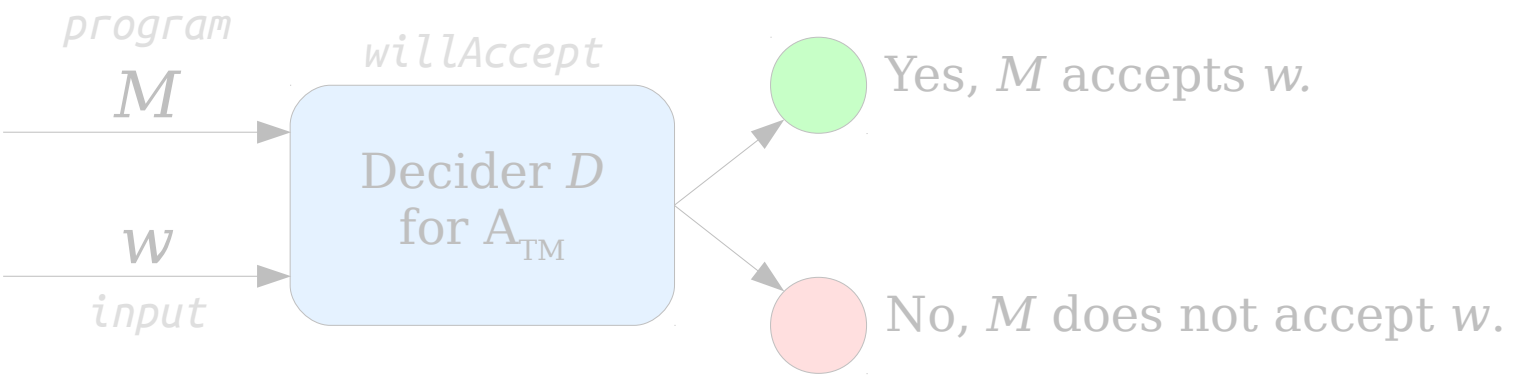


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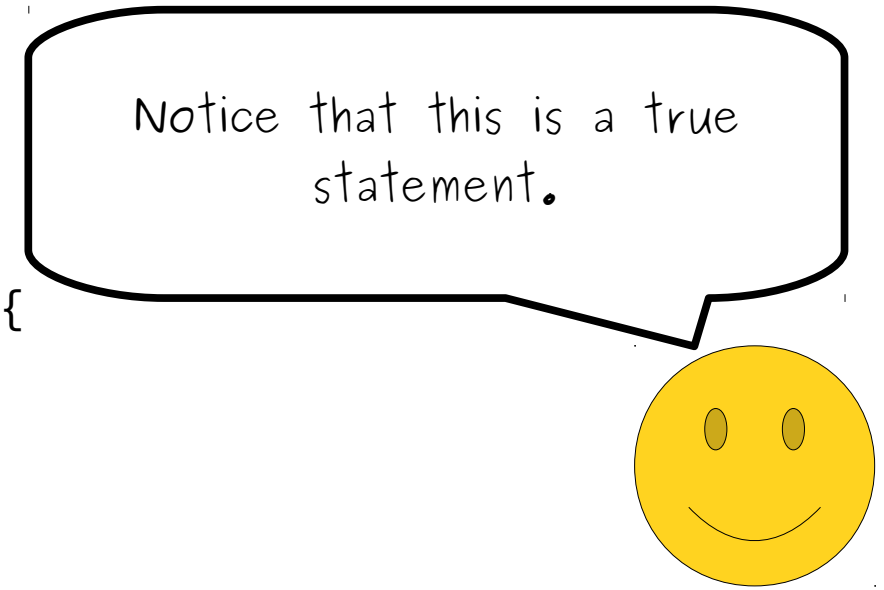

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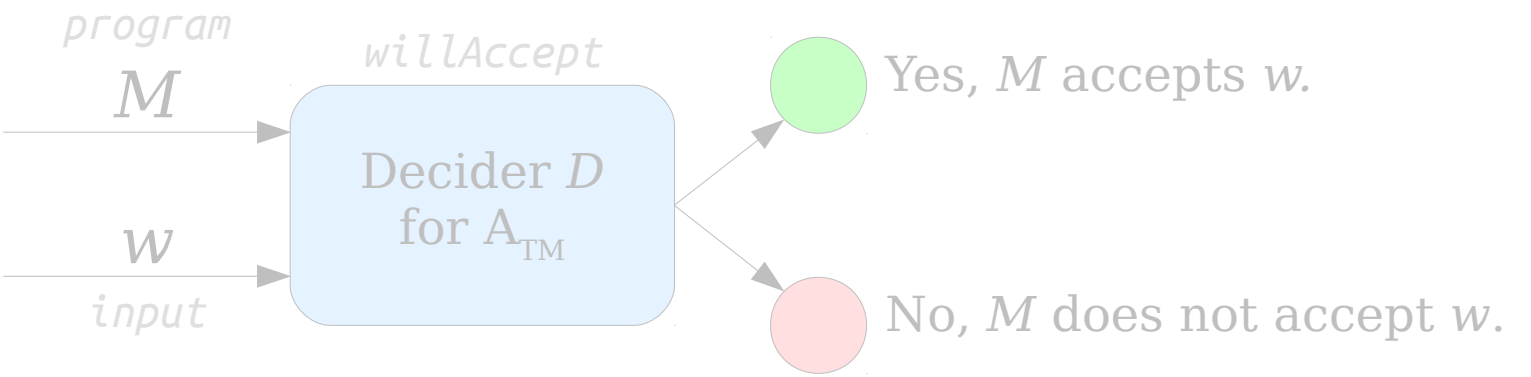


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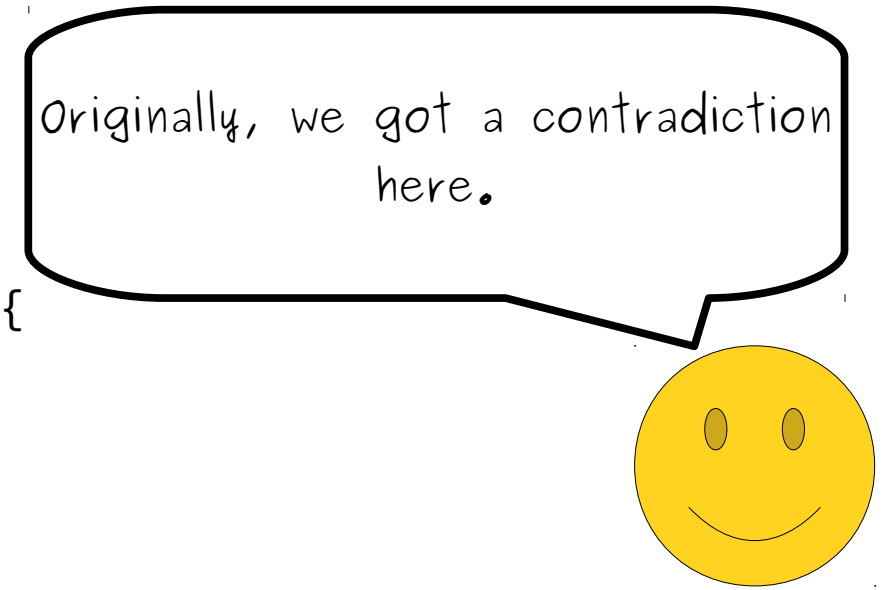

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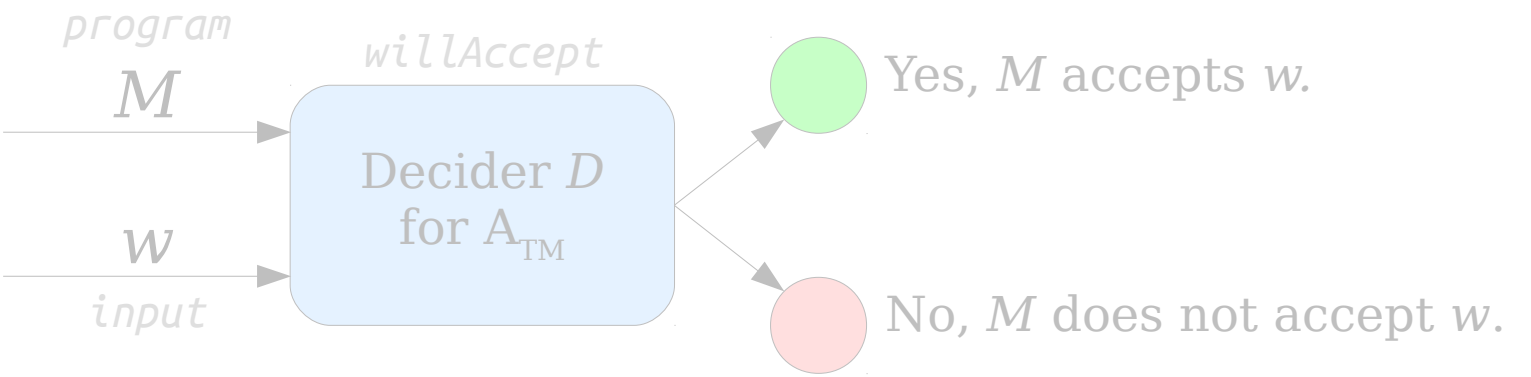


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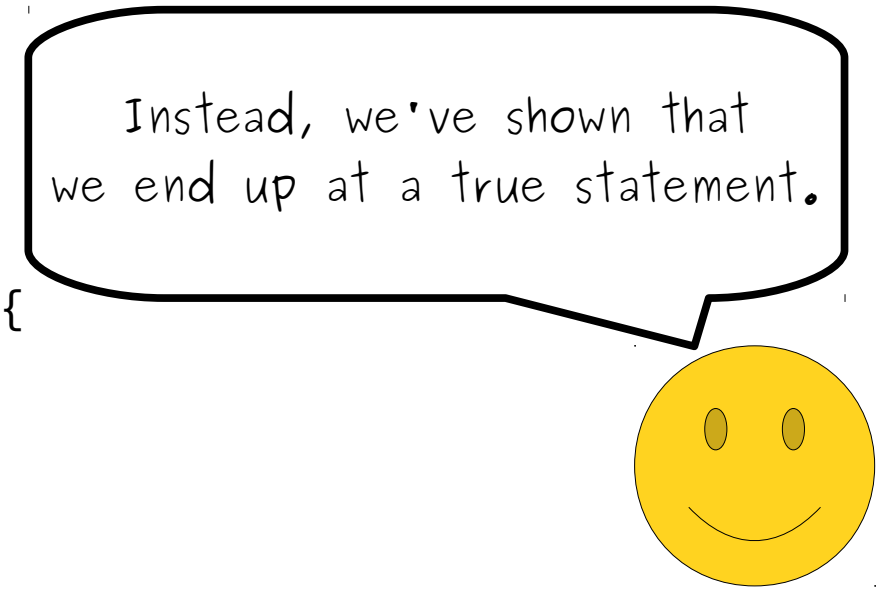

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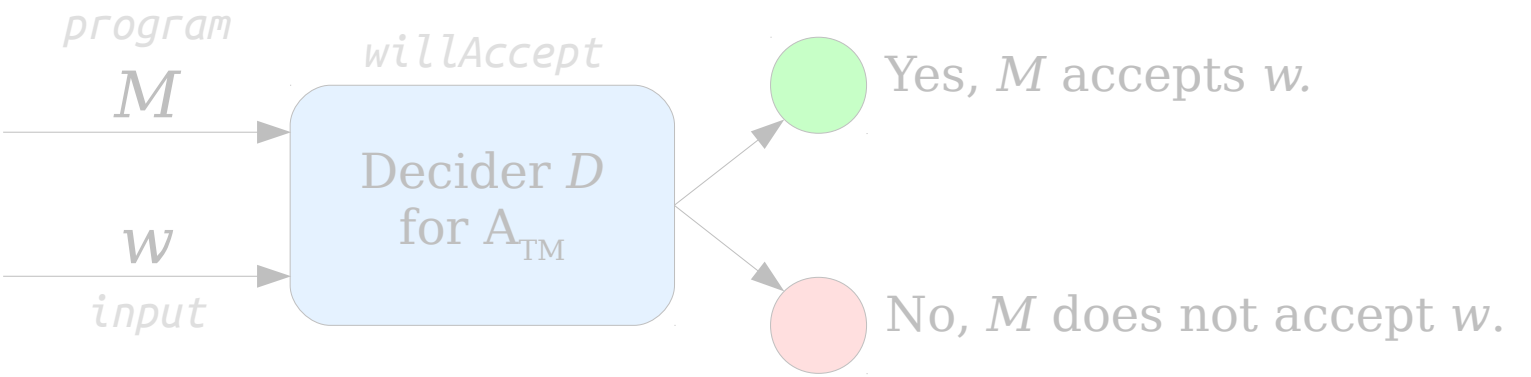


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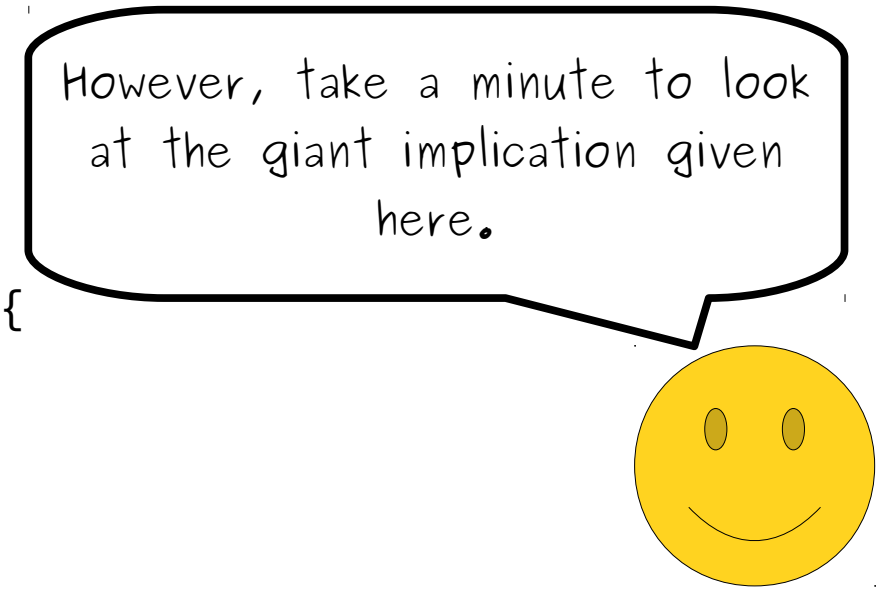

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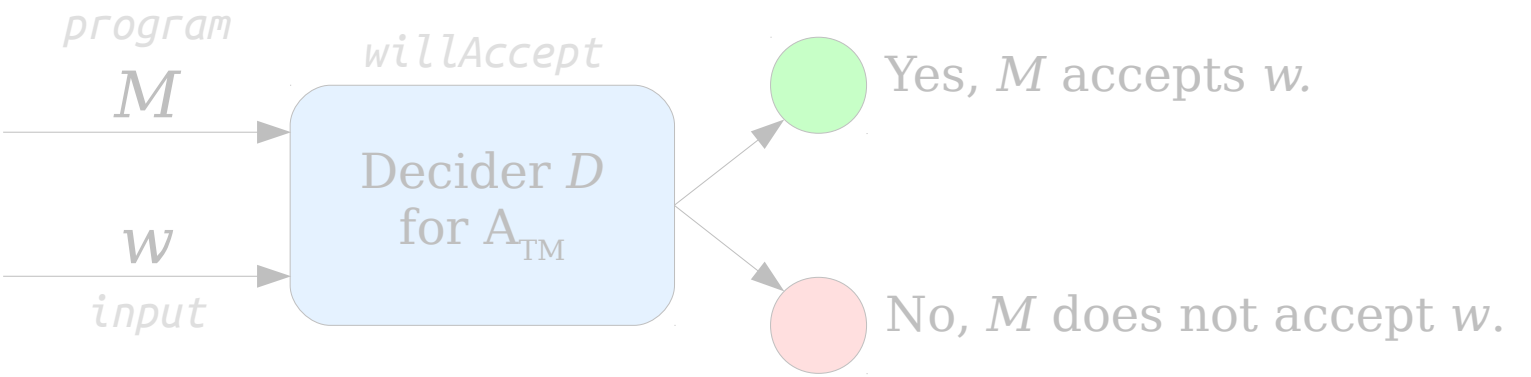


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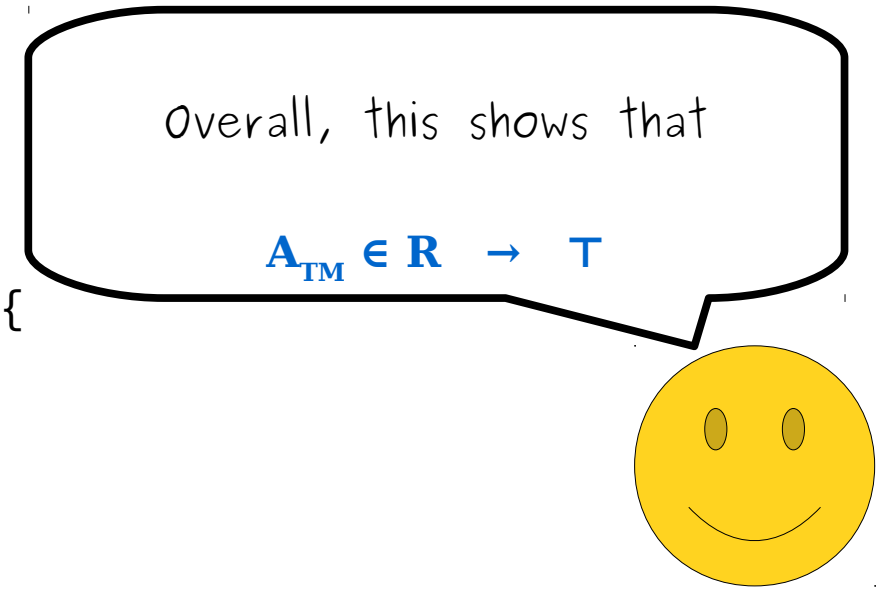

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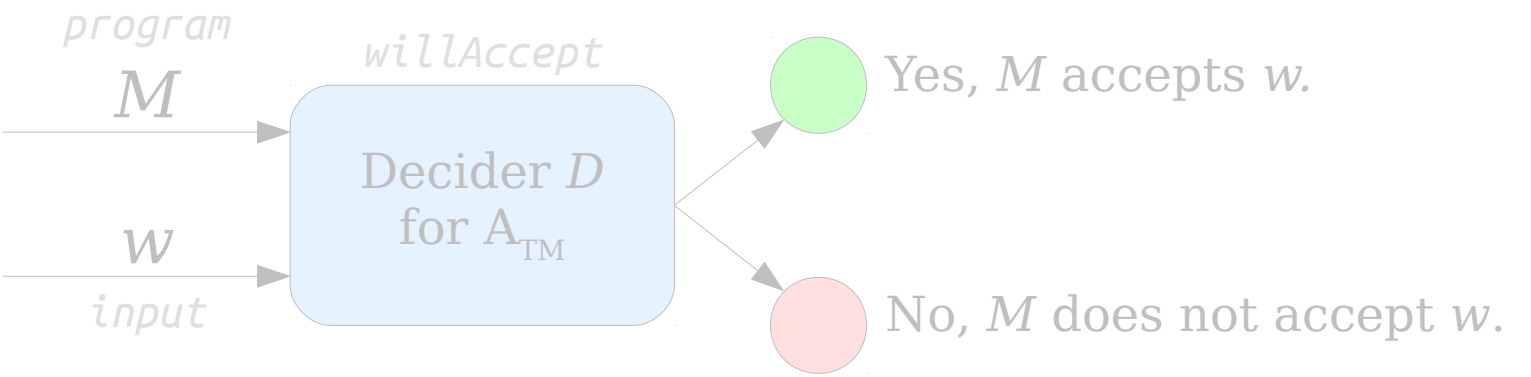


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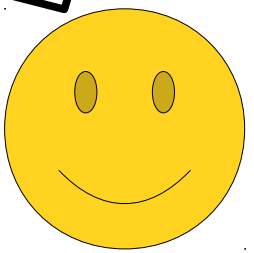
```

Does this statement say anything about whether A_{TM} is decidable?

$A_{TM} \in \mathbf{R} \rightarrow \mathbf{T}$



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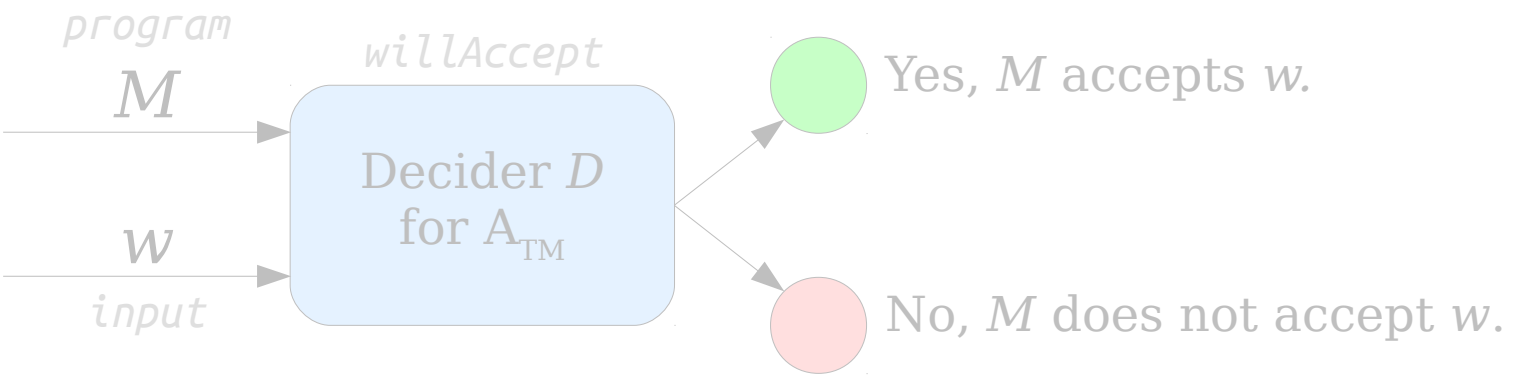


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```
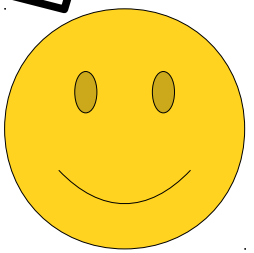

Nope! Remember, *anything* implies a true statement.

$A_{TM} \in \mathbf{R} \rightarrow \top$



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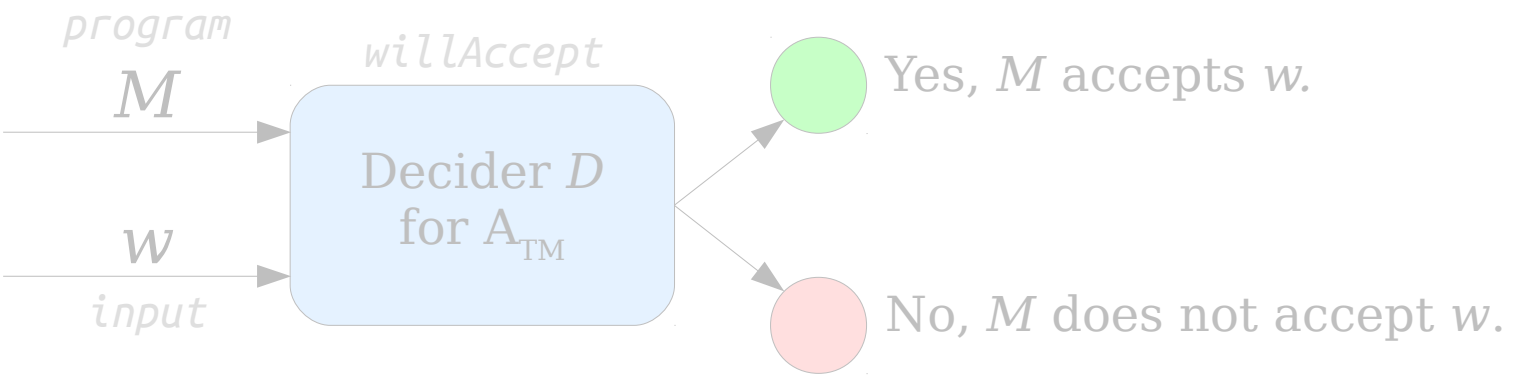


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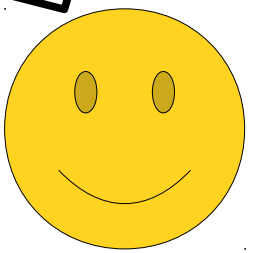
We have no way of knowing whether $A_{TM} \in R$ or not just by looking at this statement.

$A_{TM} \in R \rightarrow T$



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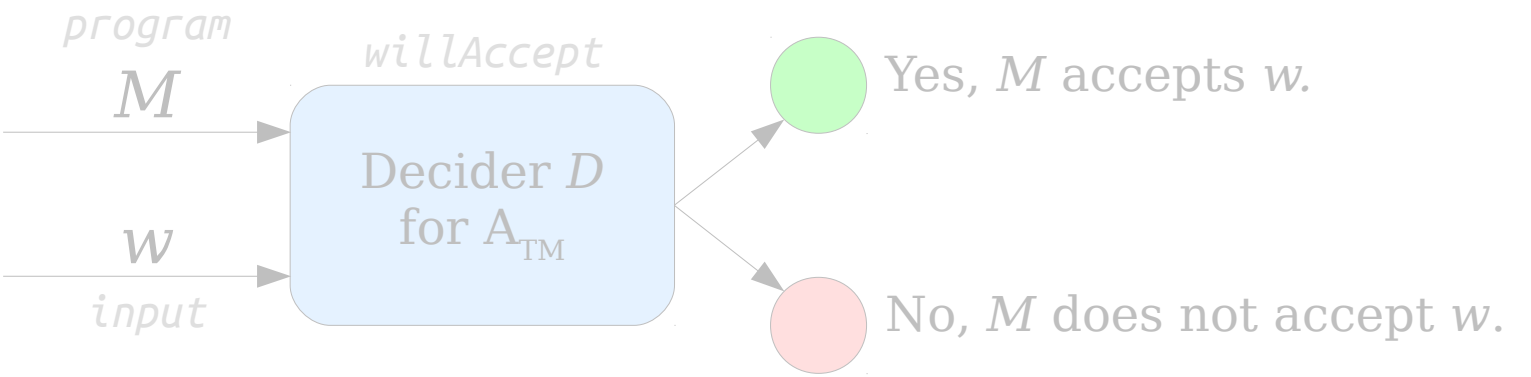


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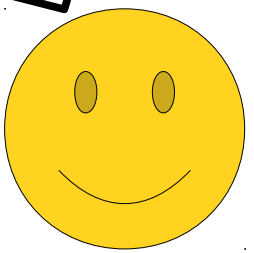
The fact that we didn't get a contradiction doesn't mean that A_{TM} is decidable.

$\mathbf{A_{TM} \in R \rightarrow T}$



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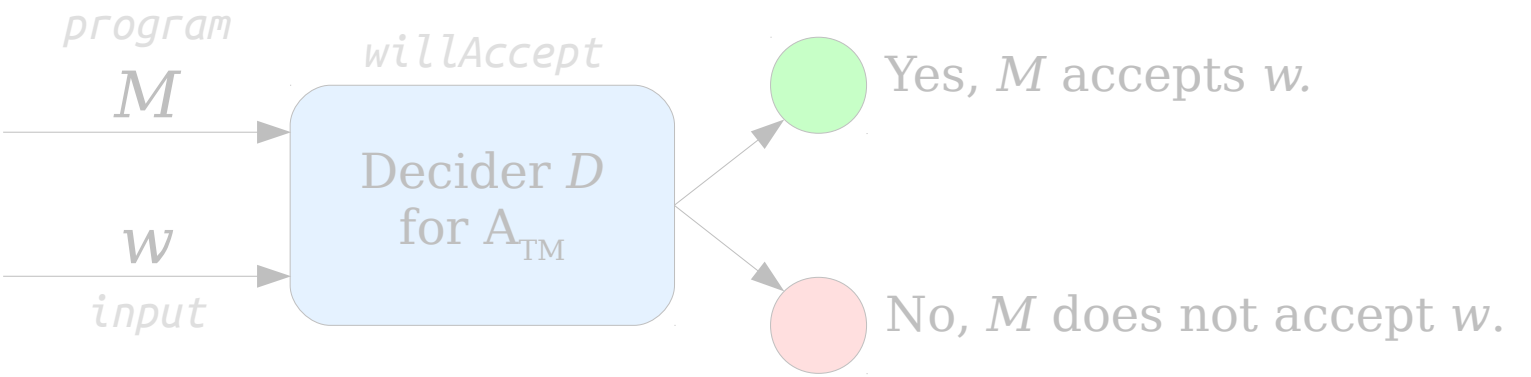


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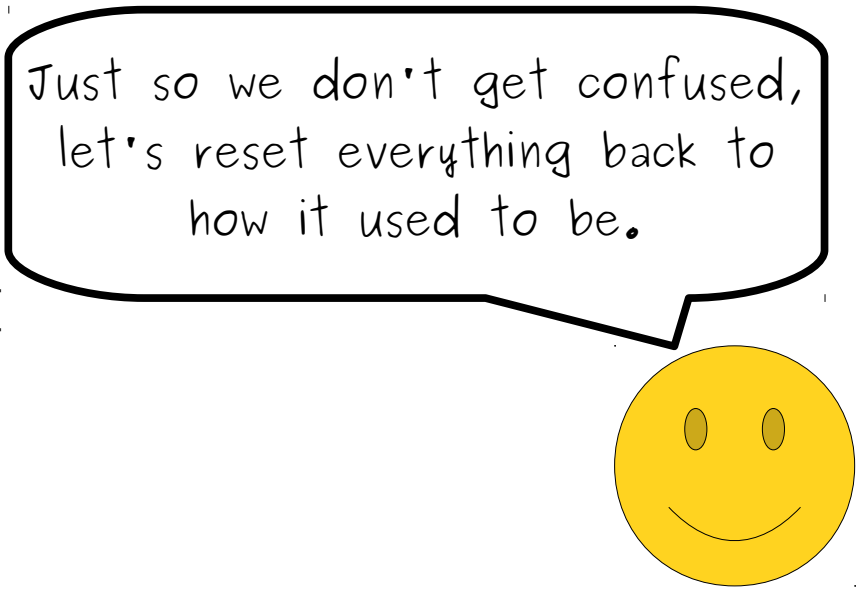

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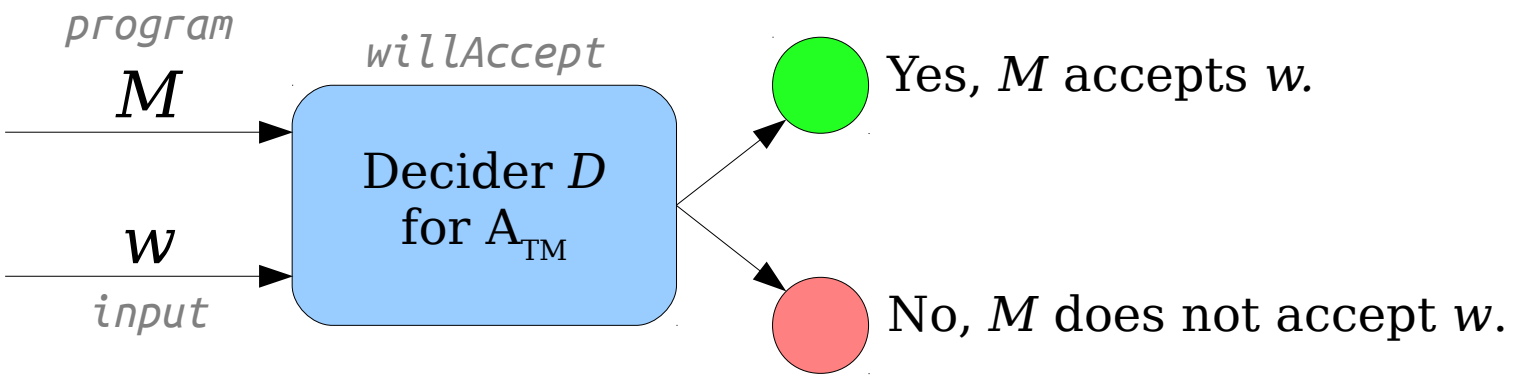


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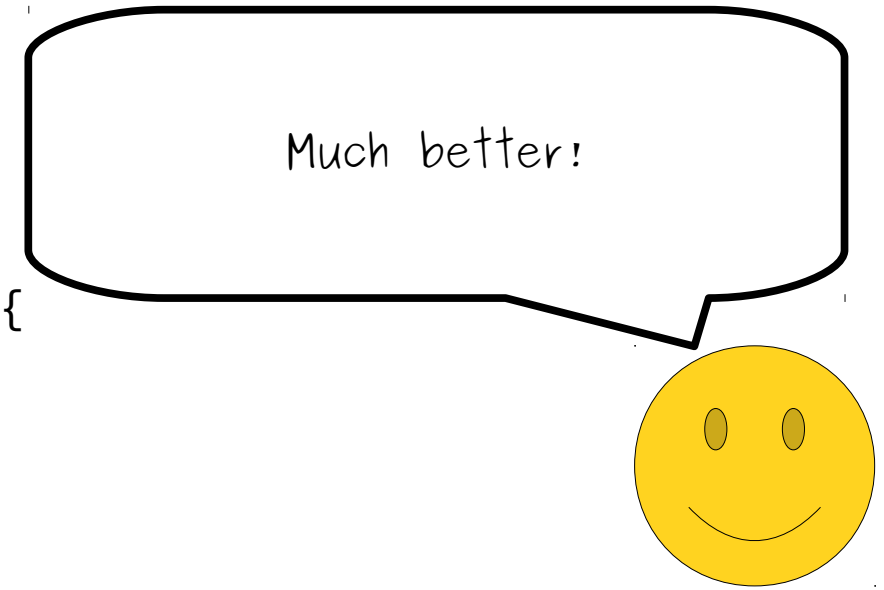

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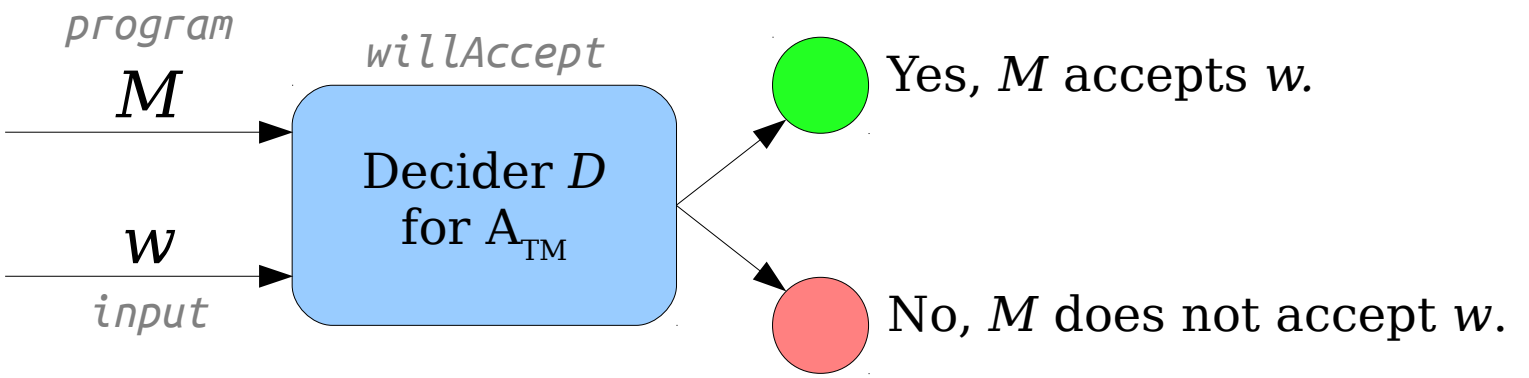


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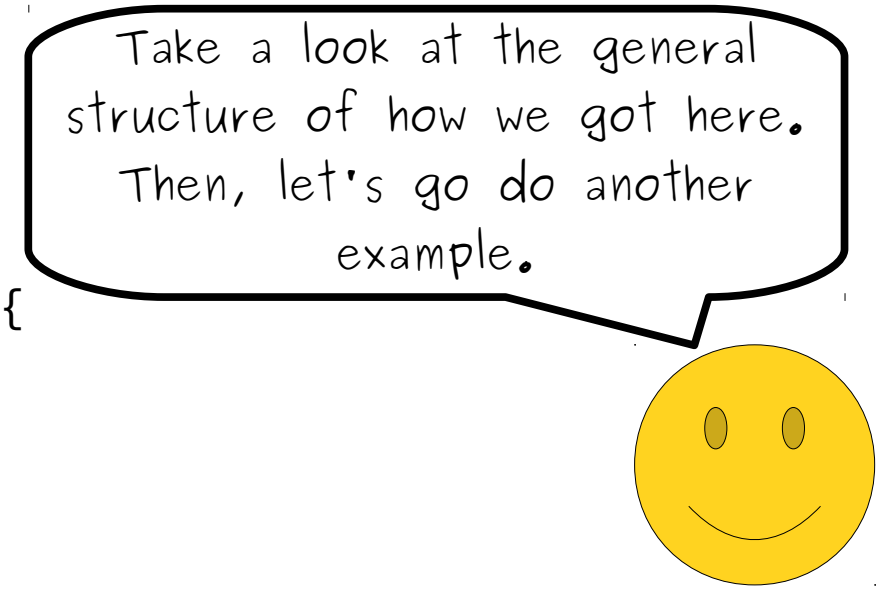
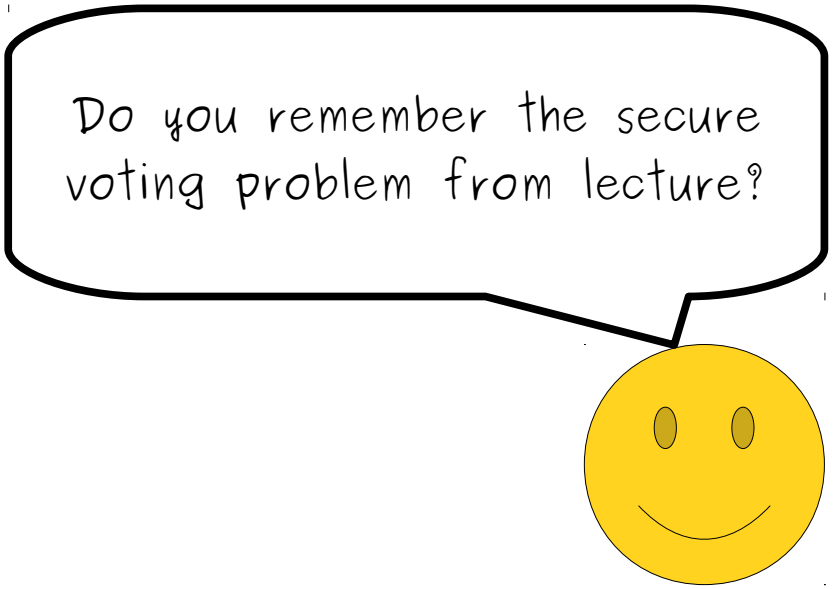
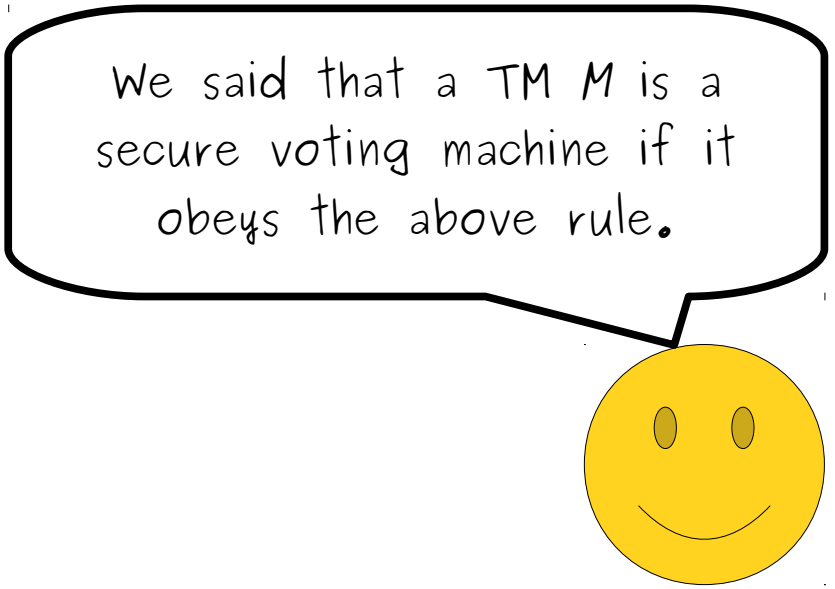



Do you remember the secure voting problem from lecture?

M is a secure voting machine

if and only if

$\mathscr{L}(M) = \{ w \in \{\texttt{r}, \texttt{d}\}^* \mid w \text{ has more } \texttt{r}\text{'s than } \texttt{d}\text{'s} \}$



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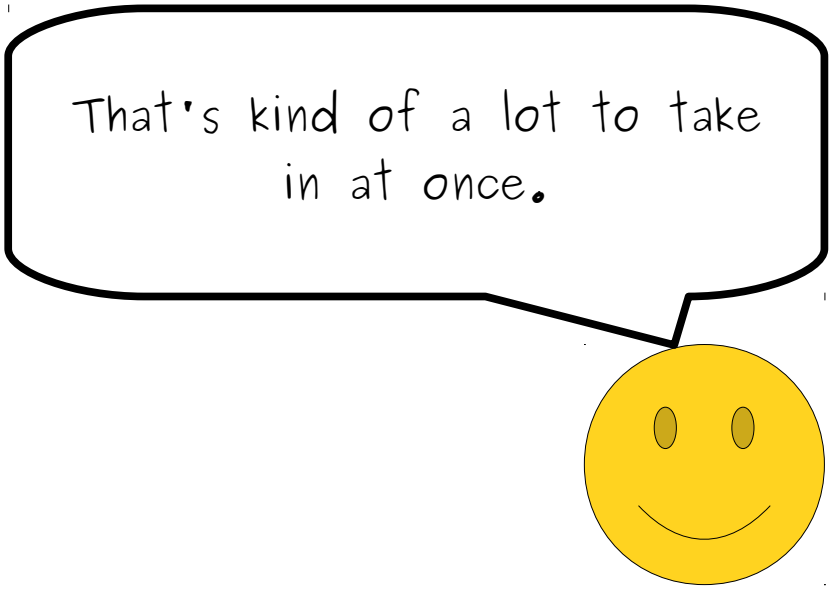

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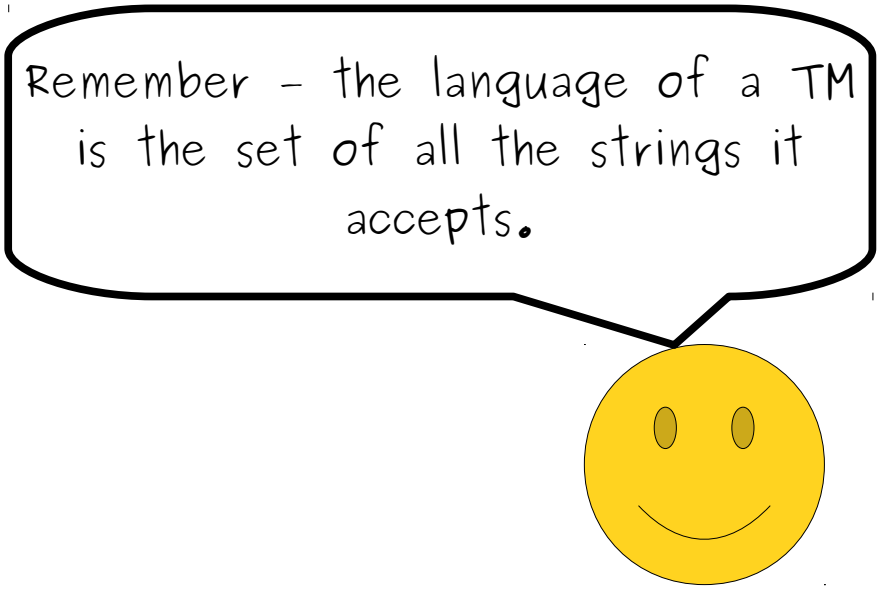

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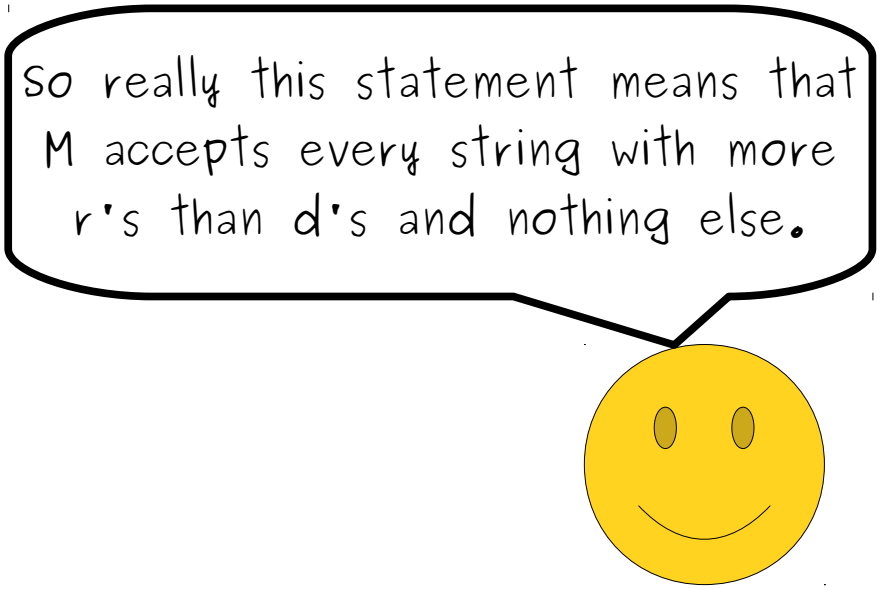

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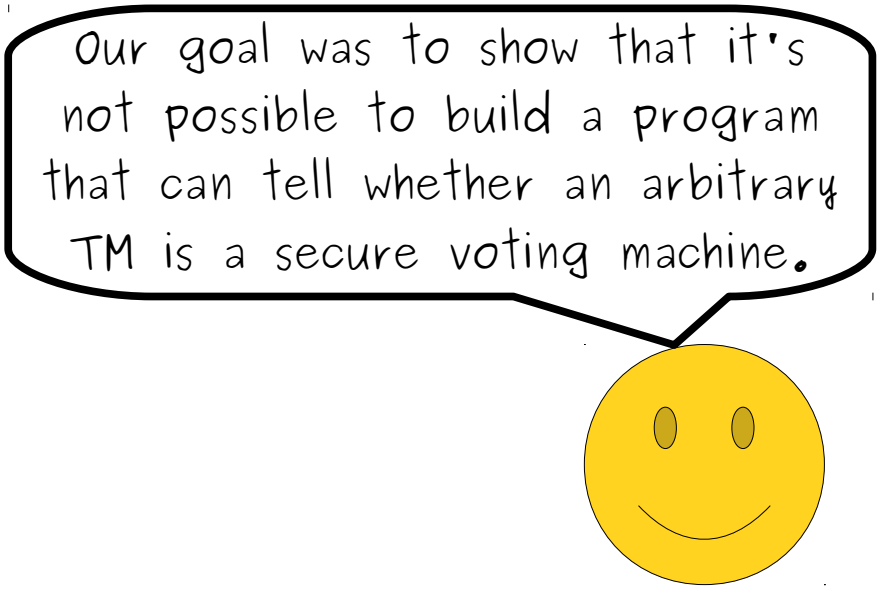

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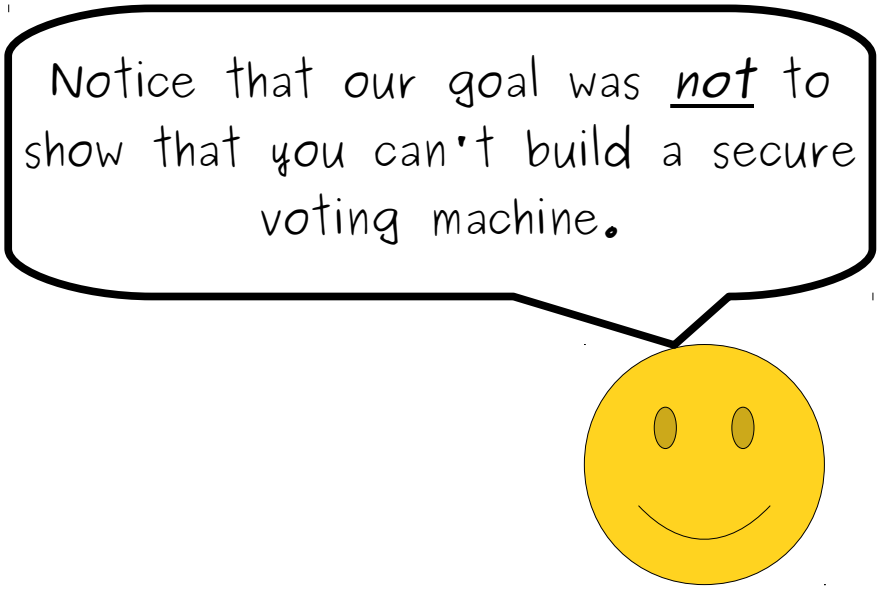

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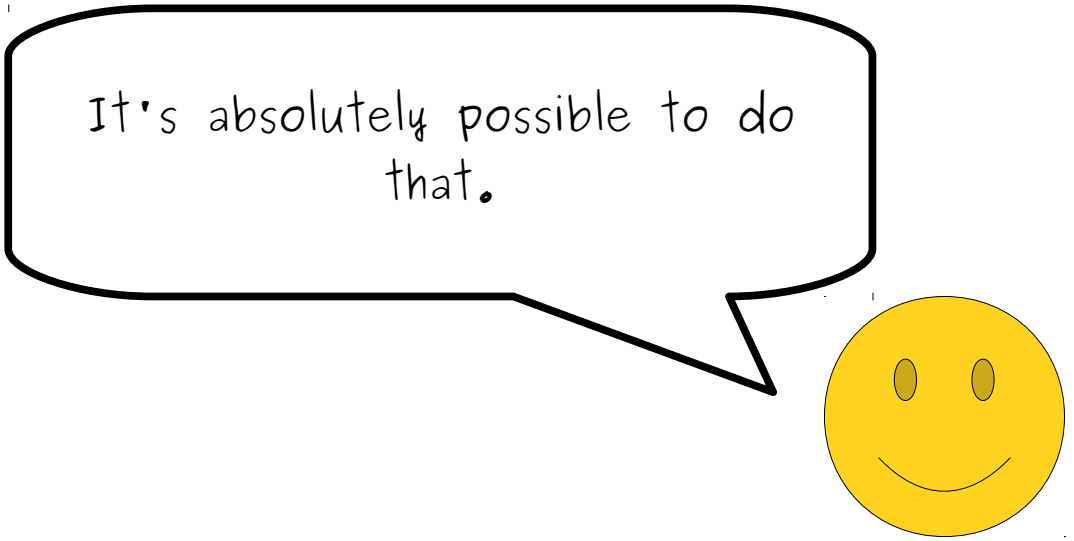


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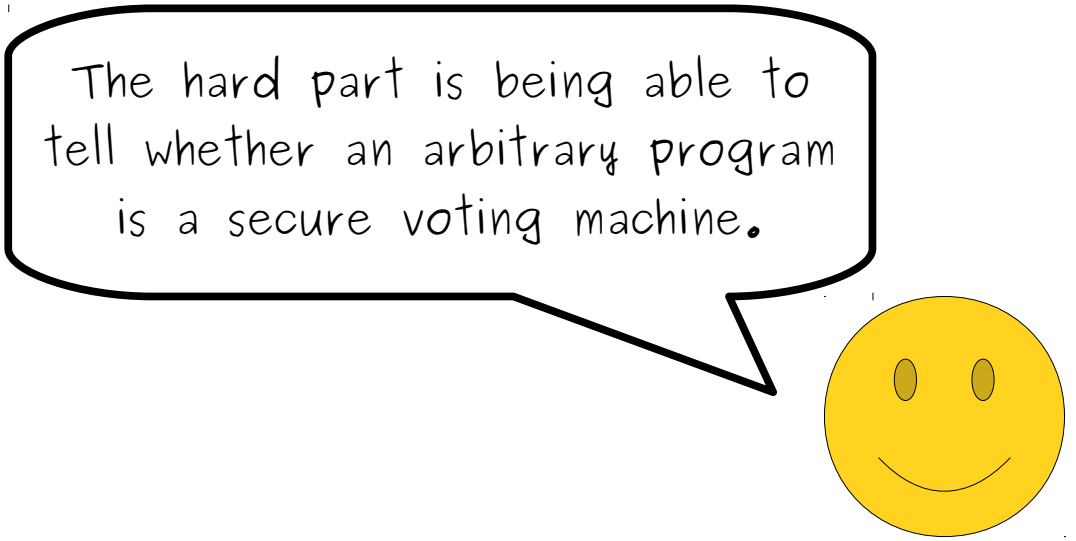


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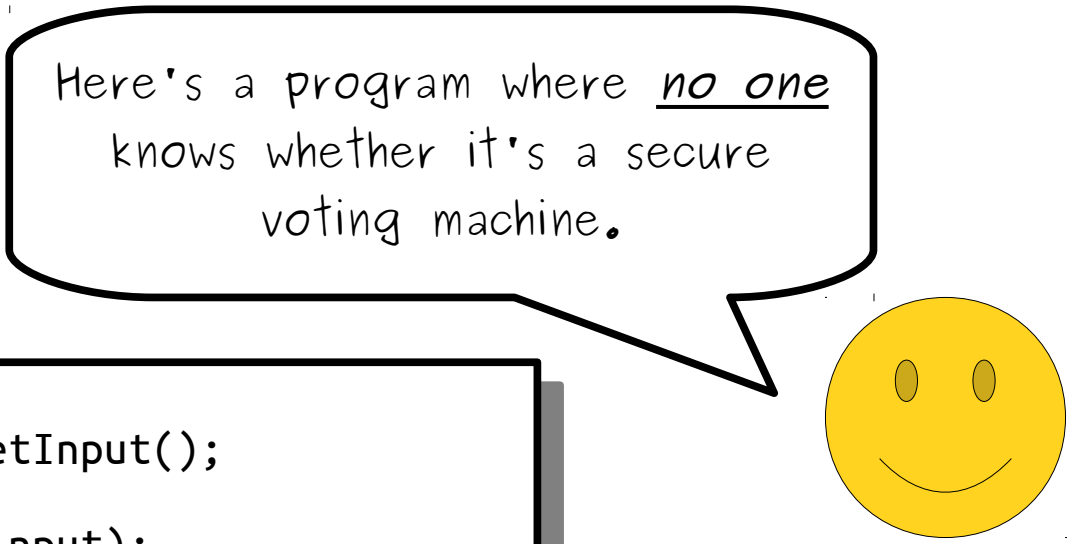


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  string input = getInput();

  int n = countRs(input);
  while (n > 1) {
    if (n % 2 == 0) n = n / 2;
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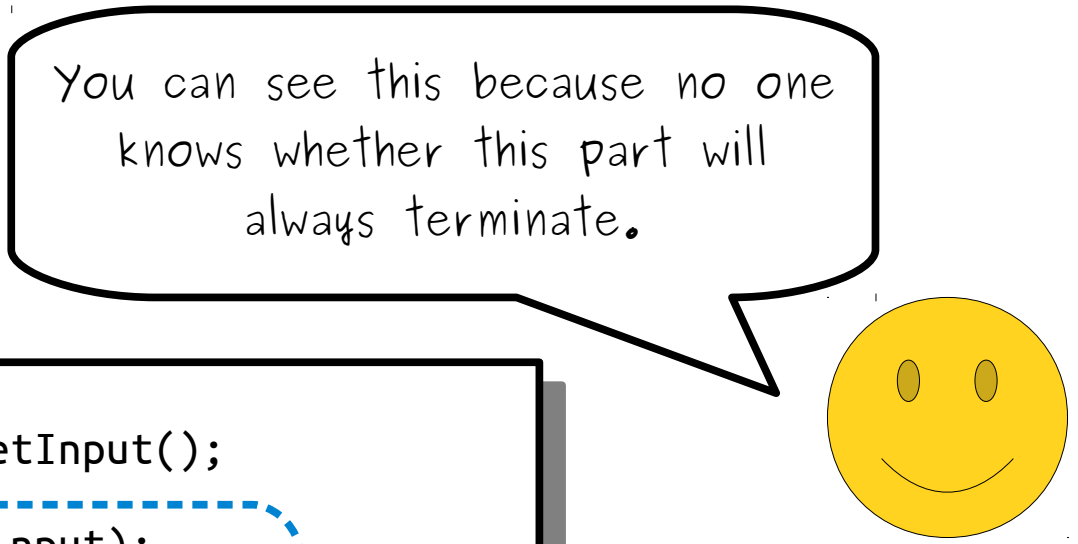


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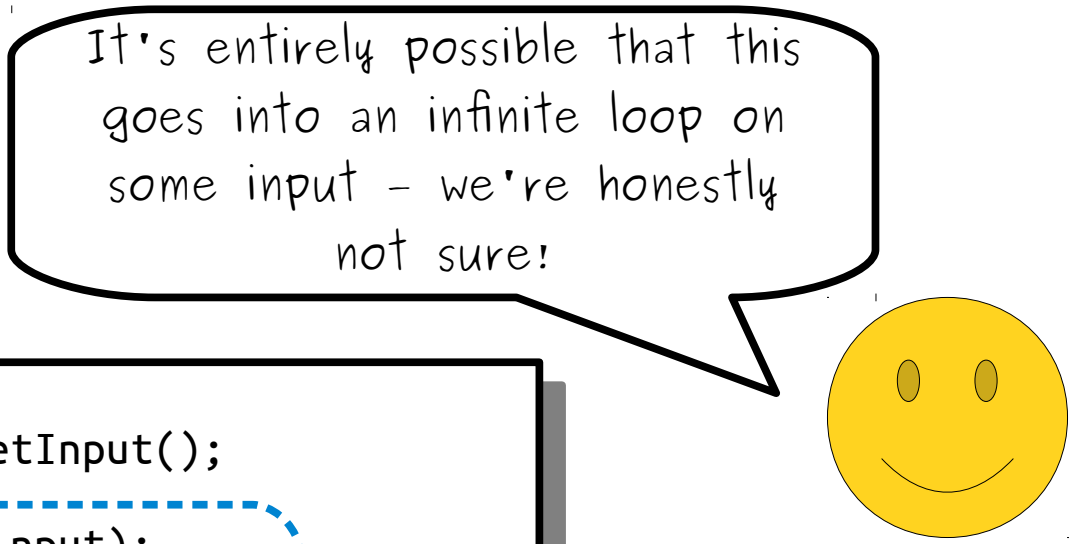


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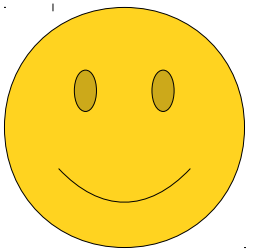
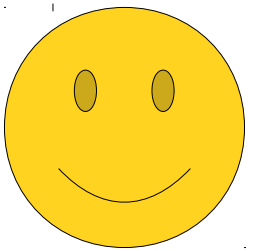
So, to recap:
Building a secure voting machine isn't hard.
Checking whether an arbitrary program is a secure voting machine is really hard.

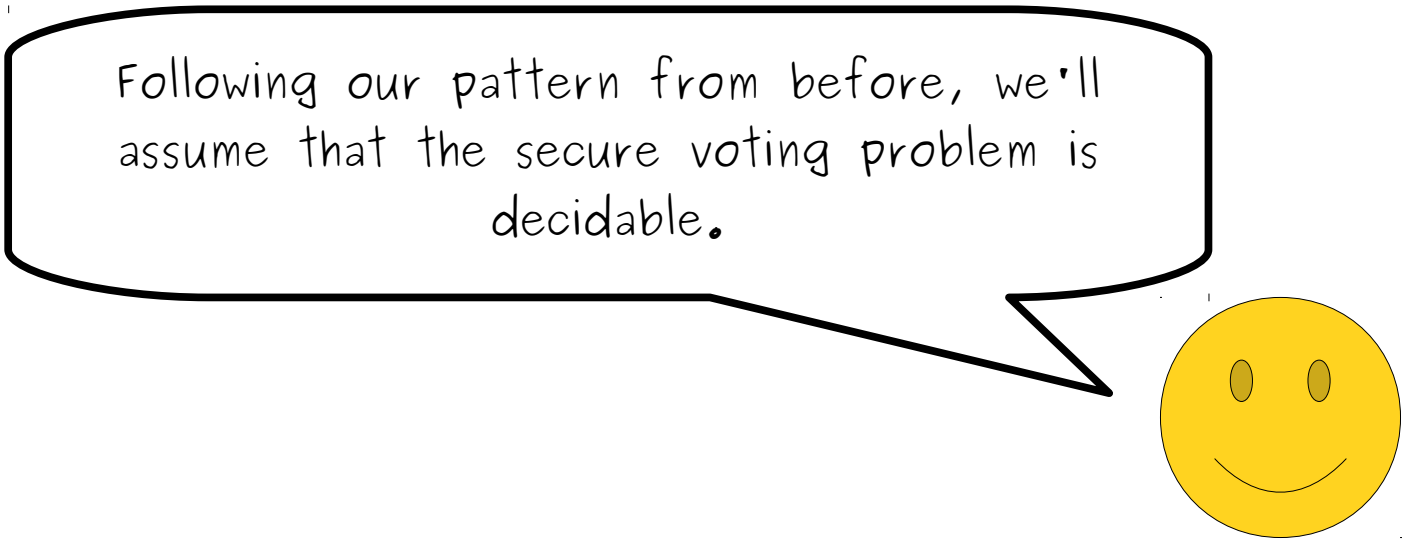


```
int main() {
  string input = getInput();

  int n = countRs(input);
  while (n > 1) {
    if (n % 2 == 0) n = n / 2;
    else n = 3*n + 1;
  }

  if (countRs(input) > countDs(input)) {
    accept();
  } else {
    reject();
  }
}
```

Our goal is to show that the secure voting problem – the problem of checking whether a program is a secure voting machine – is undecidable.



The secure voting problem is decidable.



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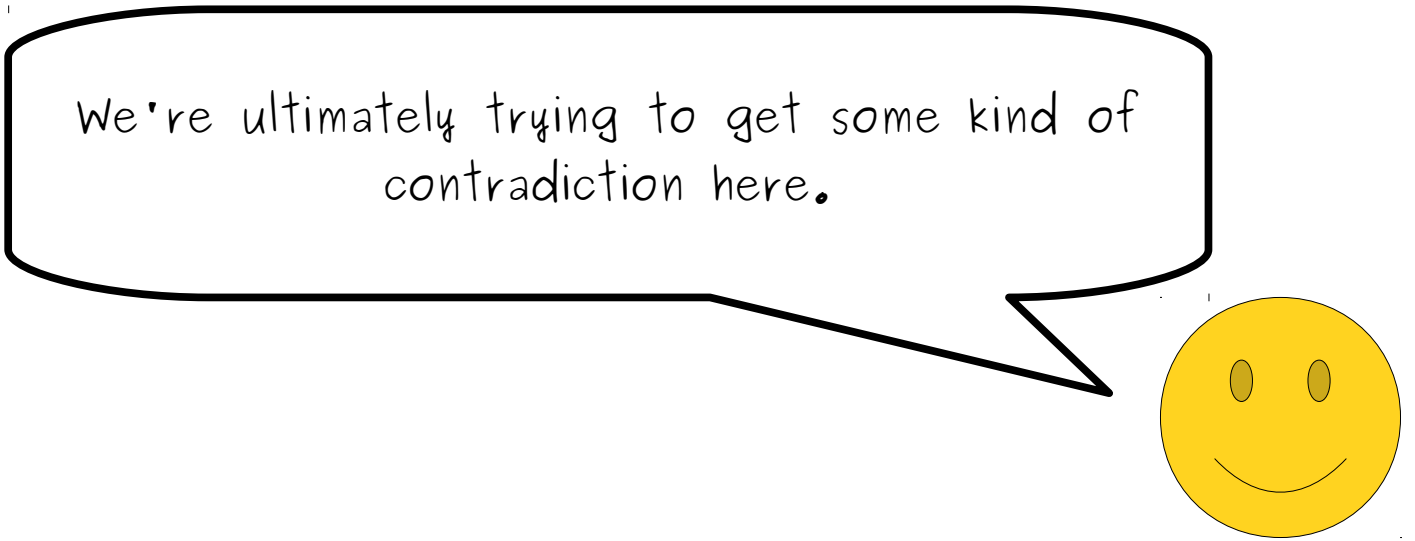

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The secure voting problem is decidable.

There is a decider D for the secure voting problem

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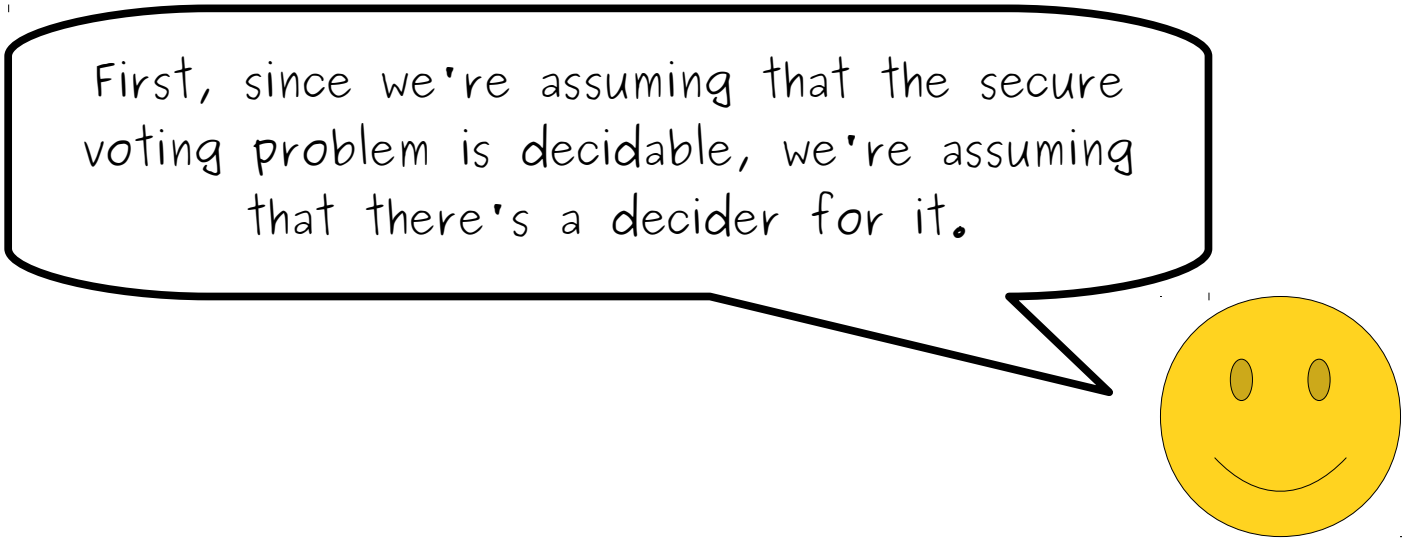

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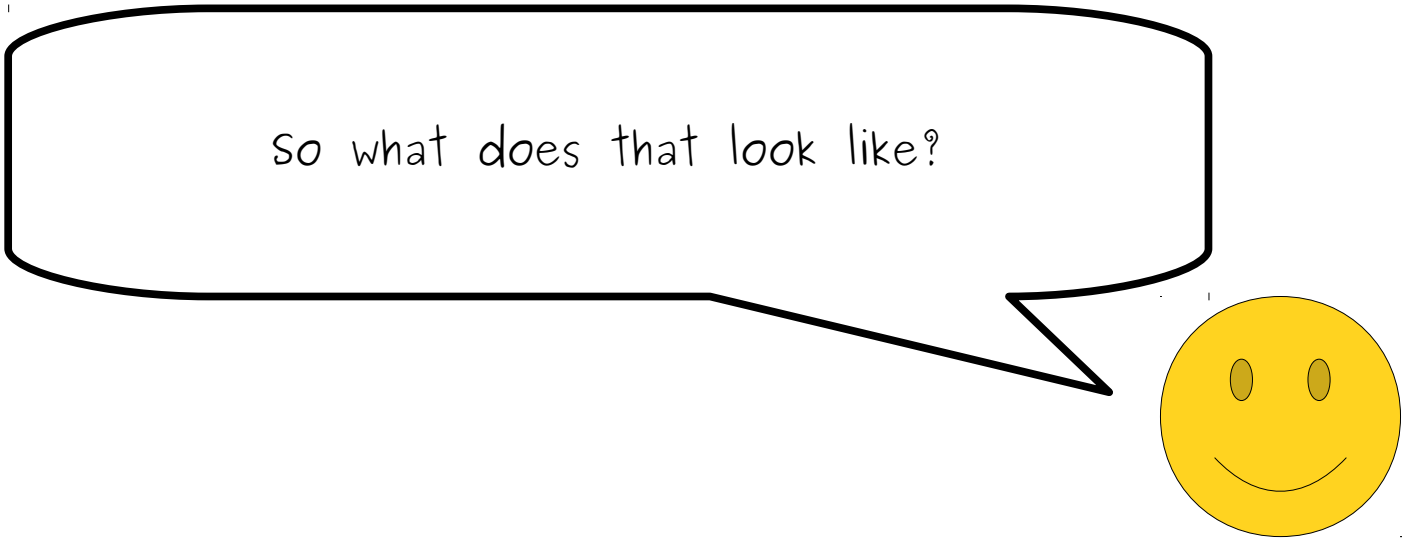

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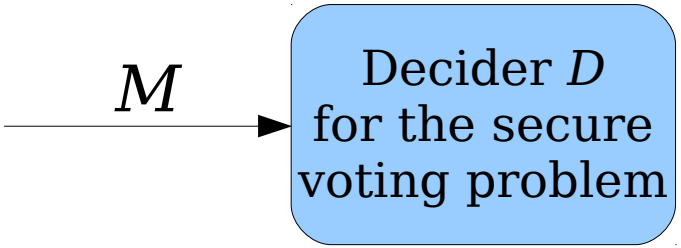

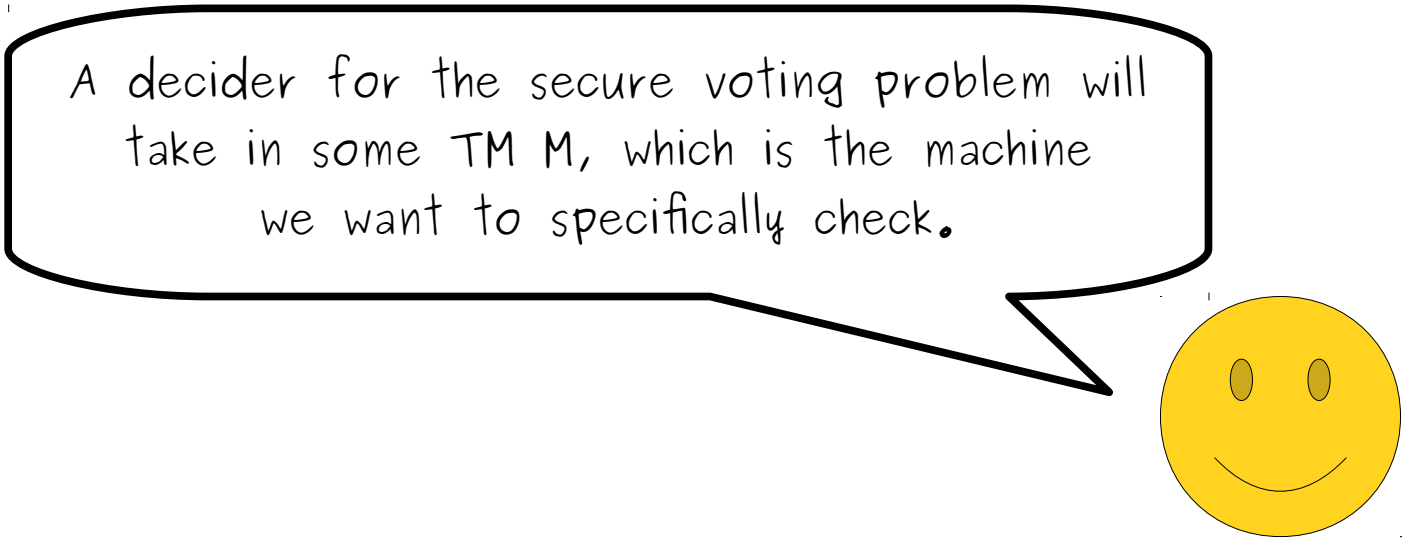

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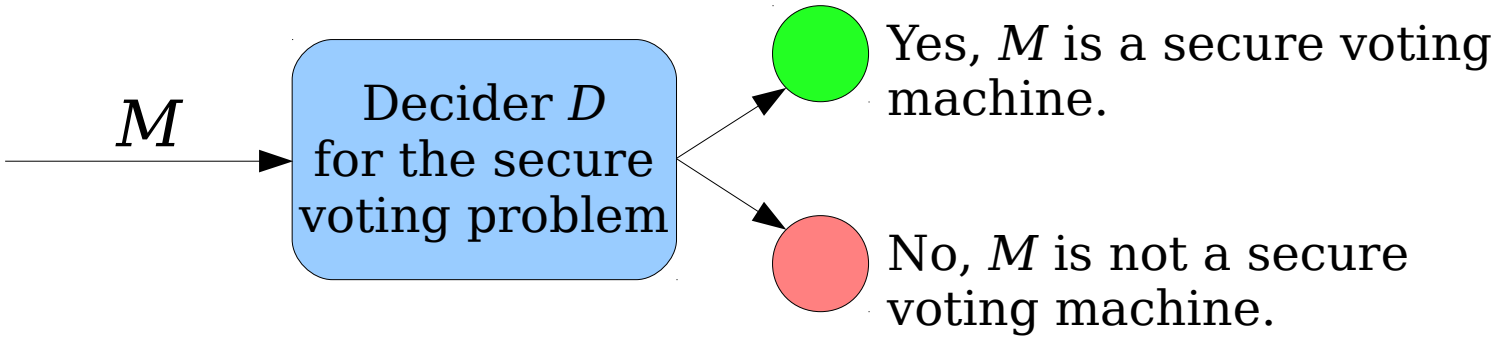

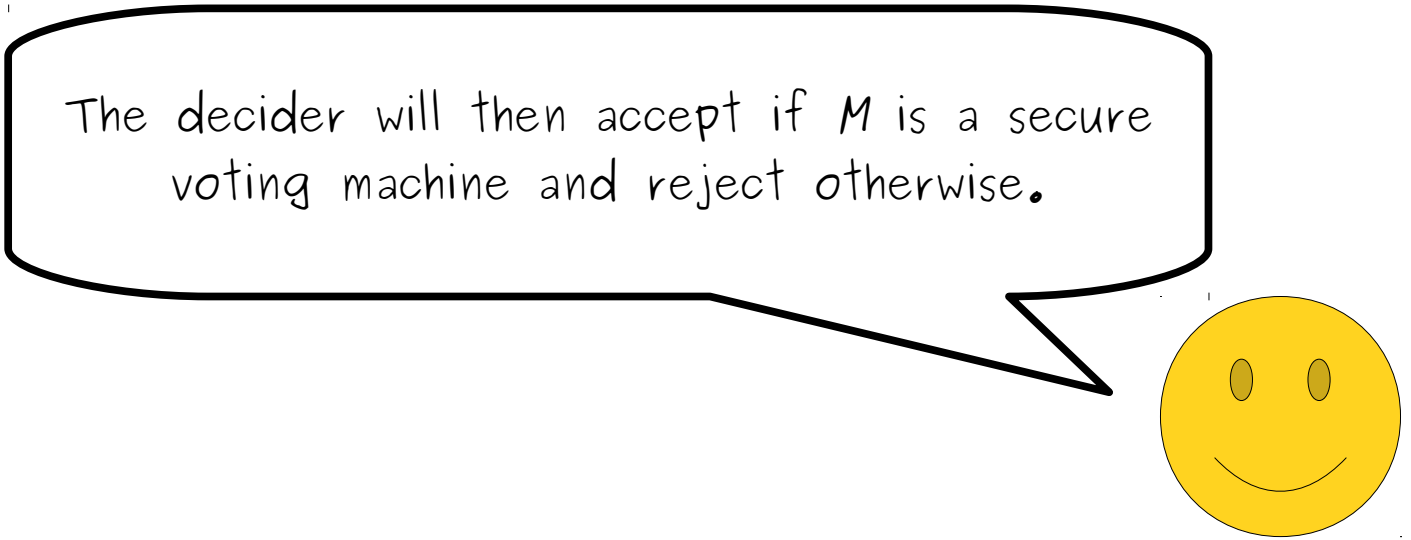

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We can write programs that use D as a helper method

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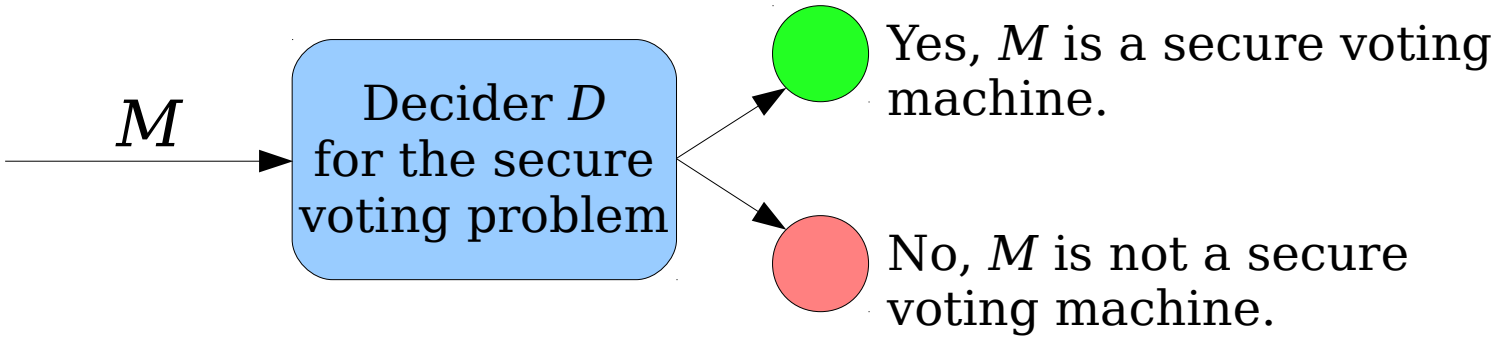

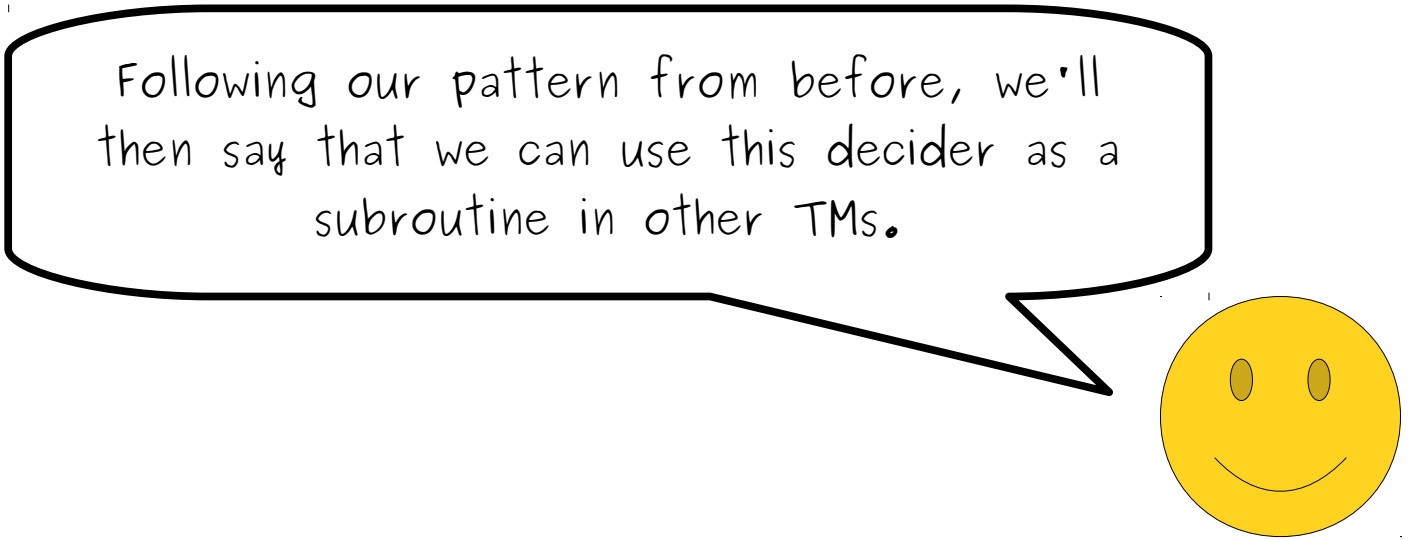

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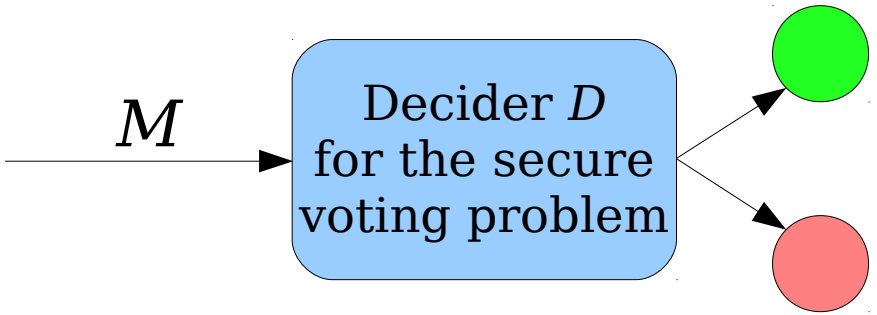


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bool isSecure(string program)
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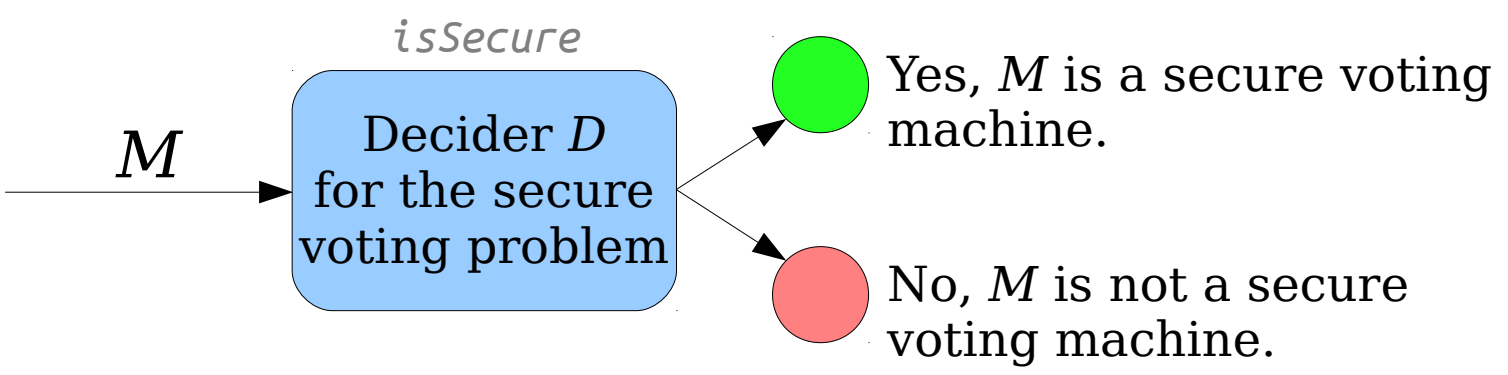


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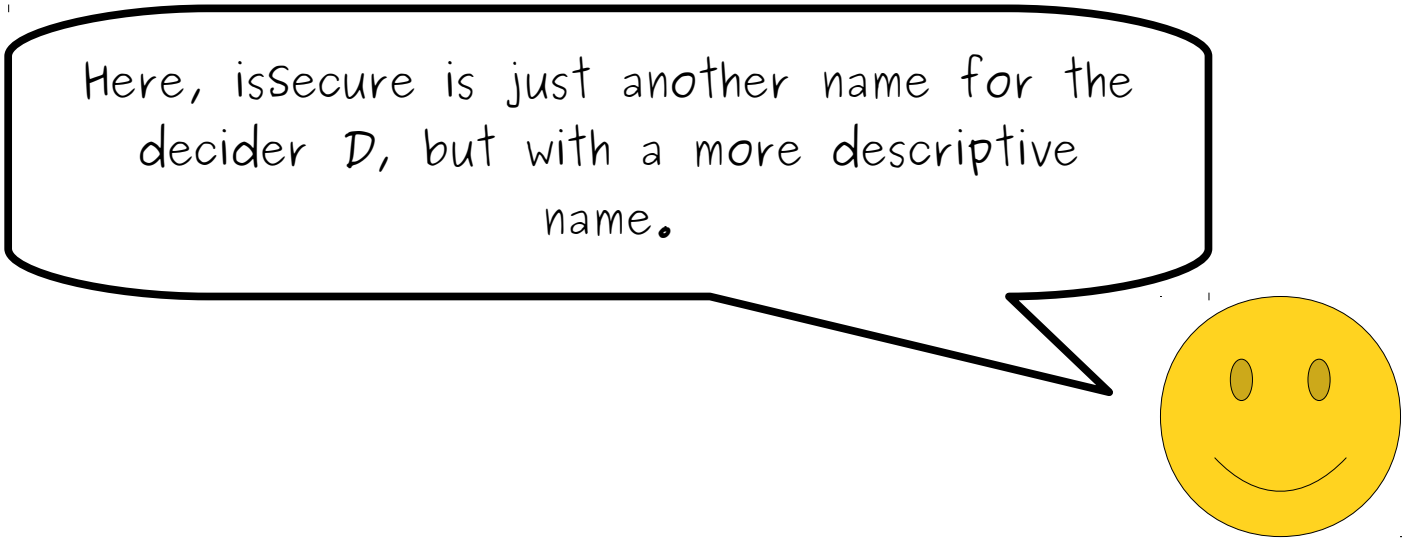

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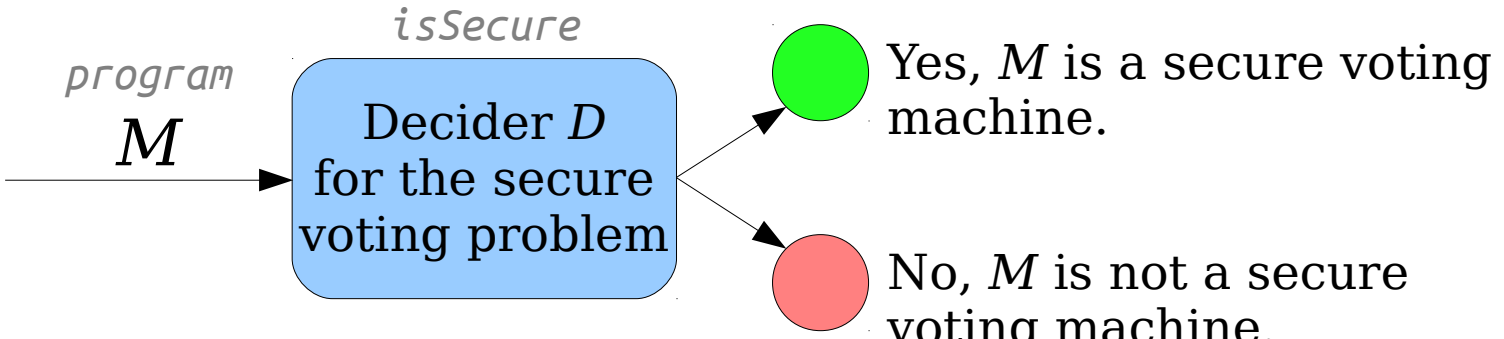


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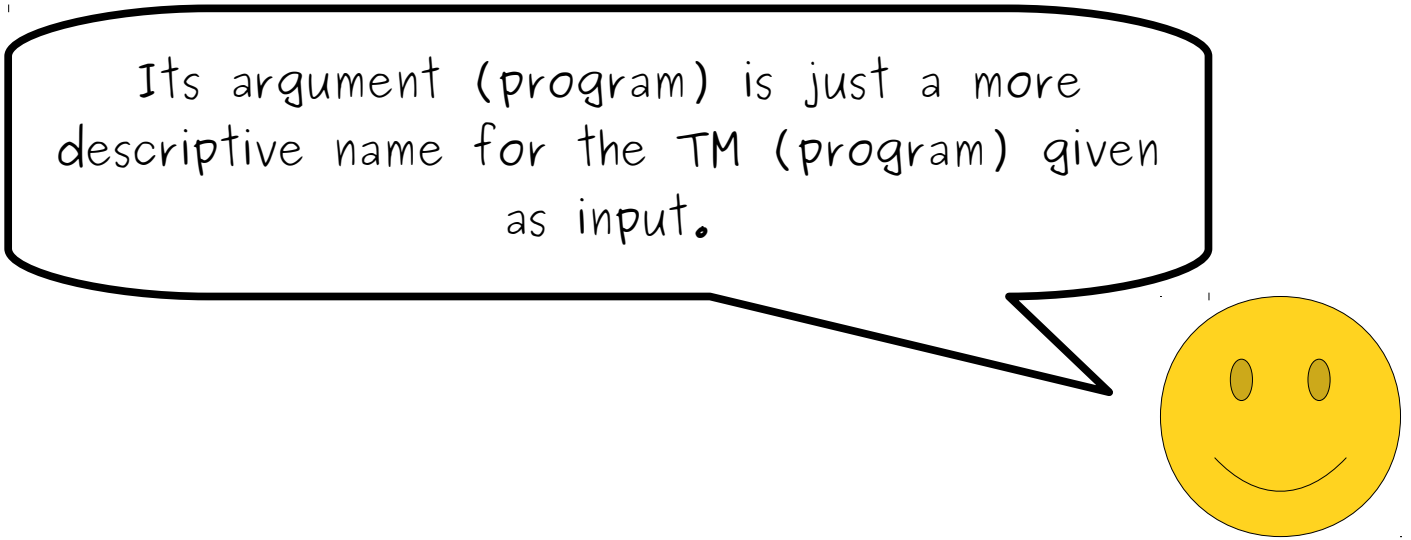

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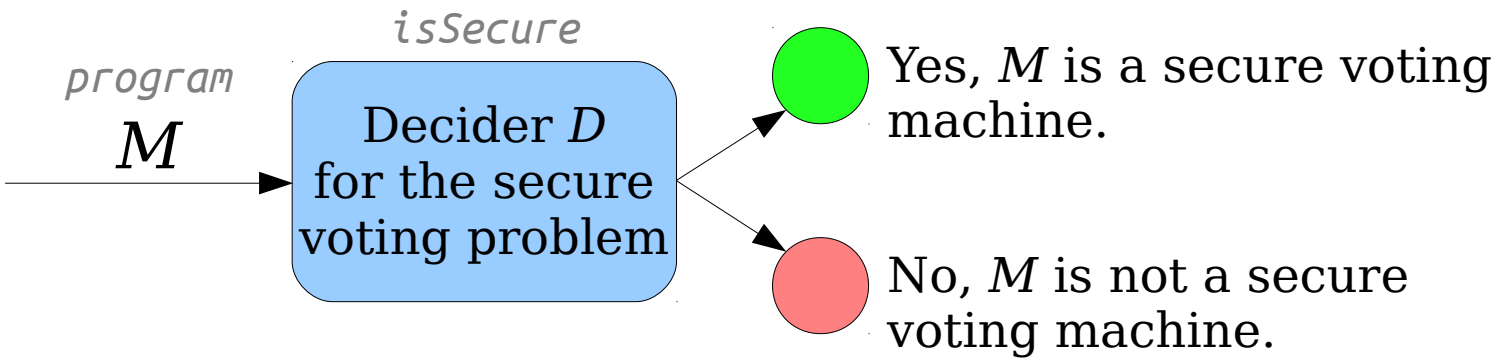


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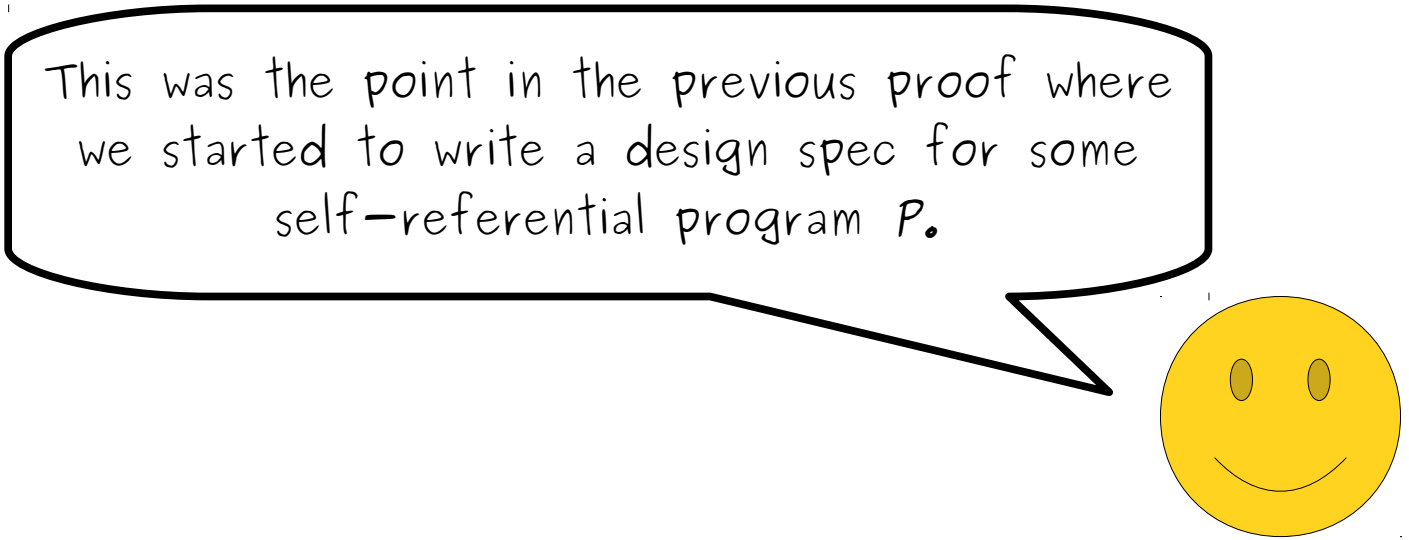

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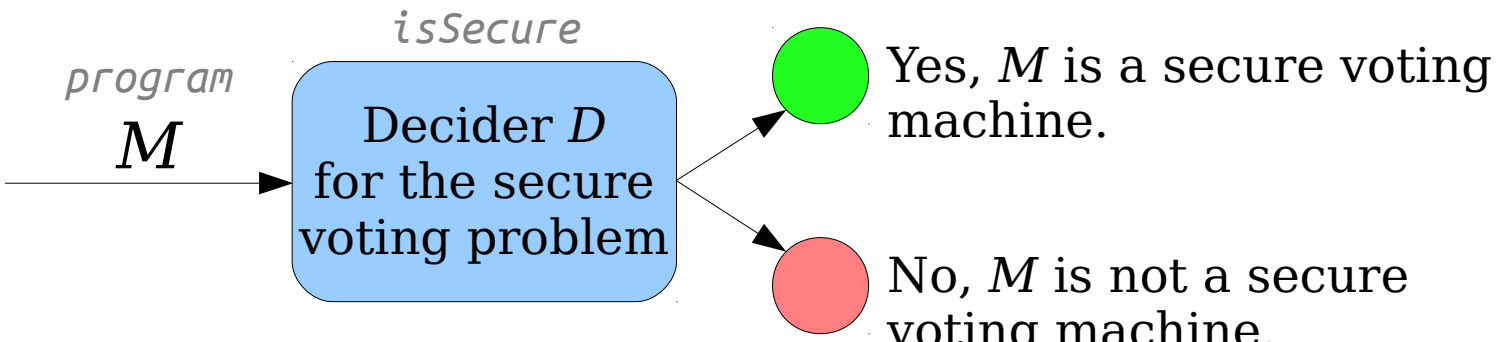


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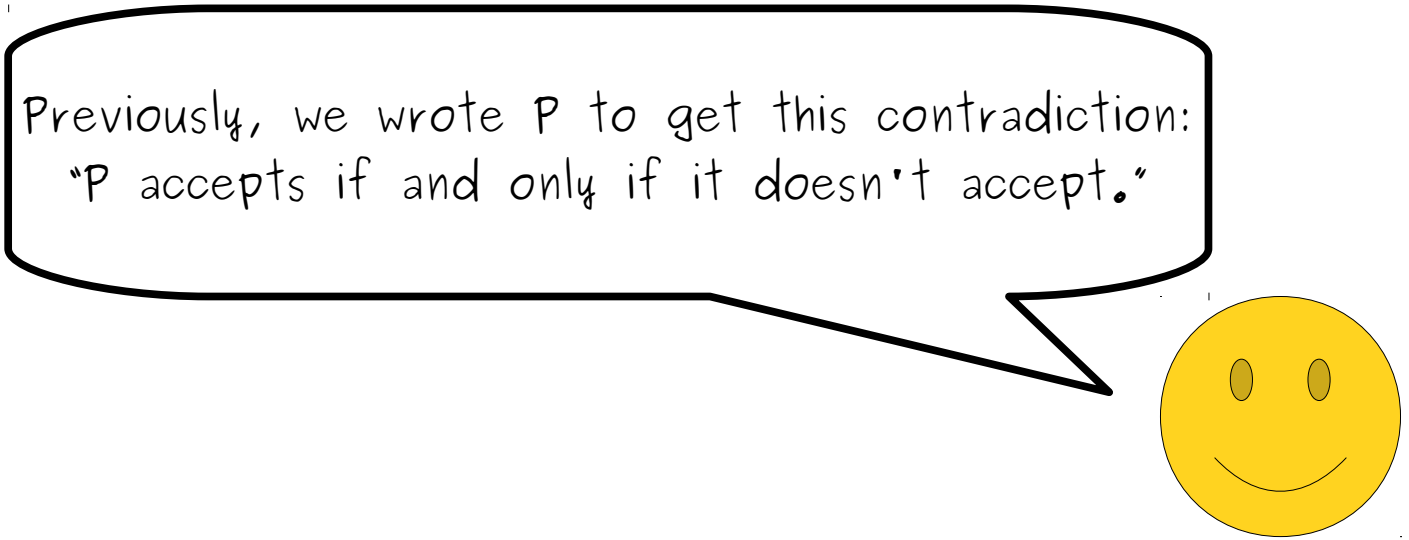

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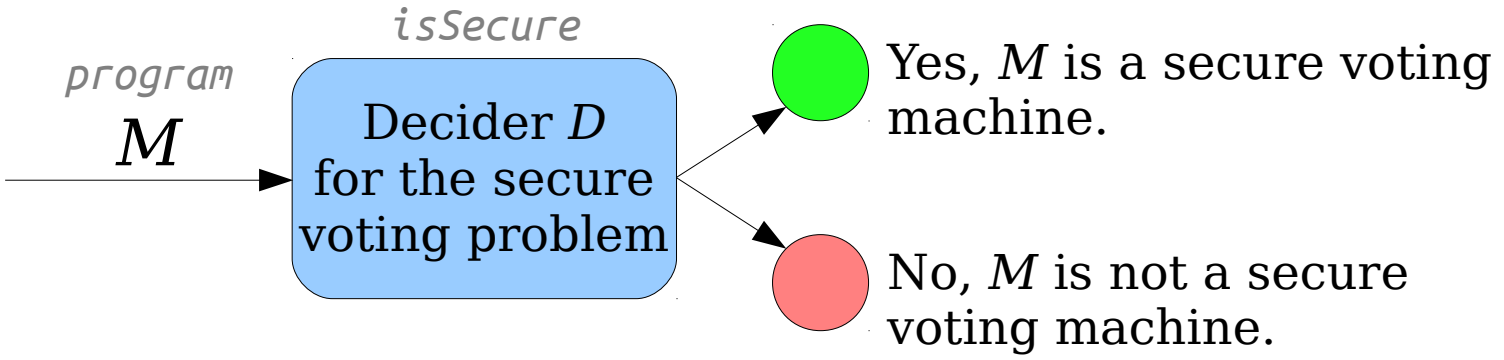


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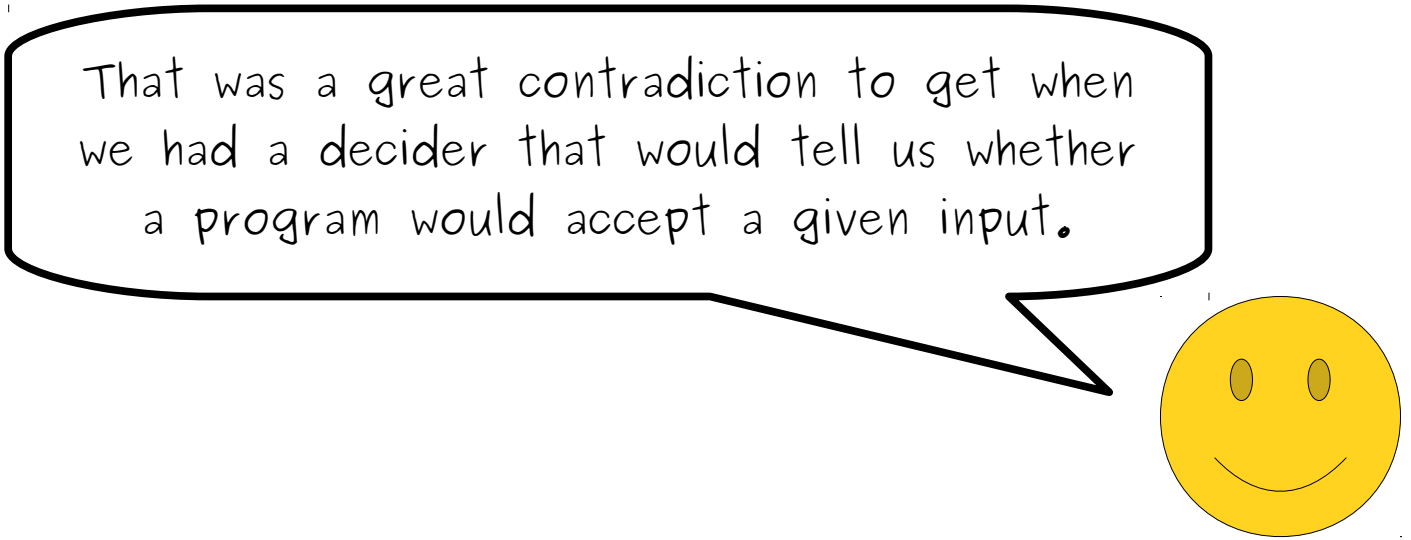

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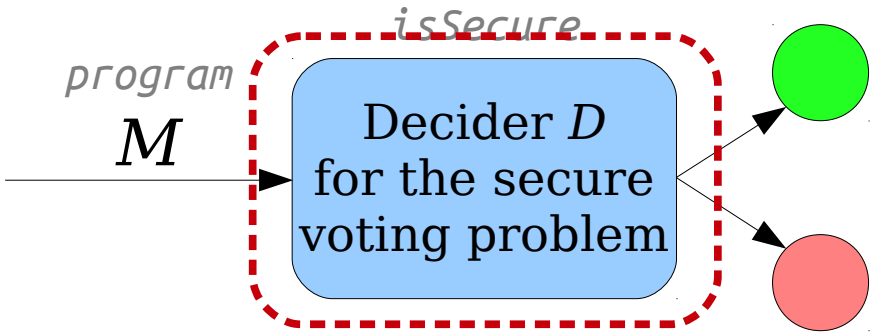


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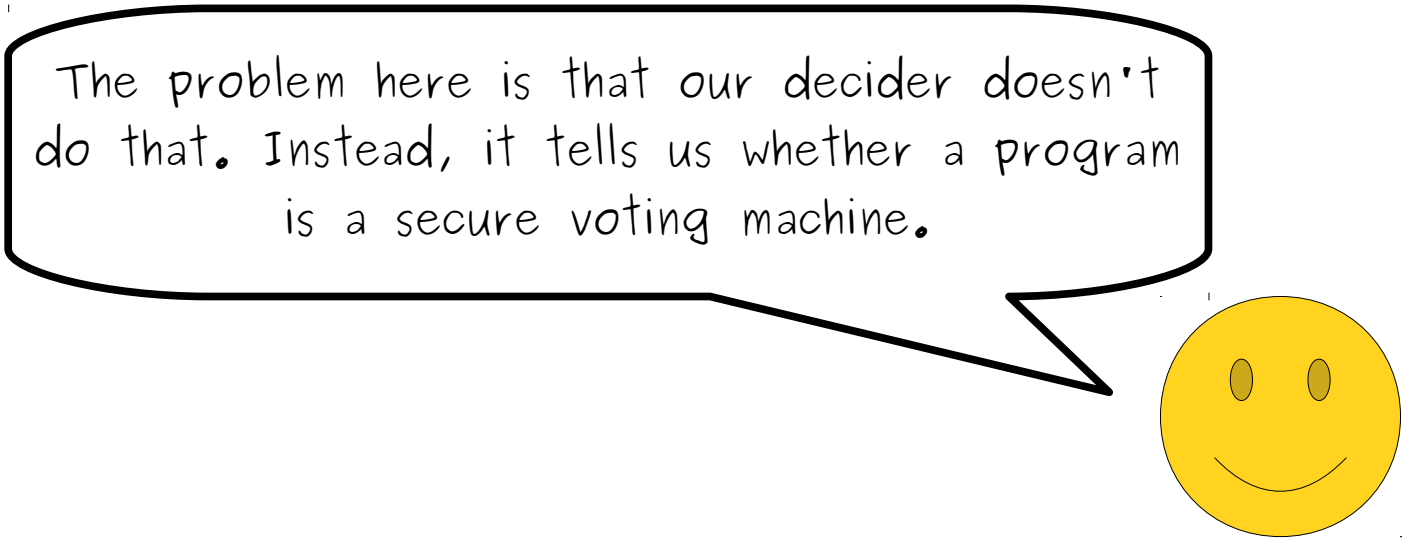

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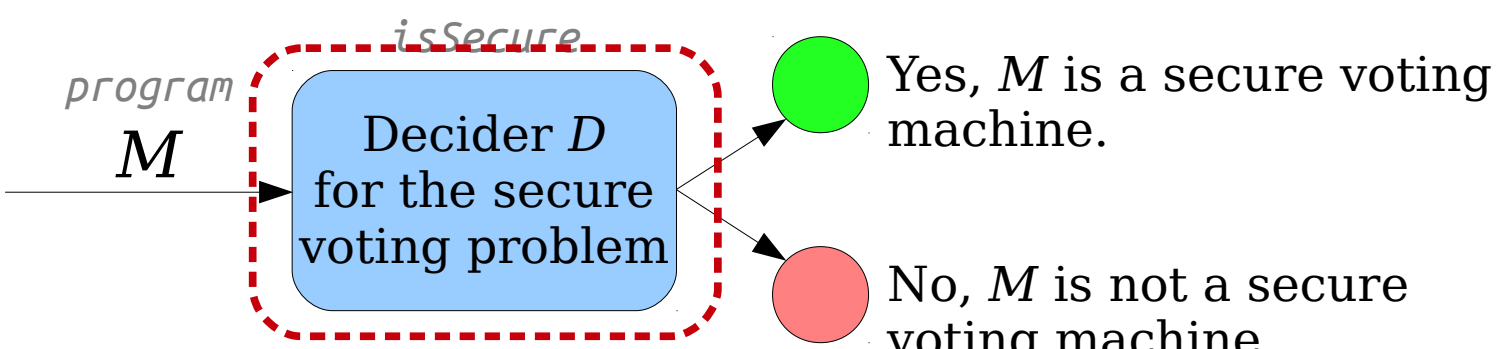


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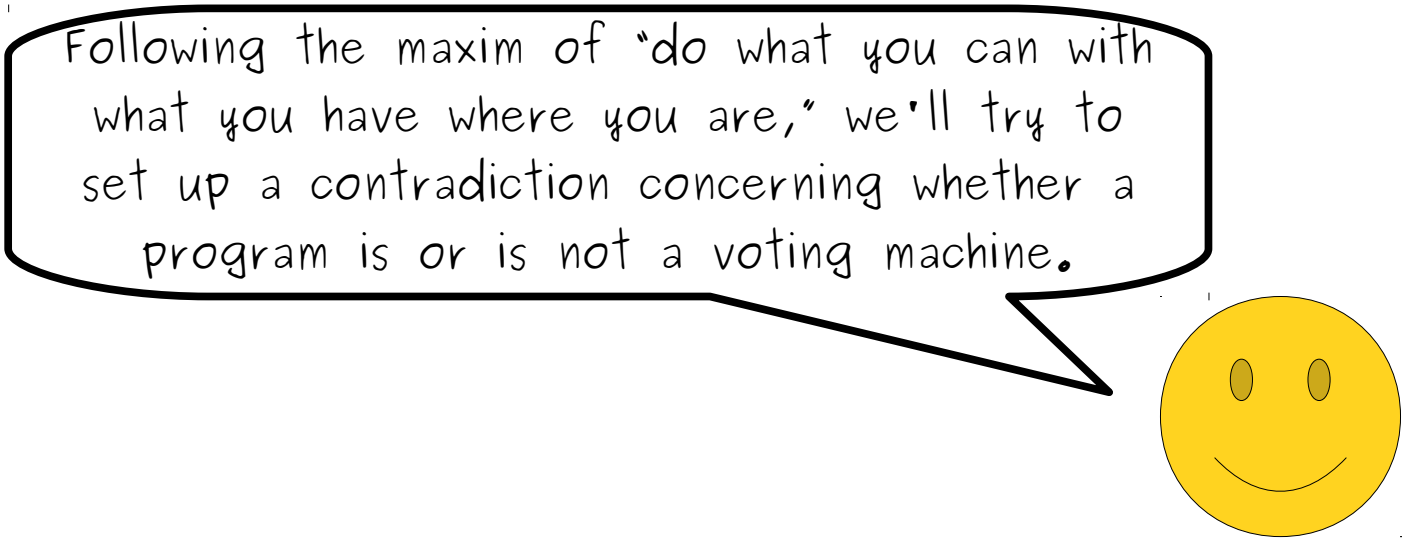

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Program P is secure if and only if program P is not secure.

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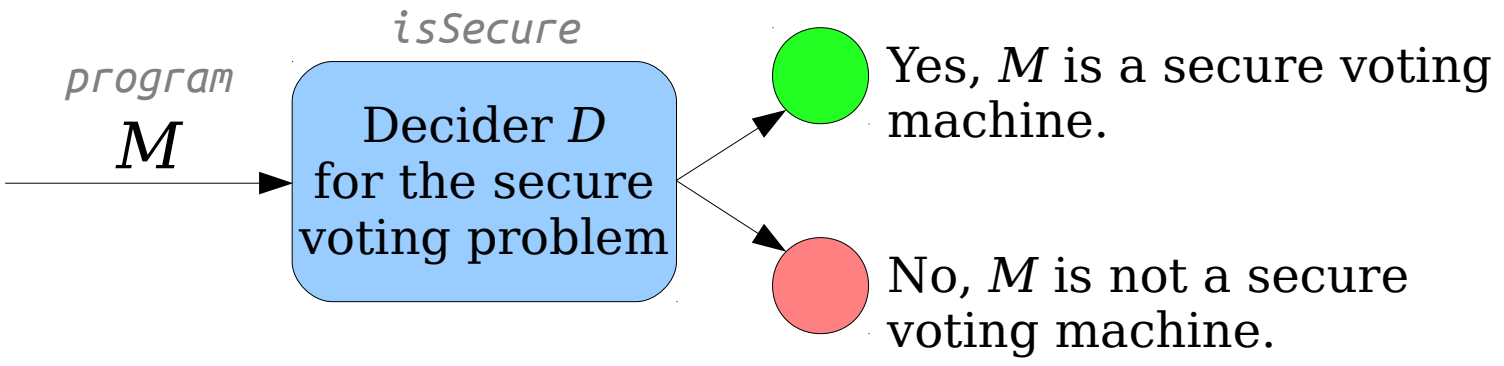


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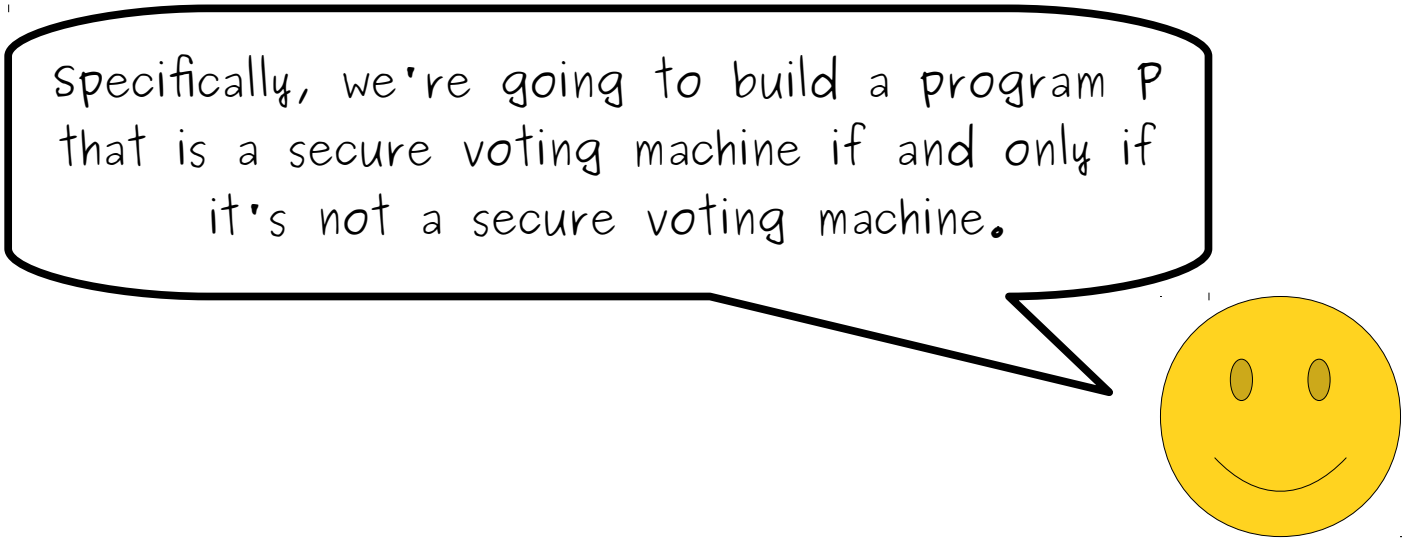

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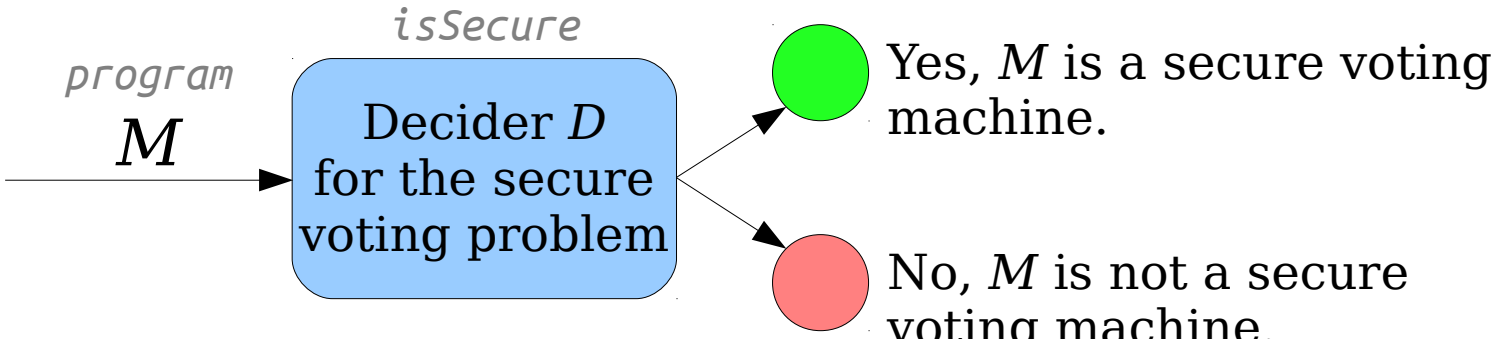


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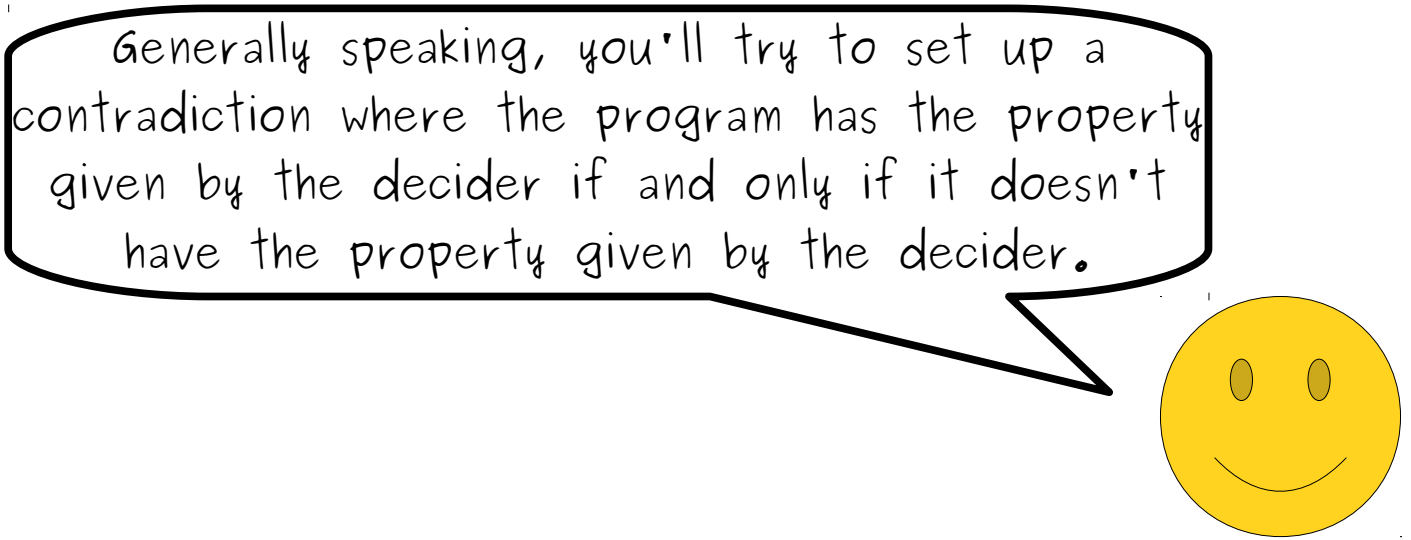

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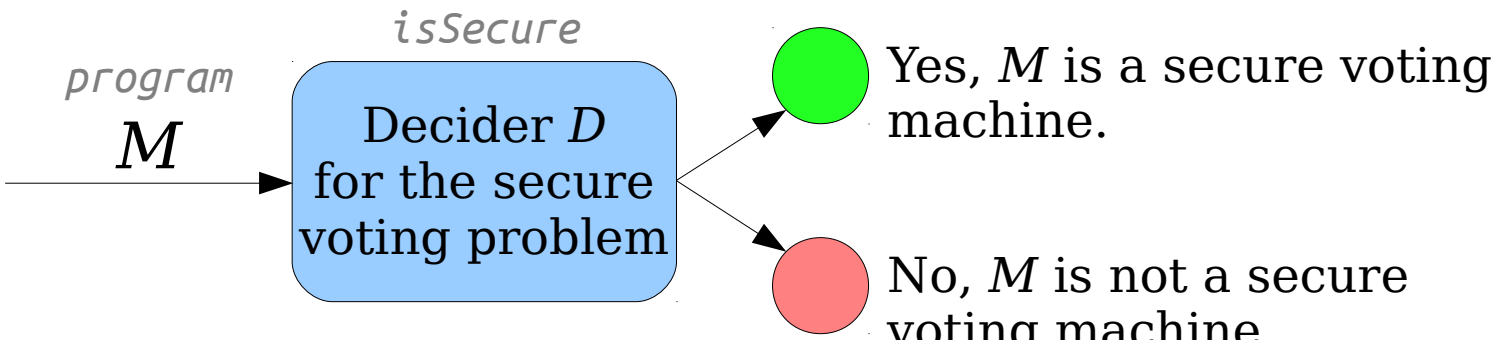


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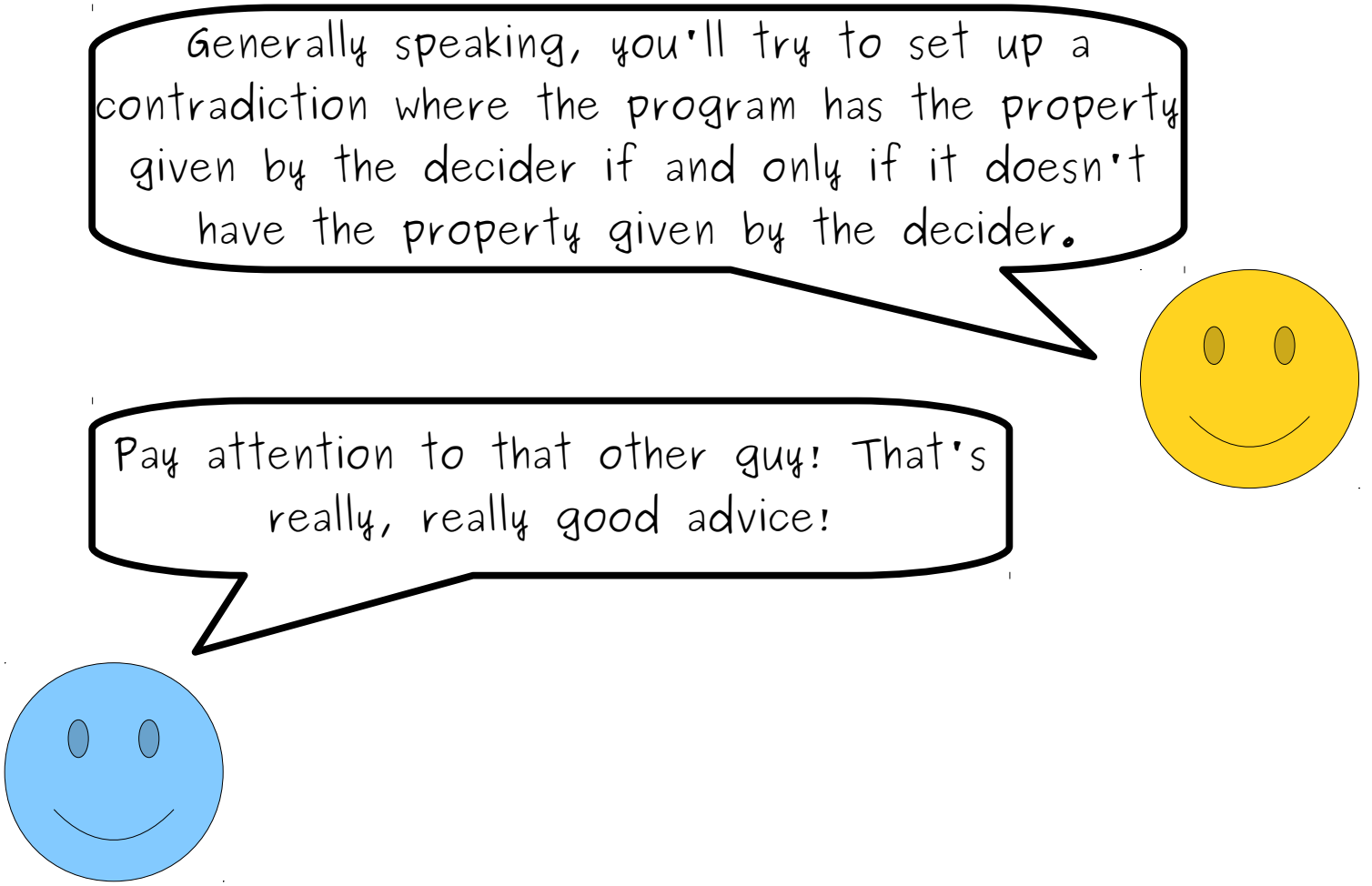

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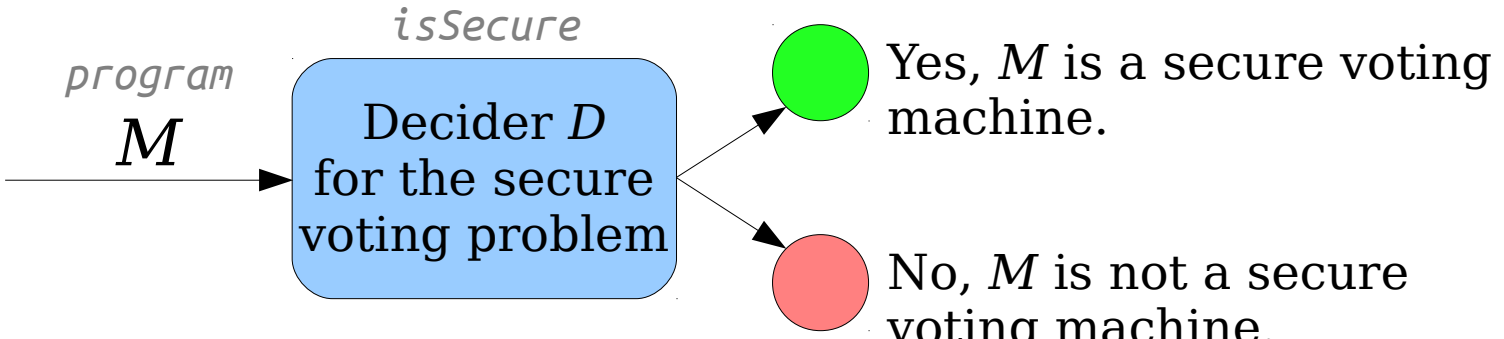


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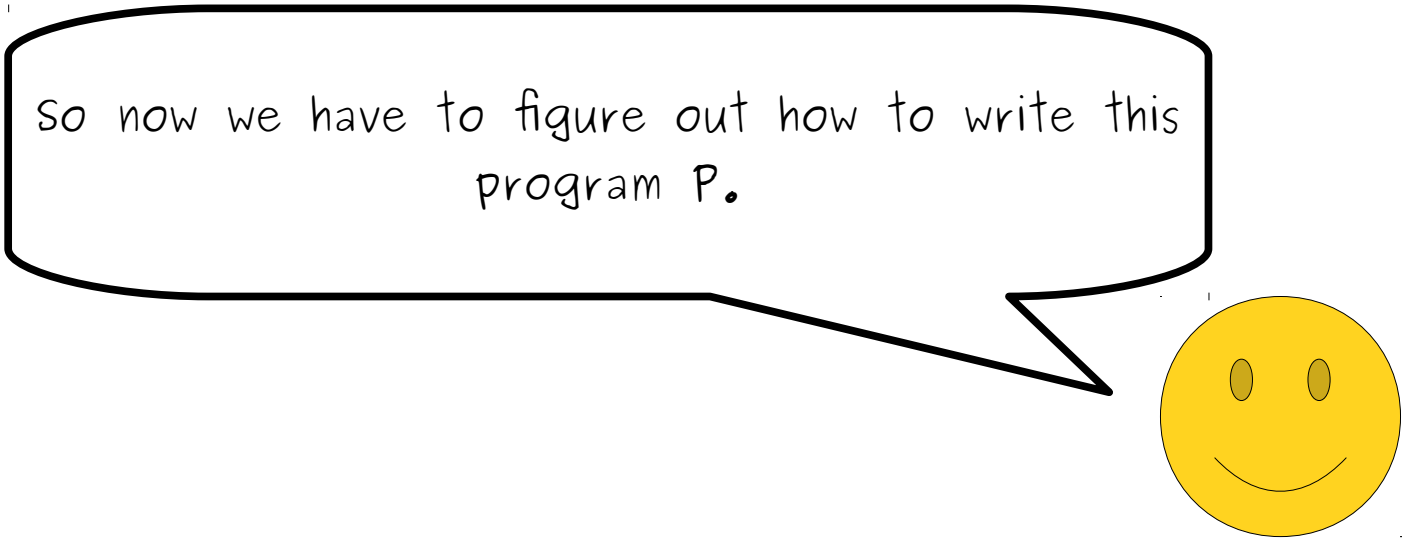

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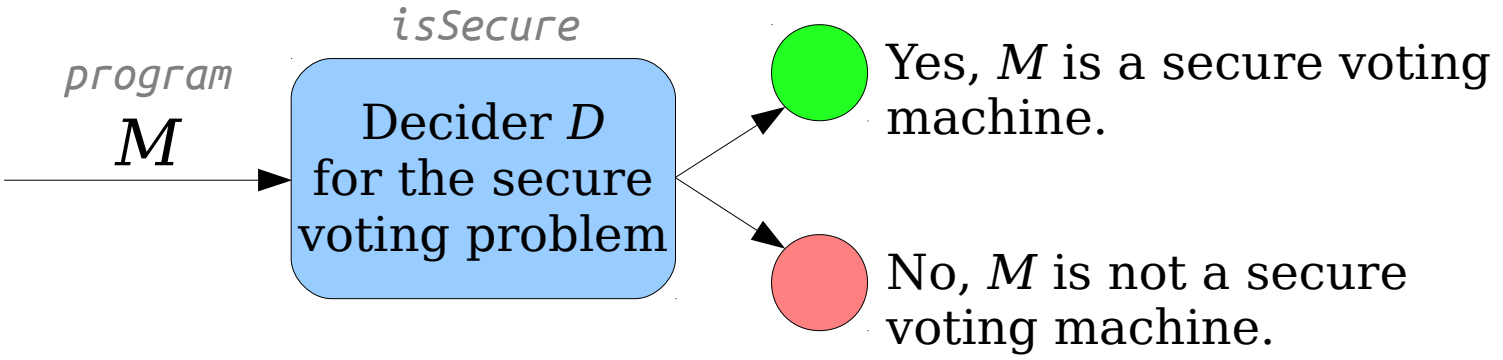


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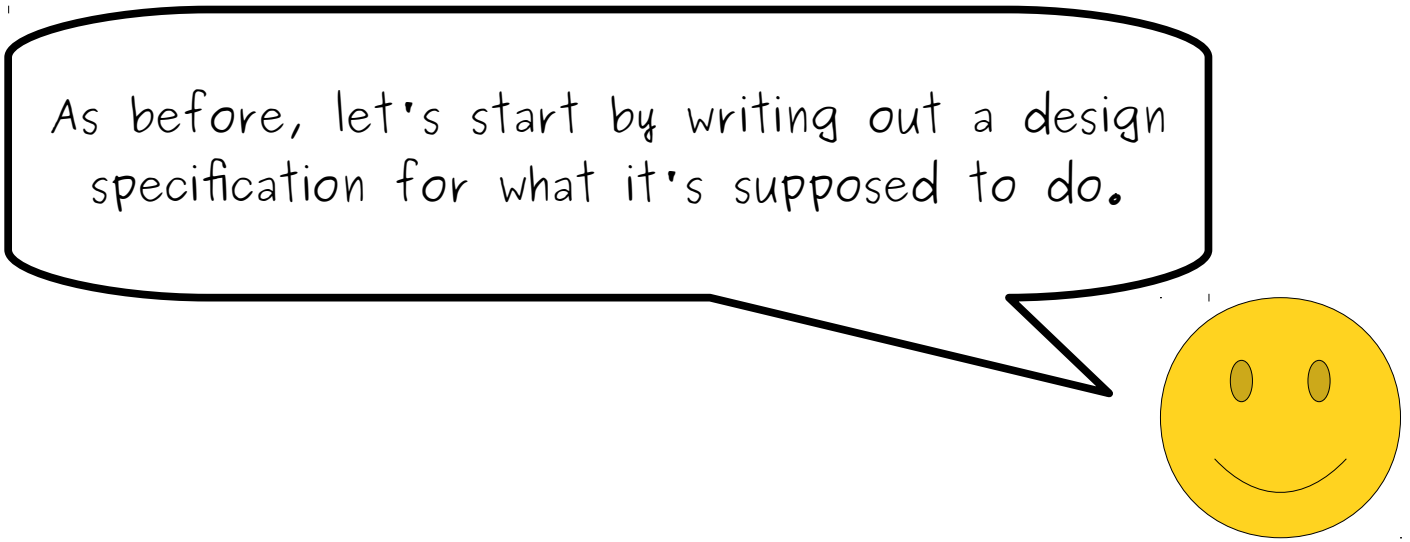

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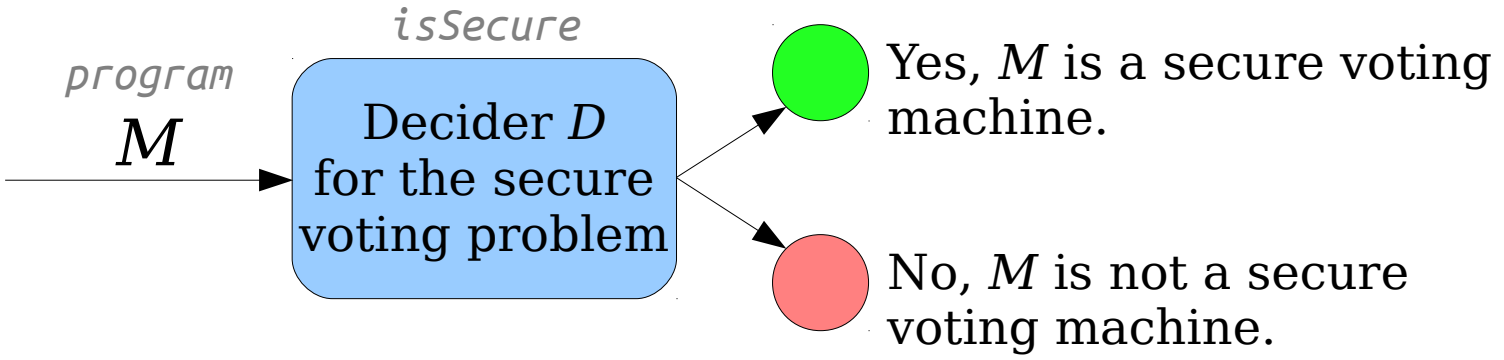


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Program P design specification:

If P is a secure voting machine, then
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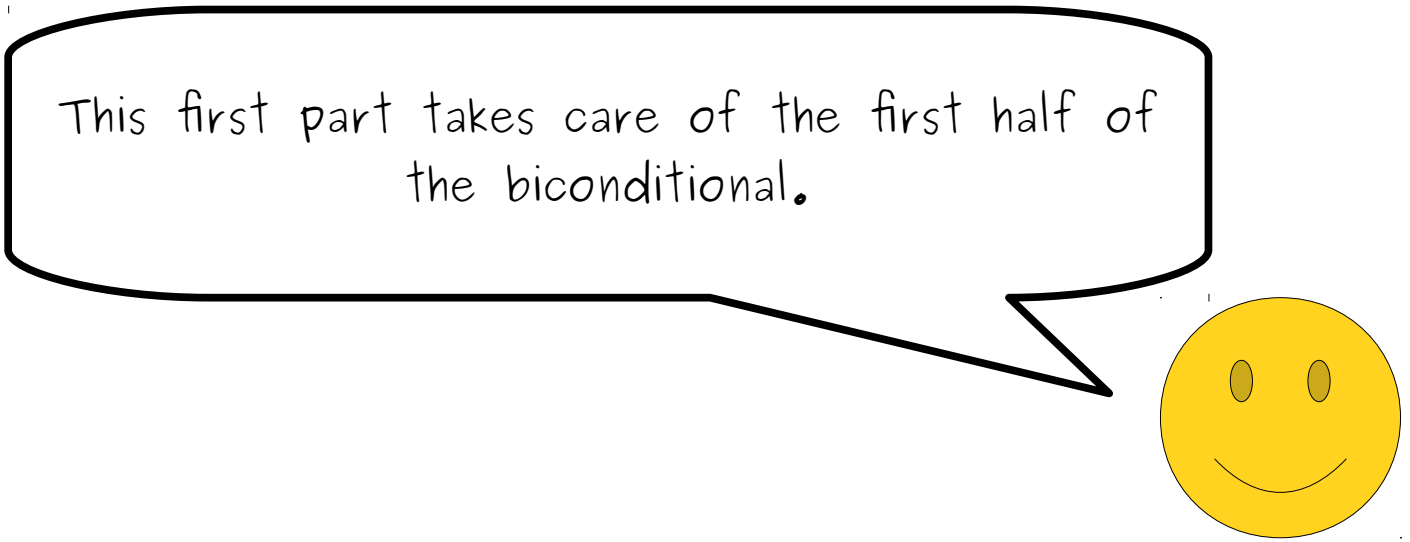

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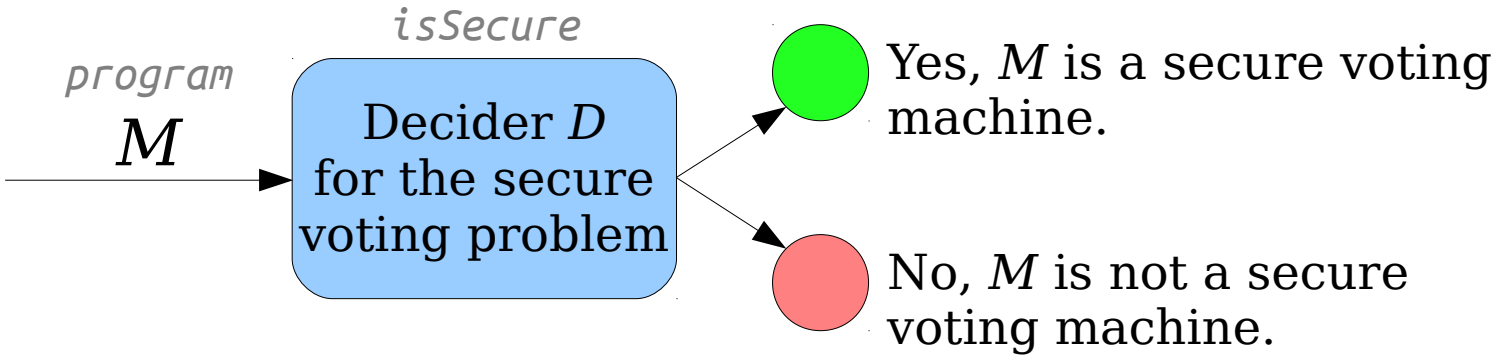


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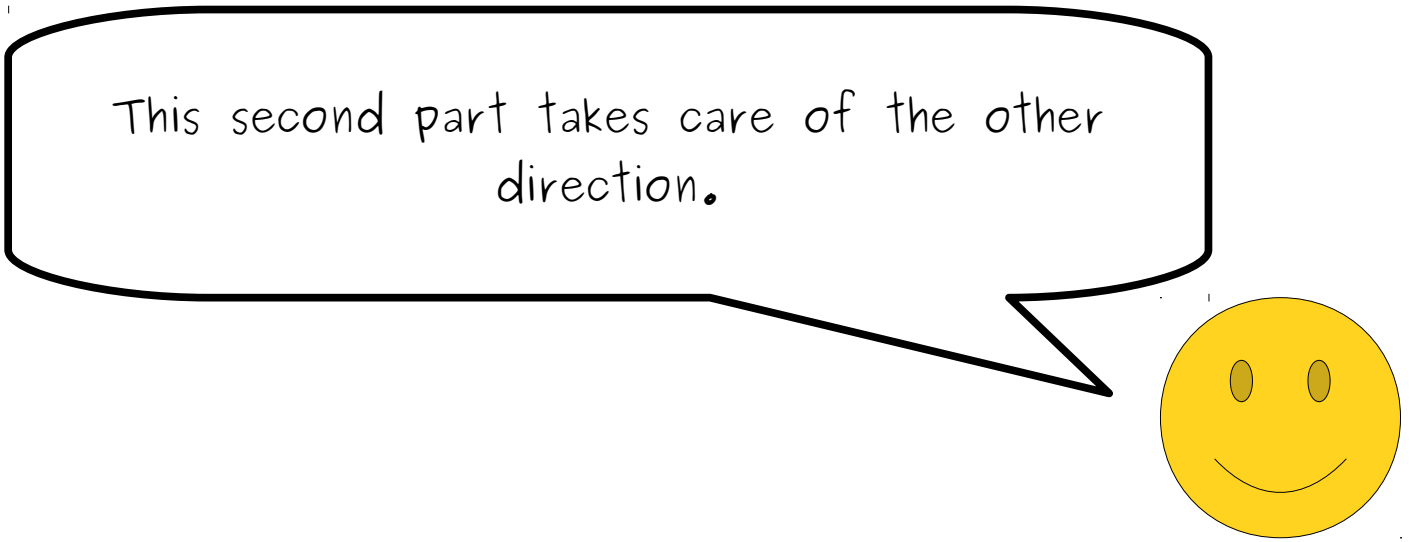

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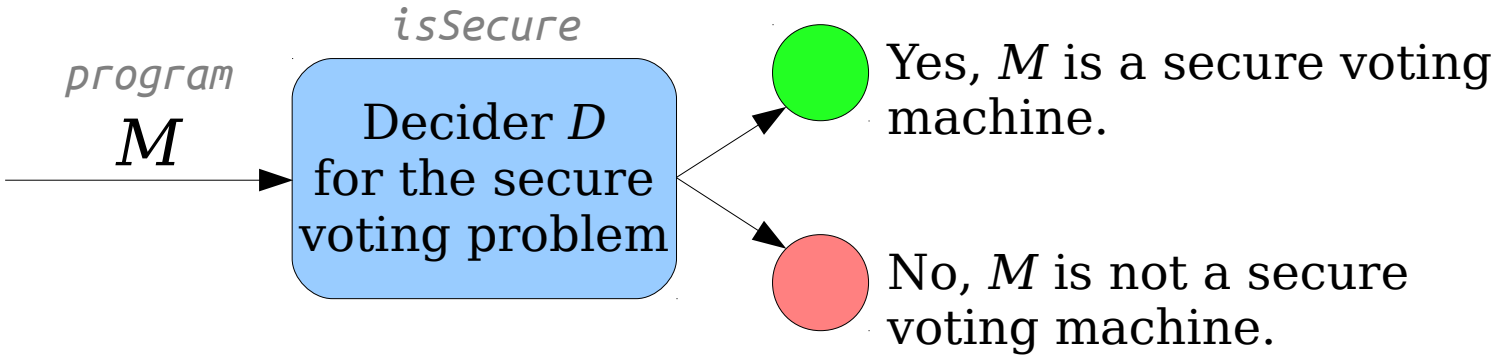


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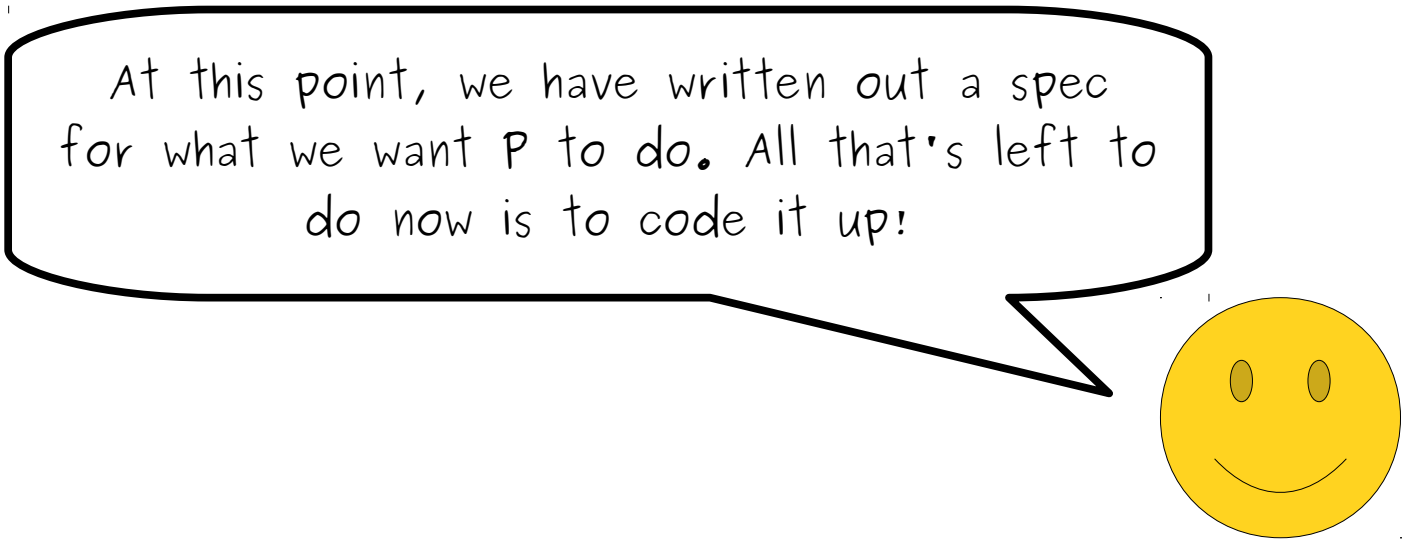

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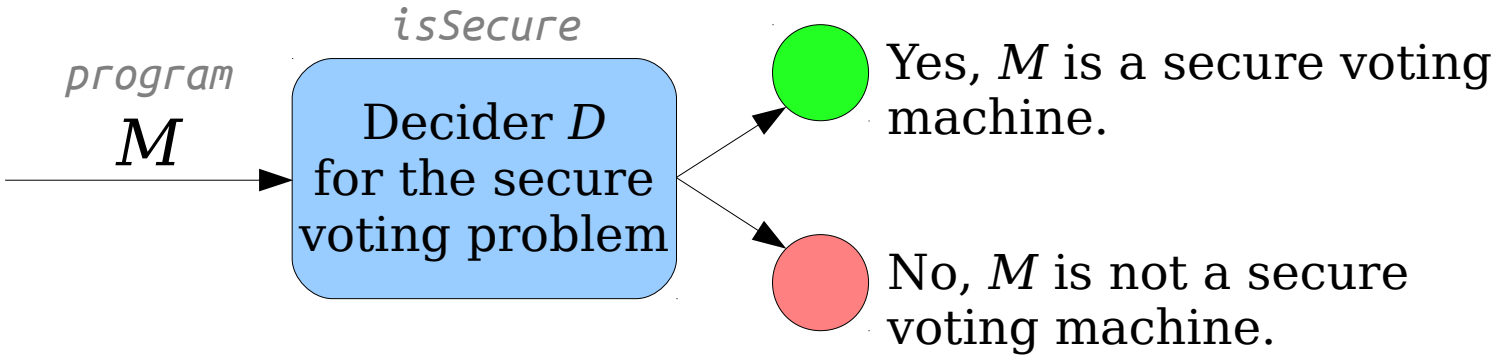


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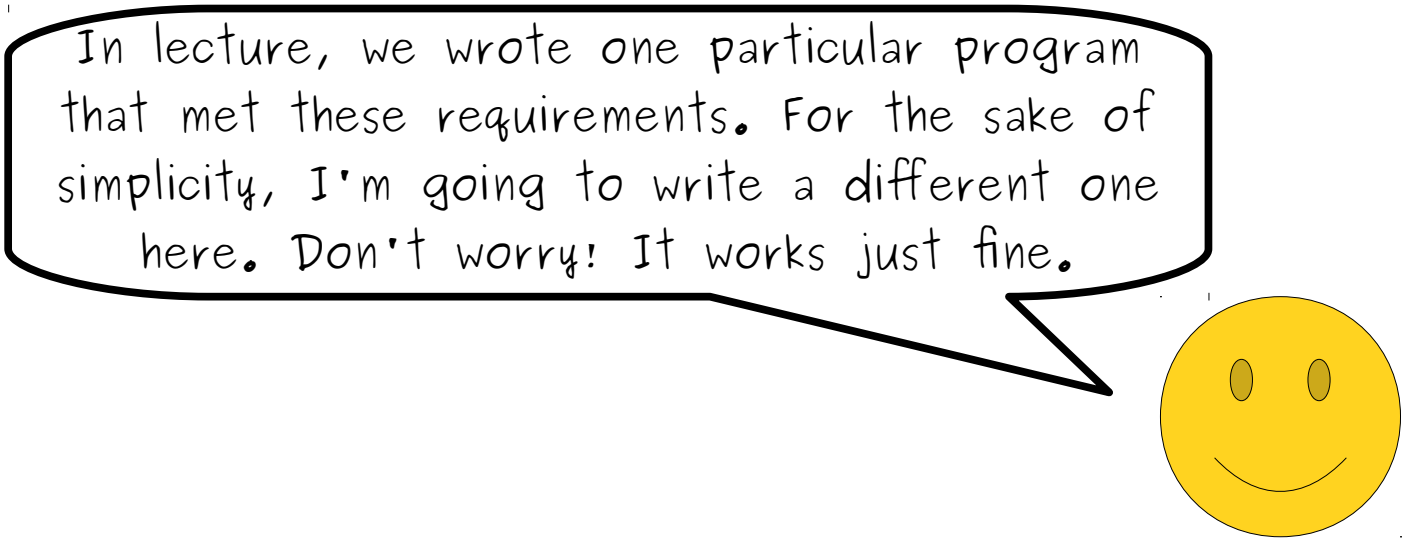

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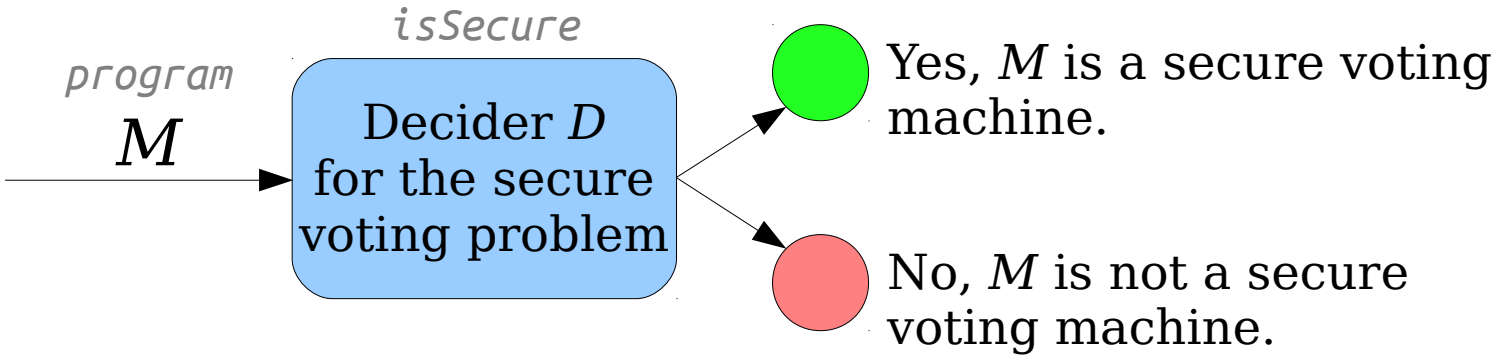


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bool isSecure(string program)
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Program P design specification:

If P is a secure voting machine, then
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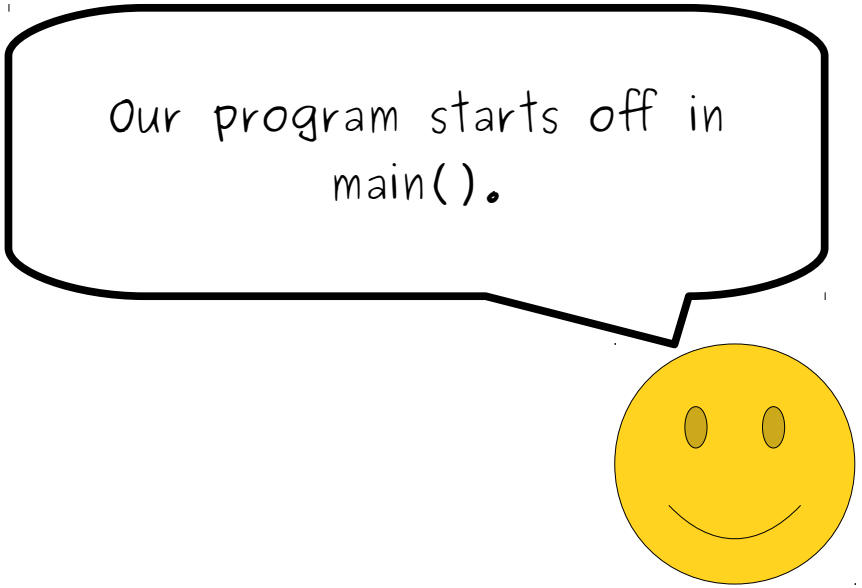
```
// Program P

int main() {



}
```



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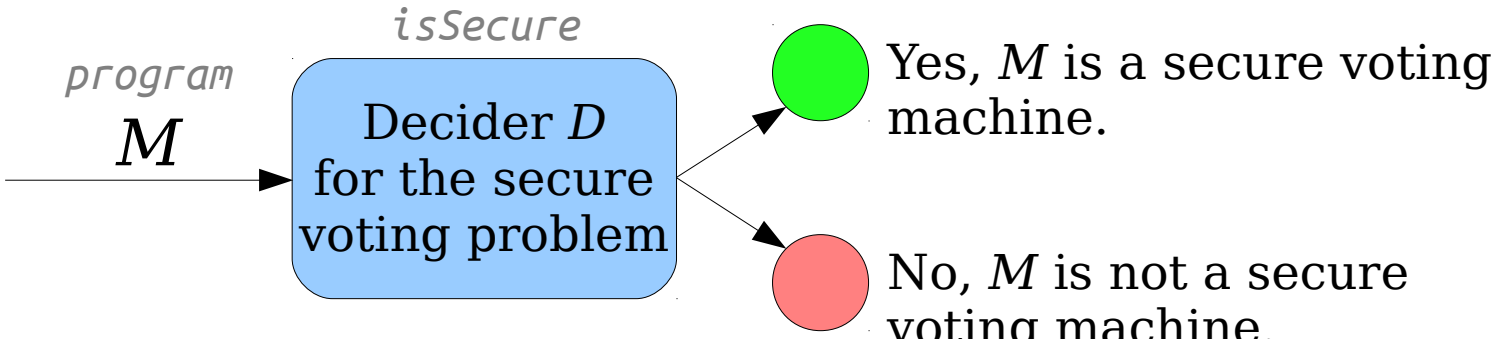


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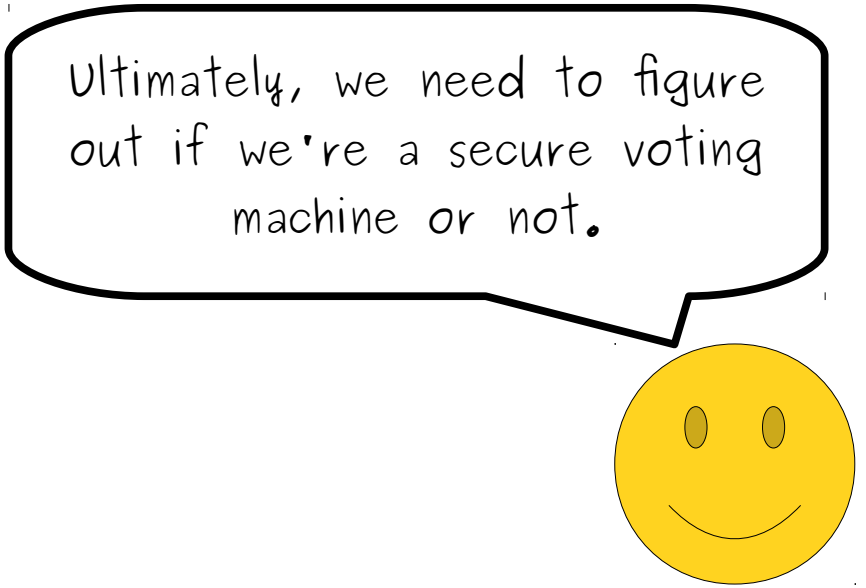

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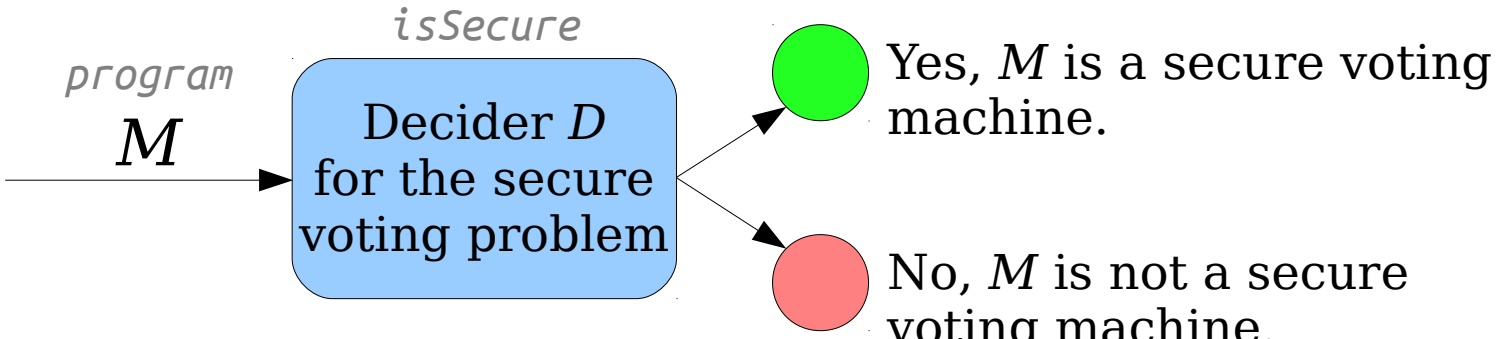


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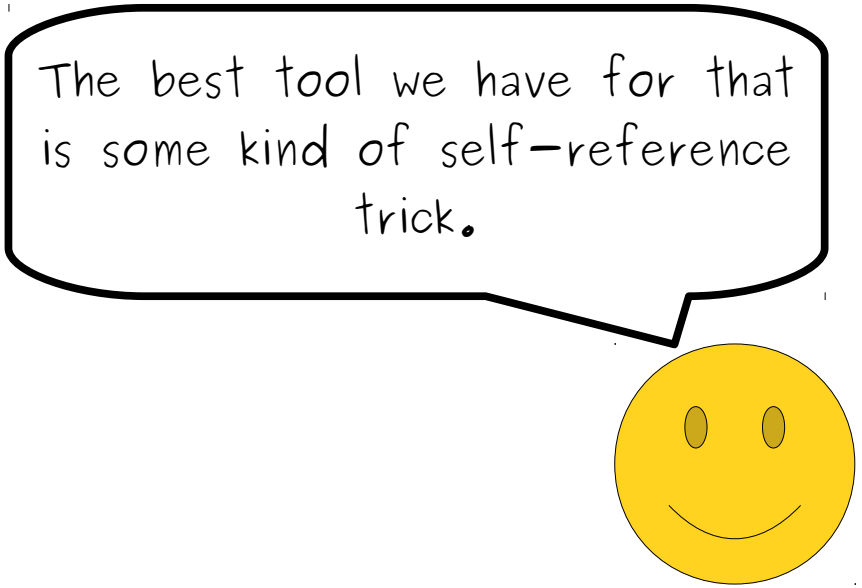

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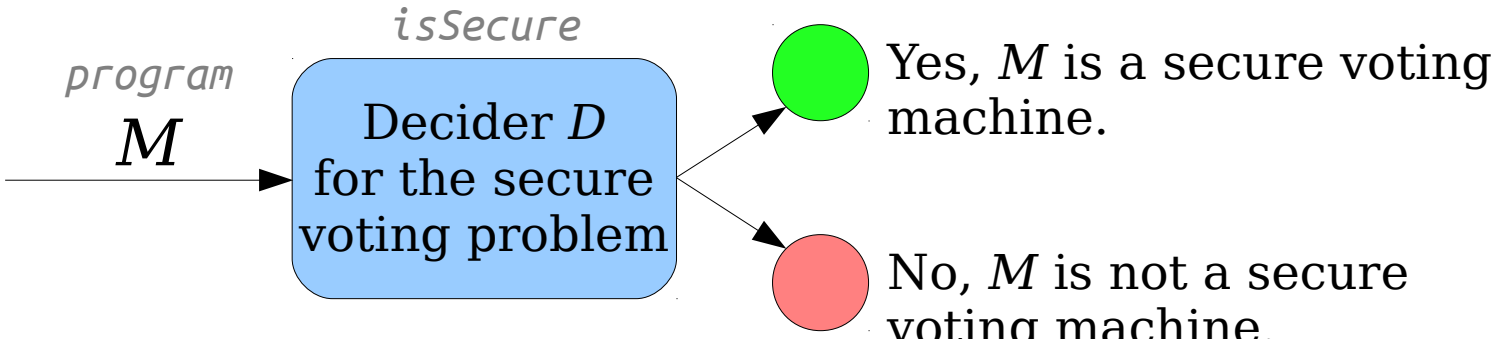


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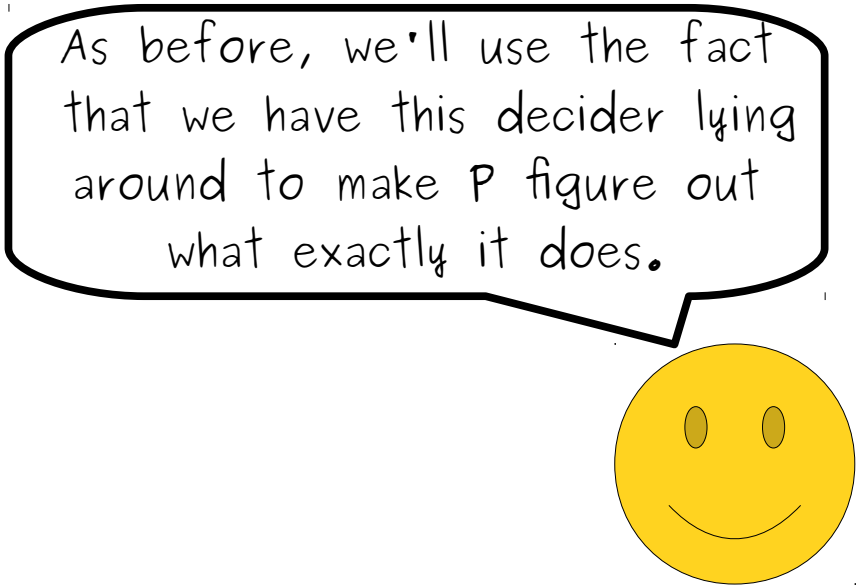

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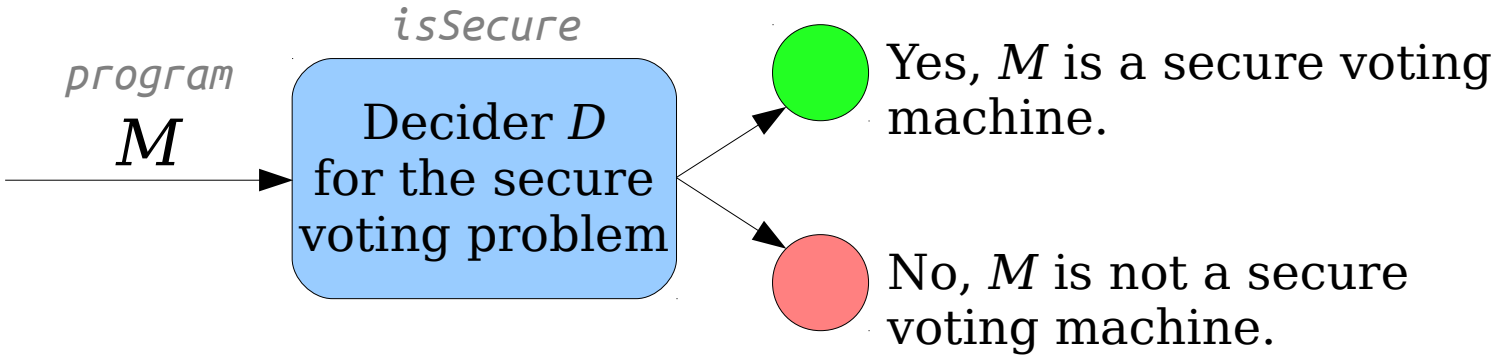


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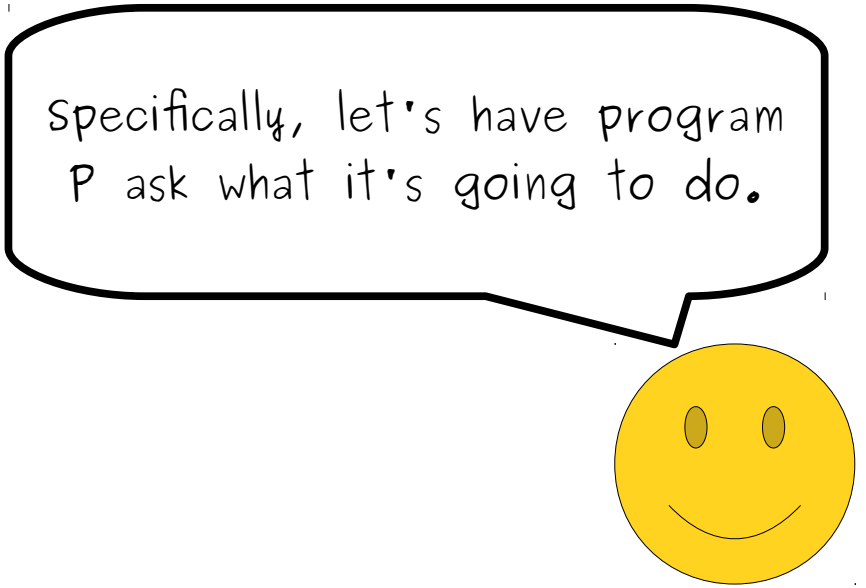

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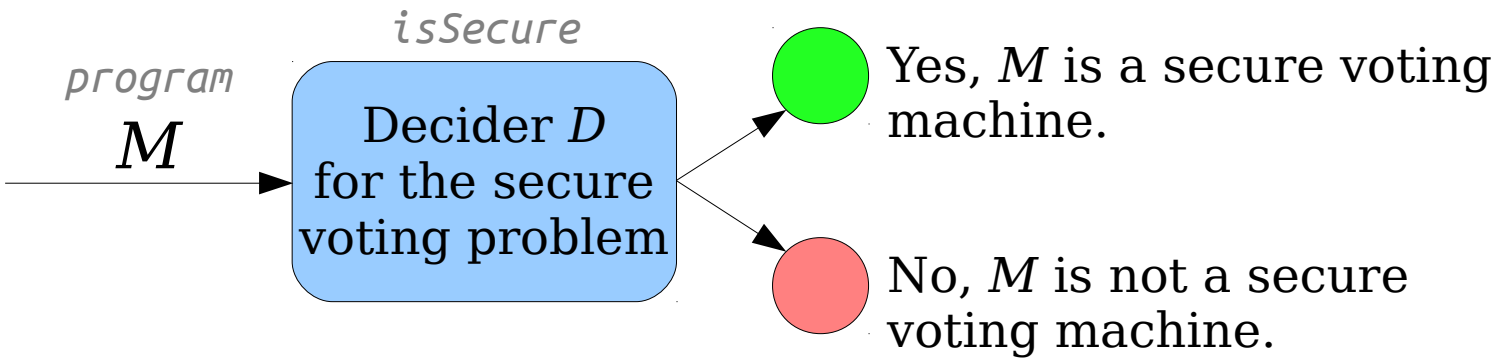


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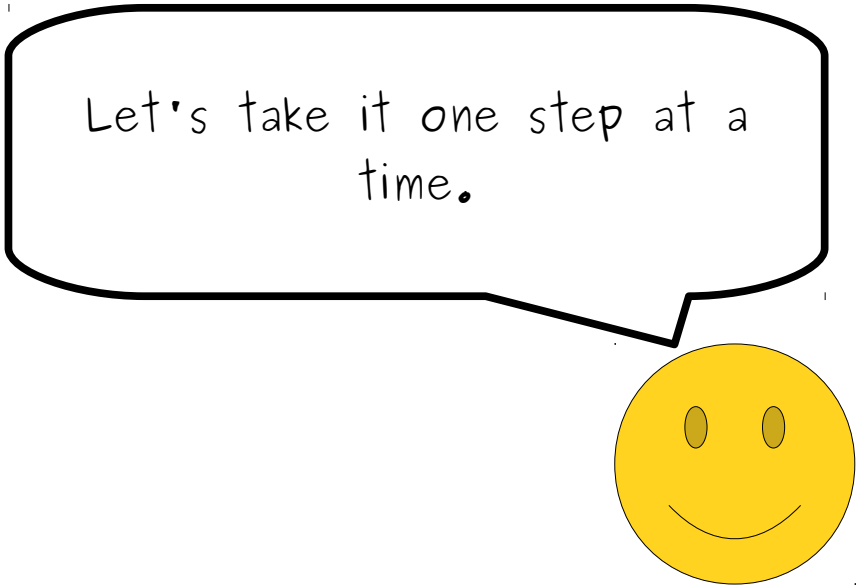

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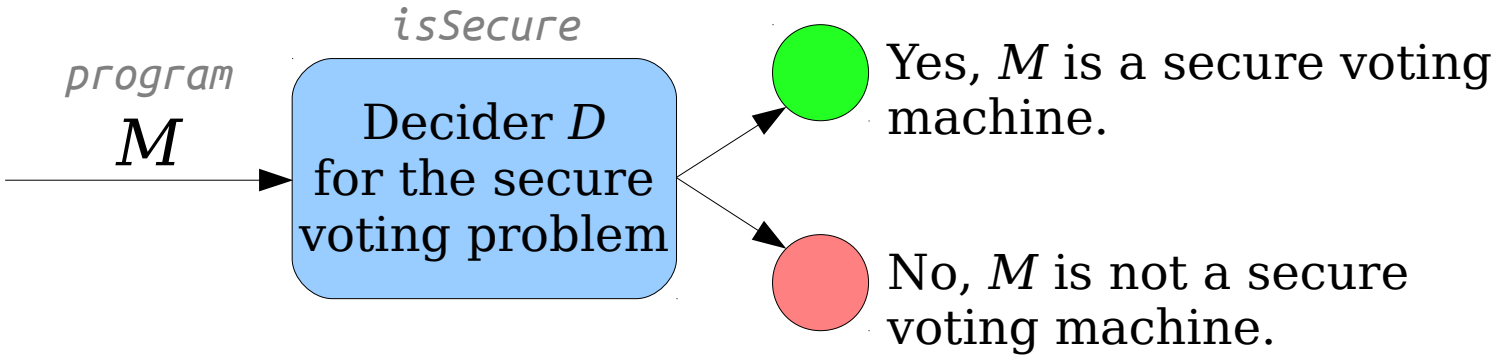


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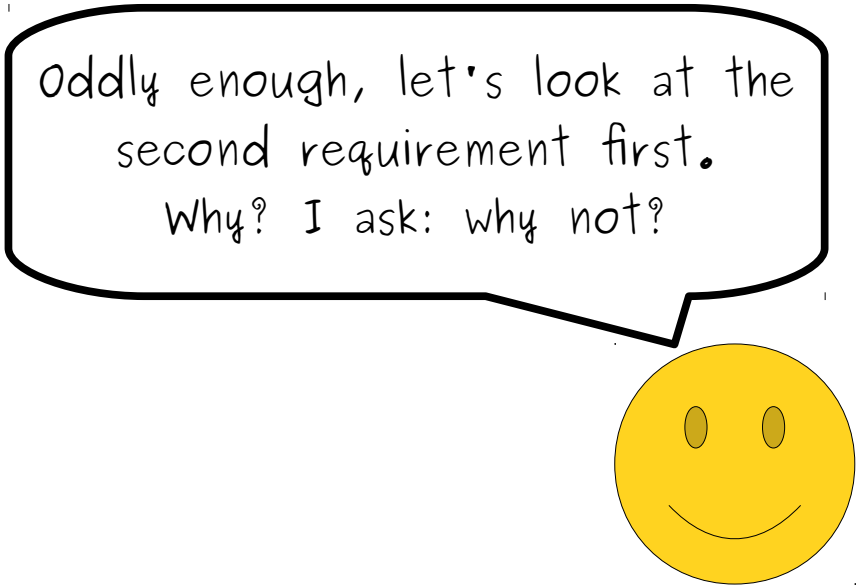

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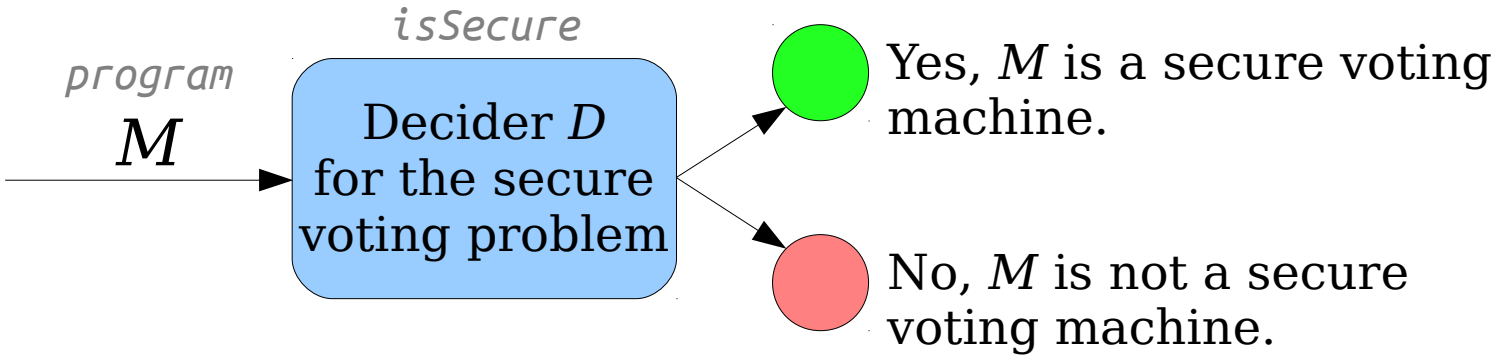


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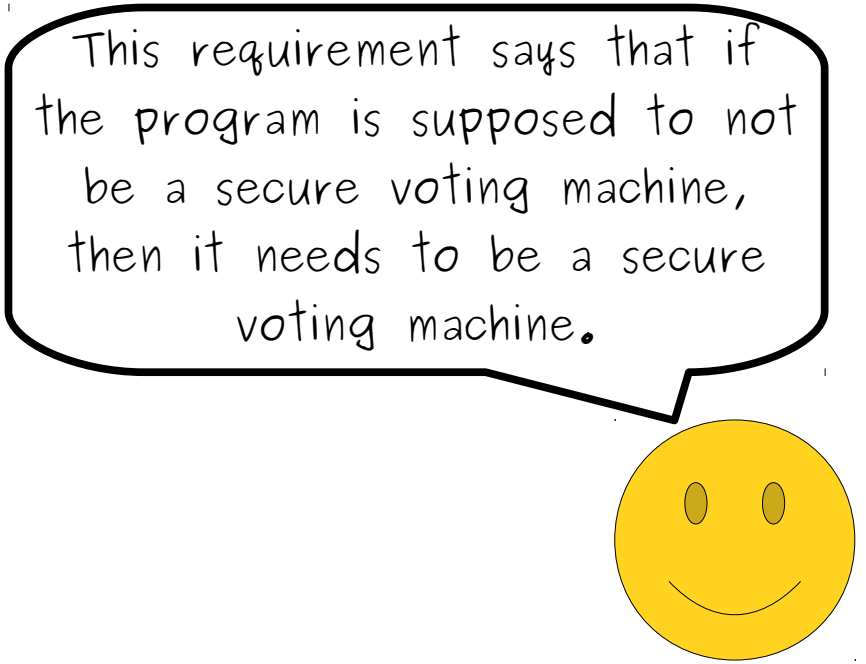

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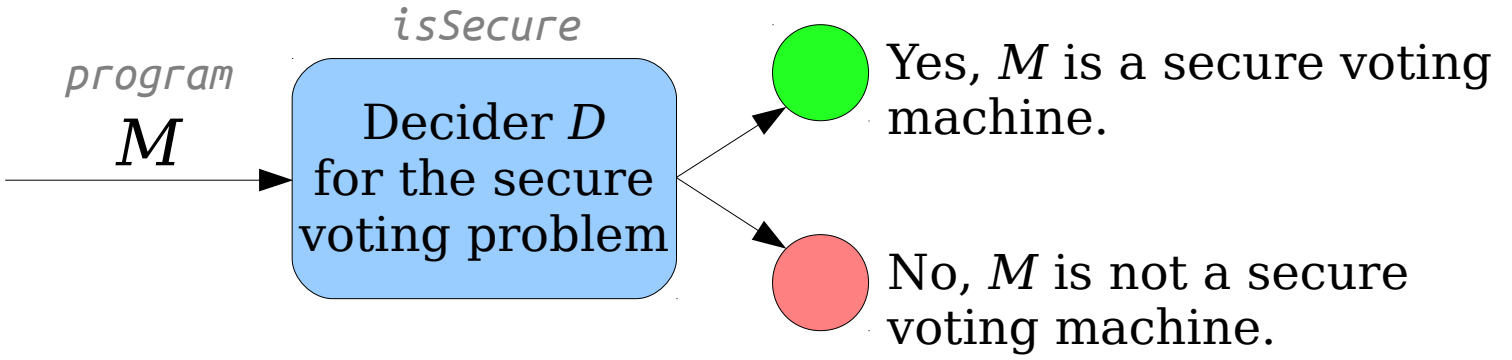


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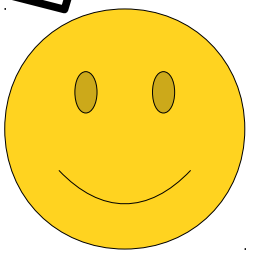

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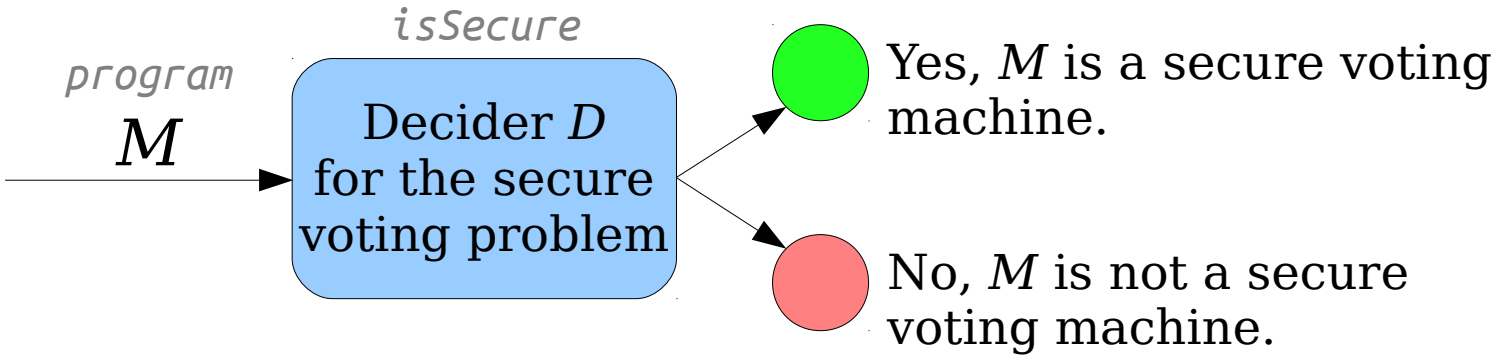


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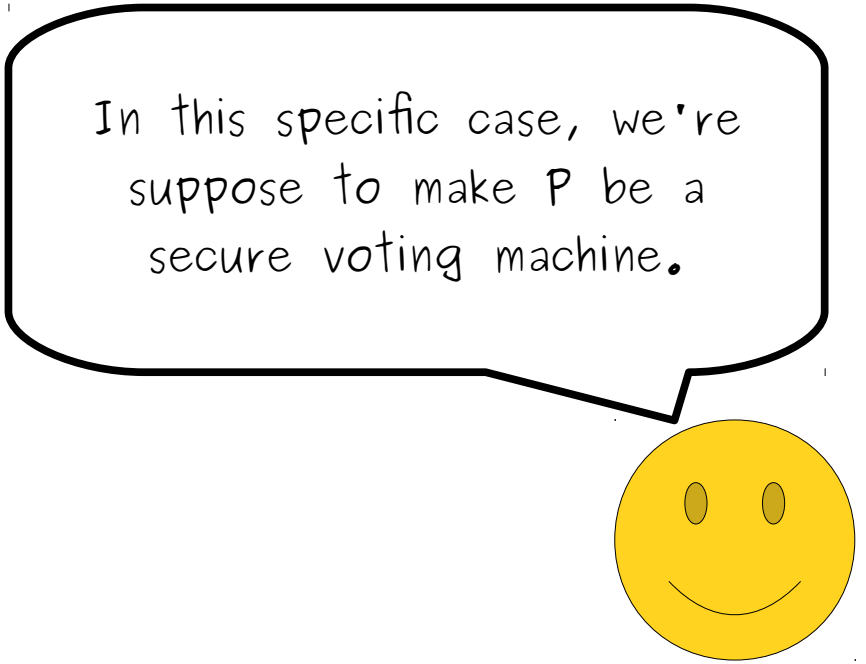

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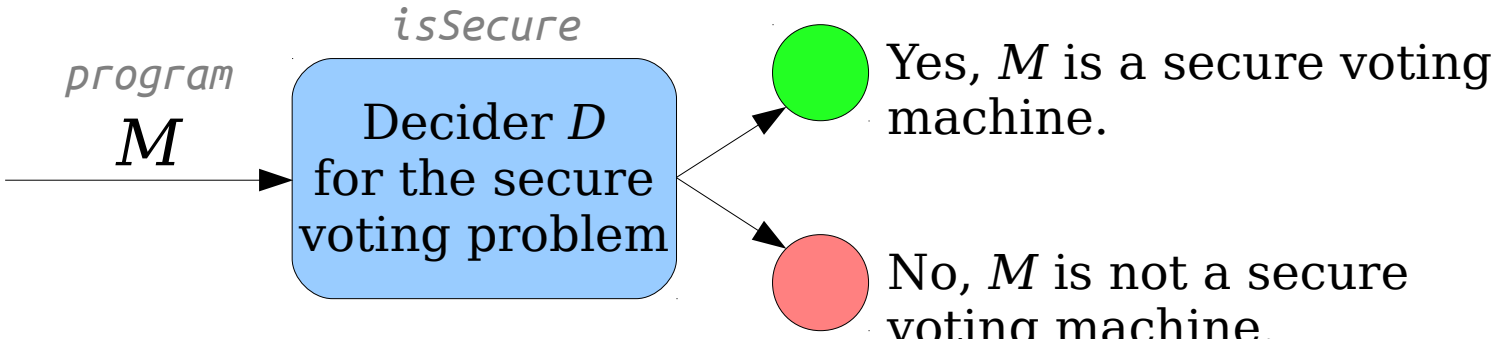


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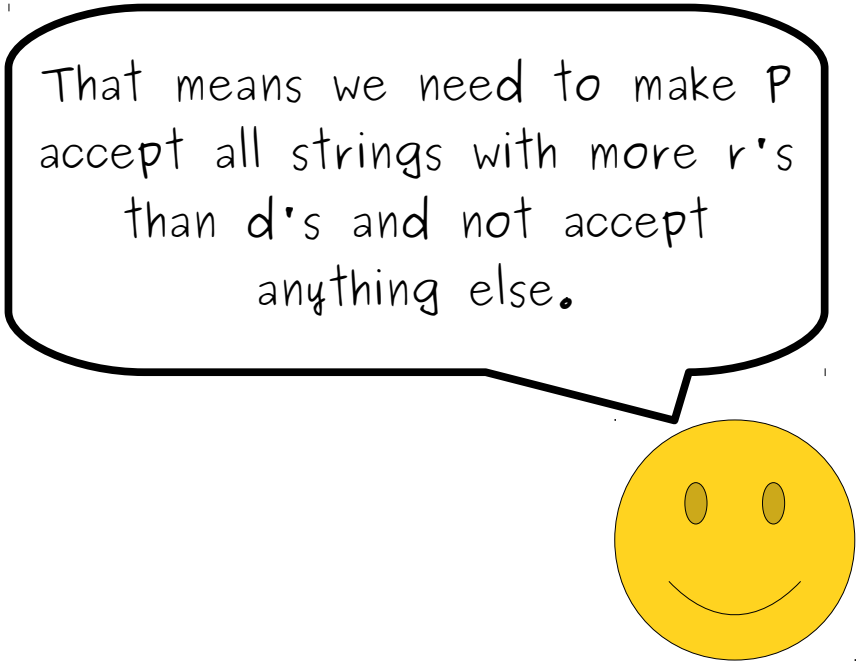

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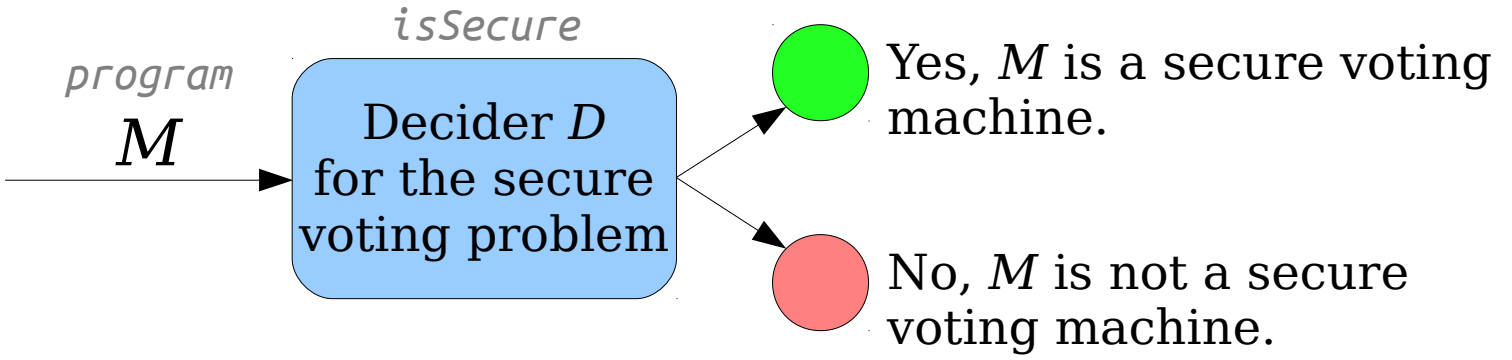


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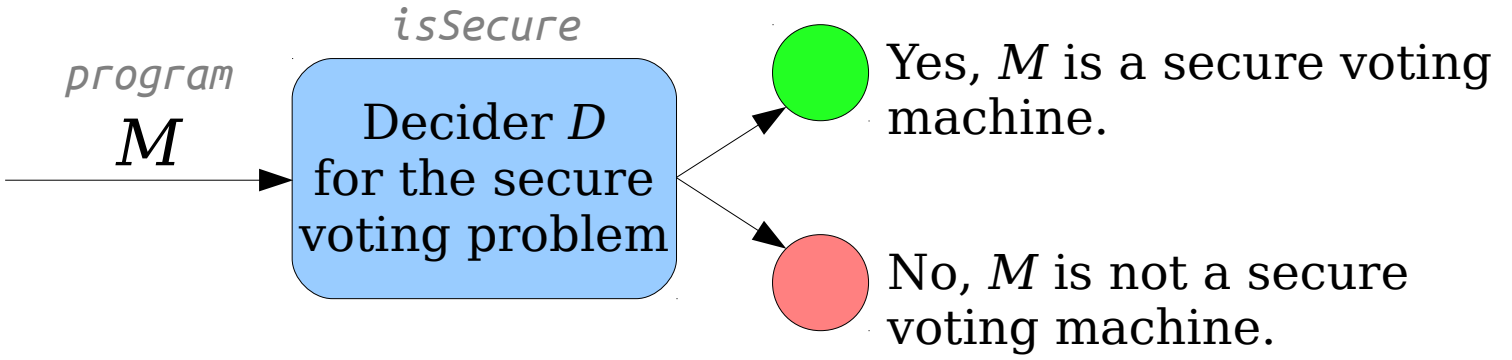


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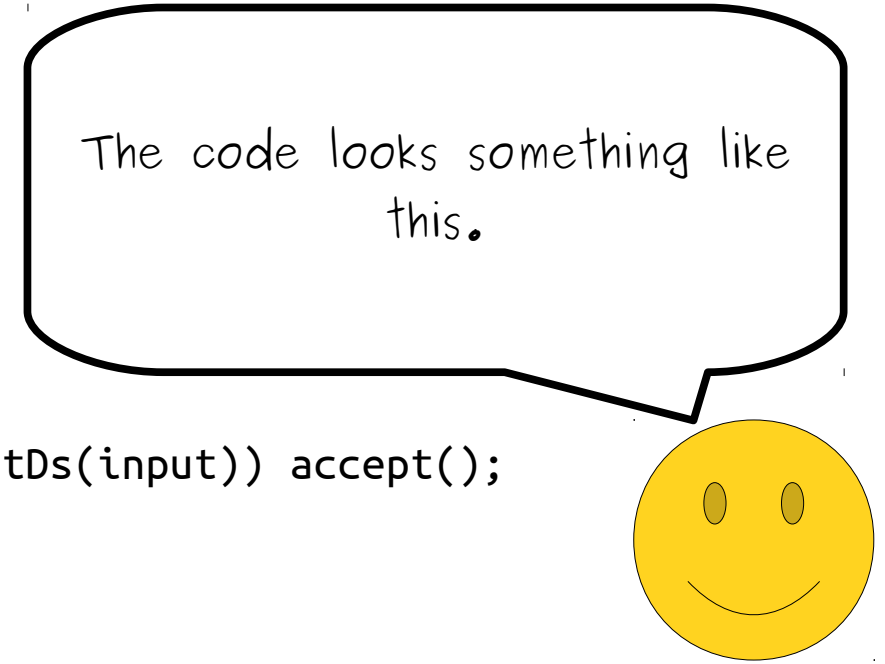

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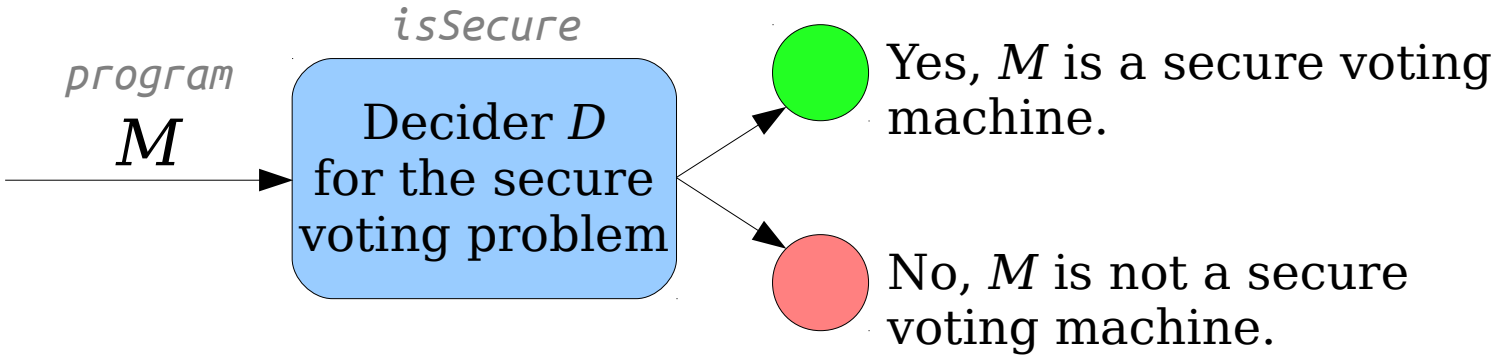


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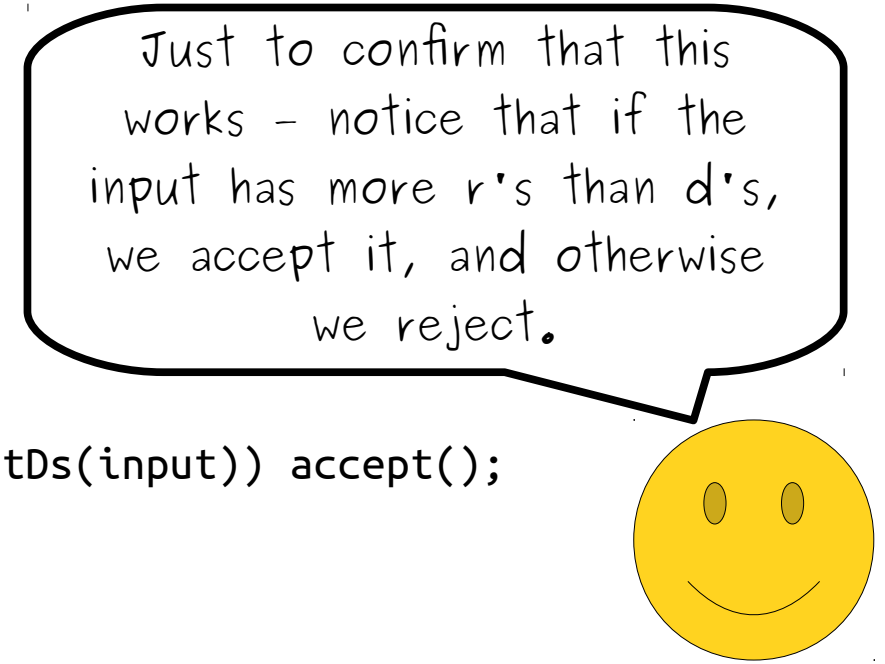

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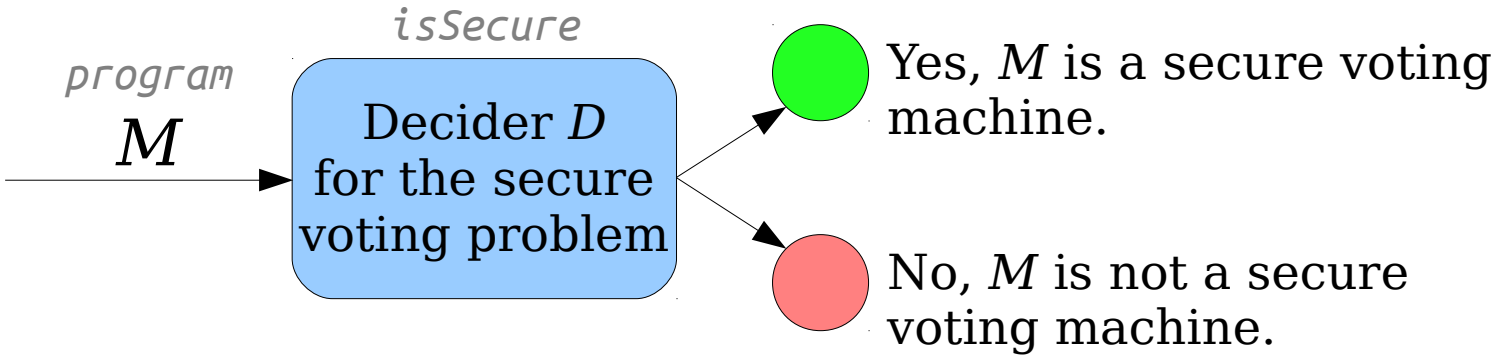


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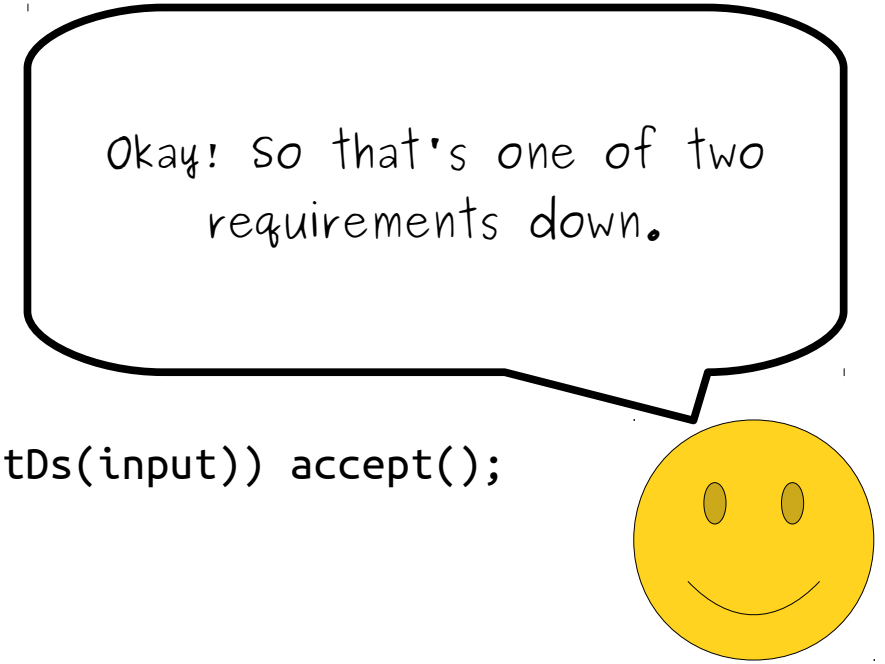

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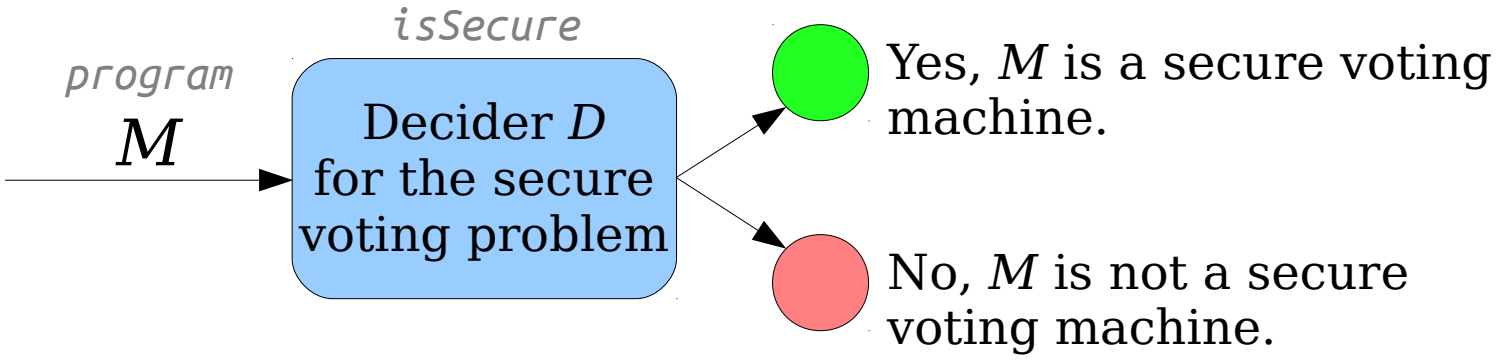


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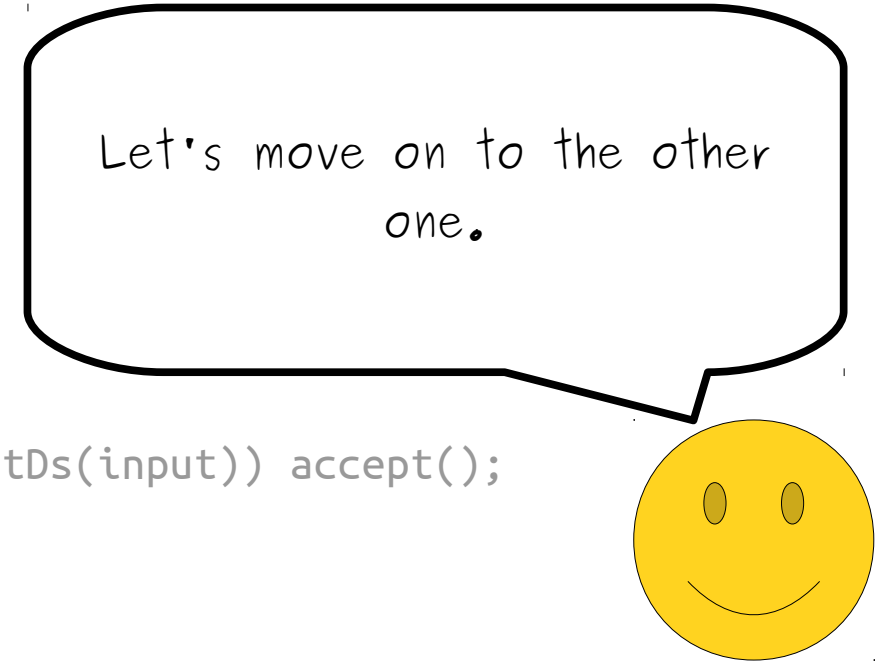

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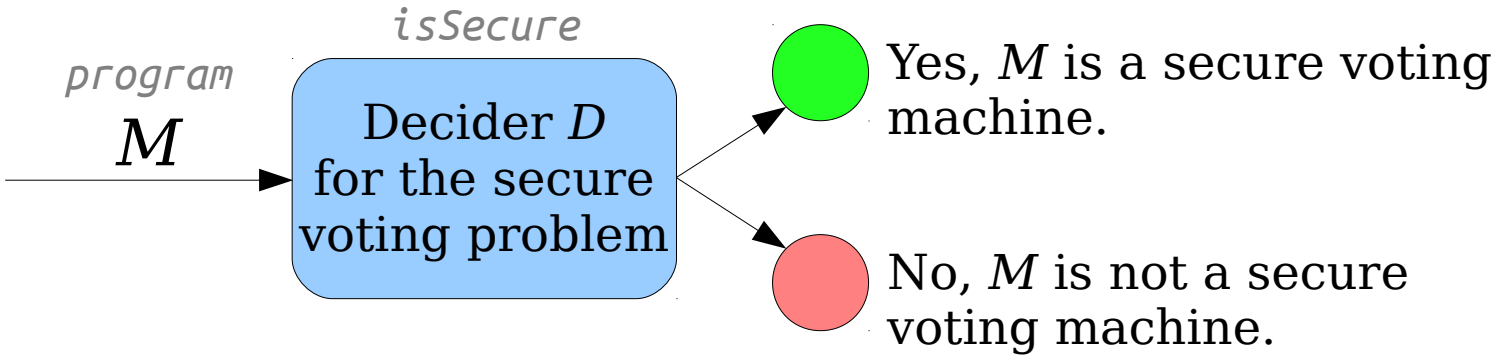


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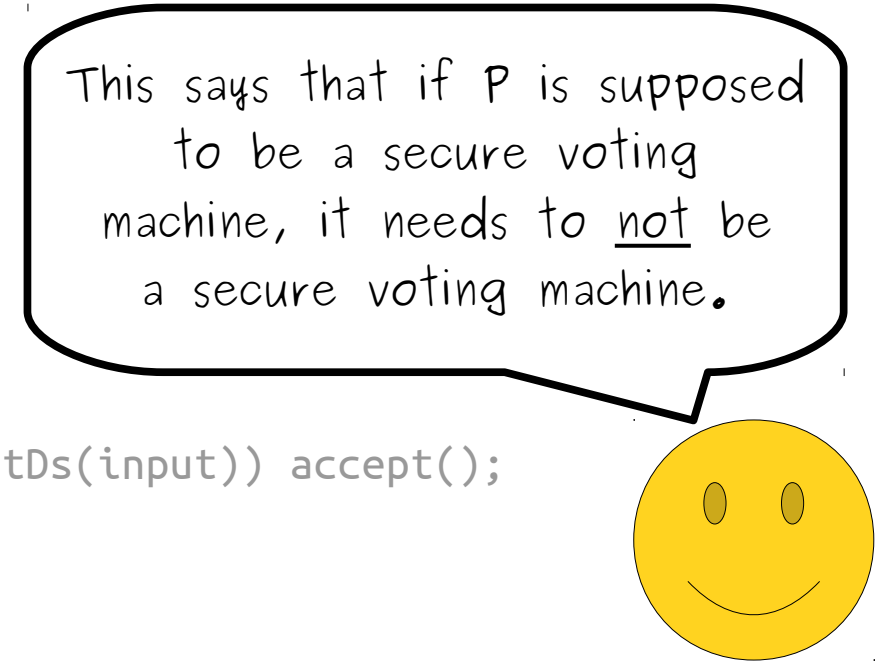

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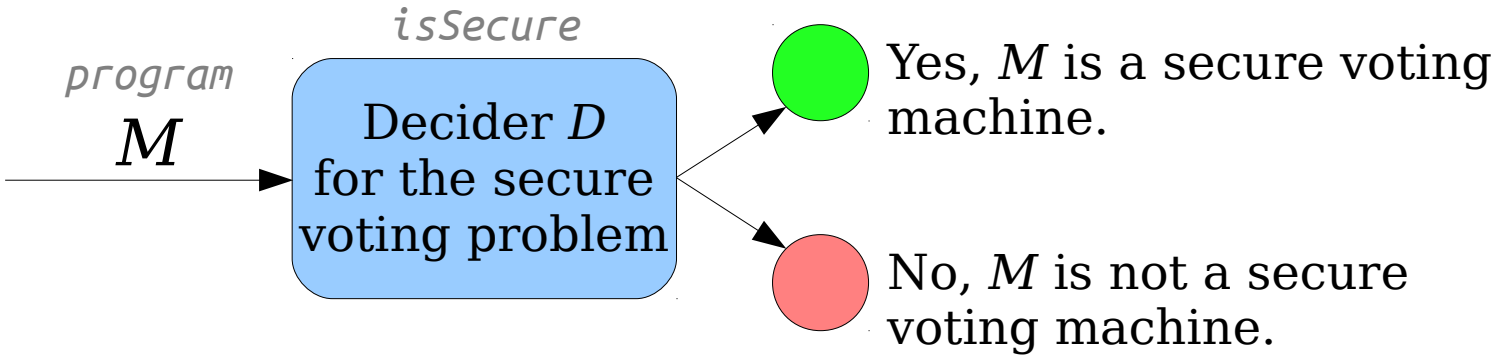


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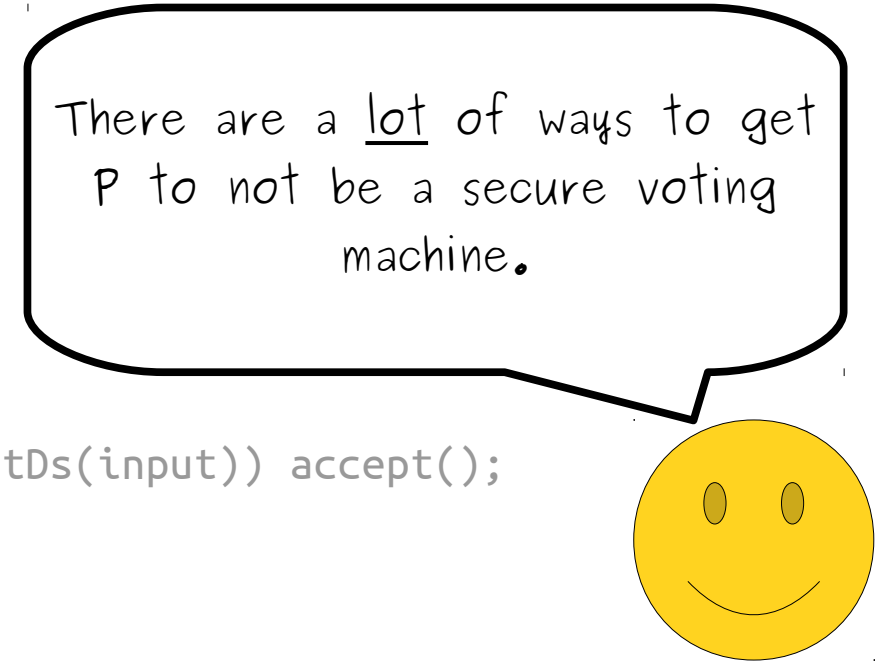

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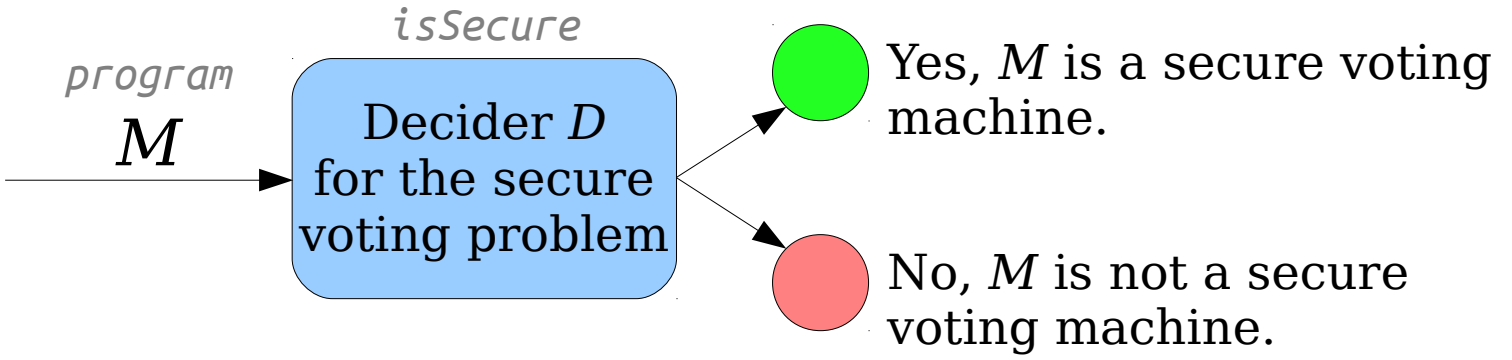


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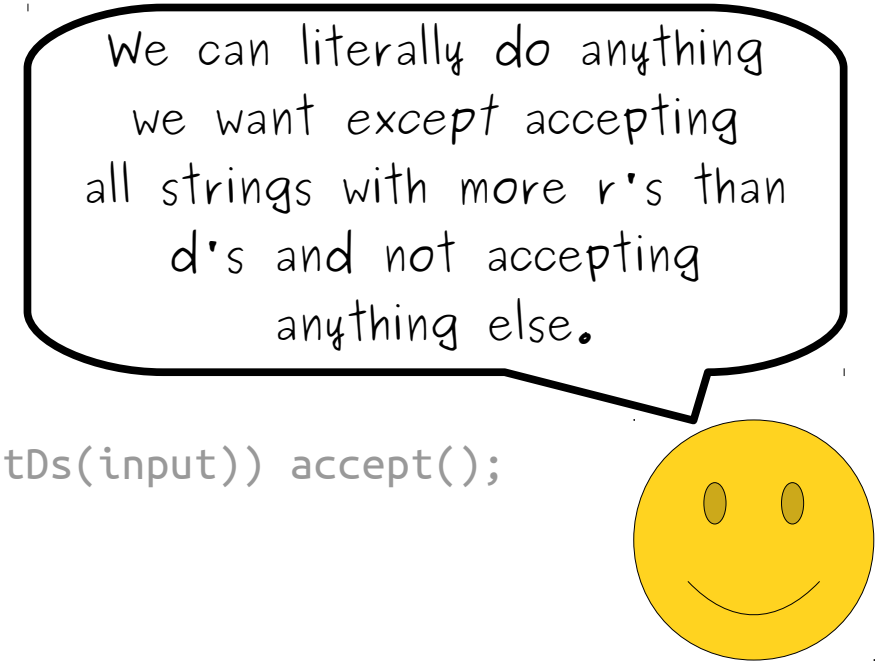

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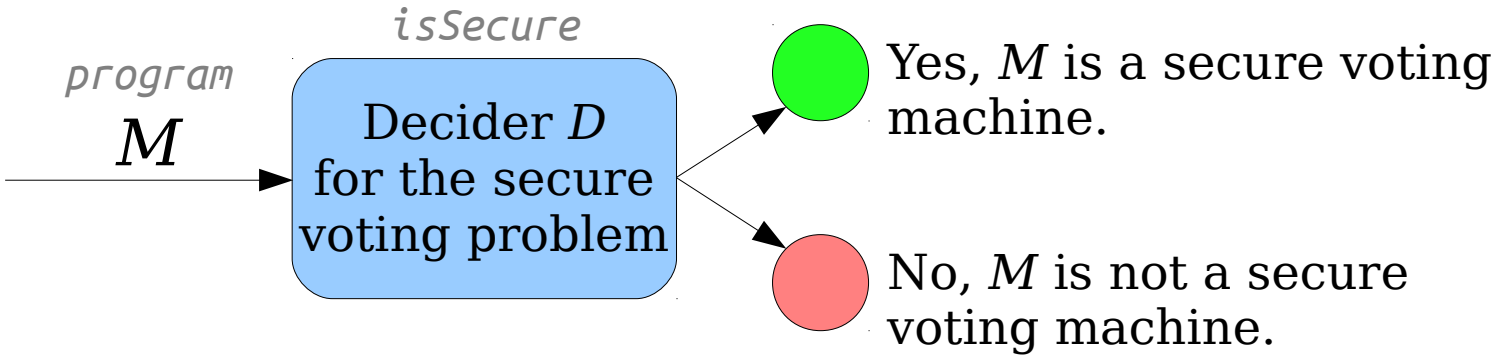


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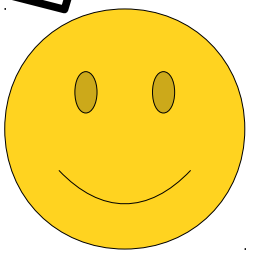
```

Among the many things we can do that falls into the "literally anything else" camp would be to just accept everything.



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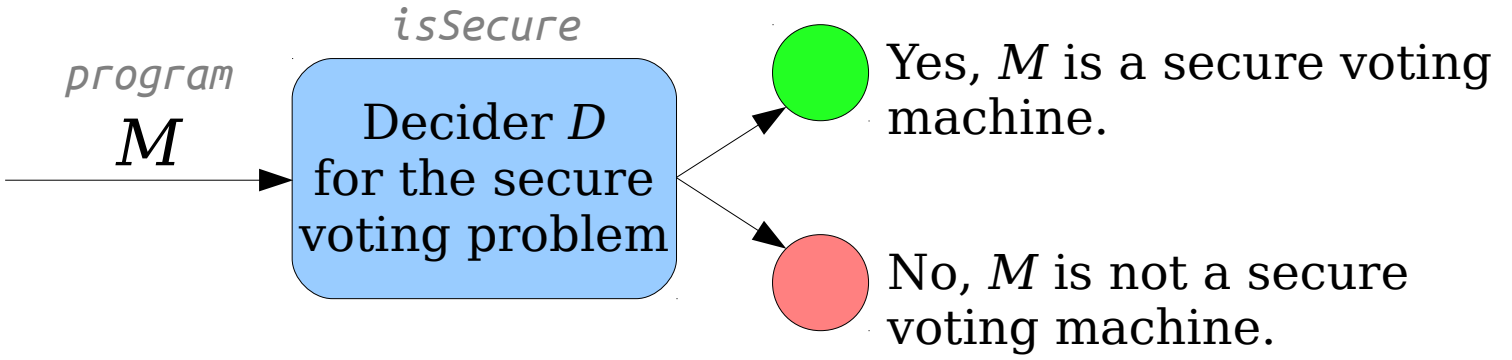


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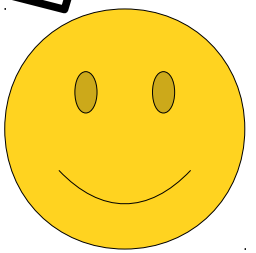
Notice that in this case, P is not a secure voting machine: it accepts everything, including a ton of strings it's not supposed to.



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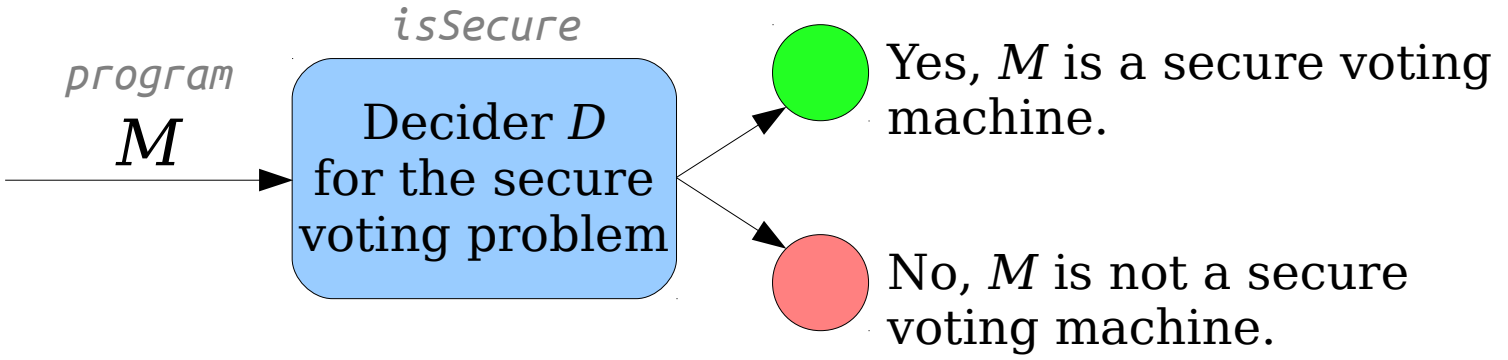


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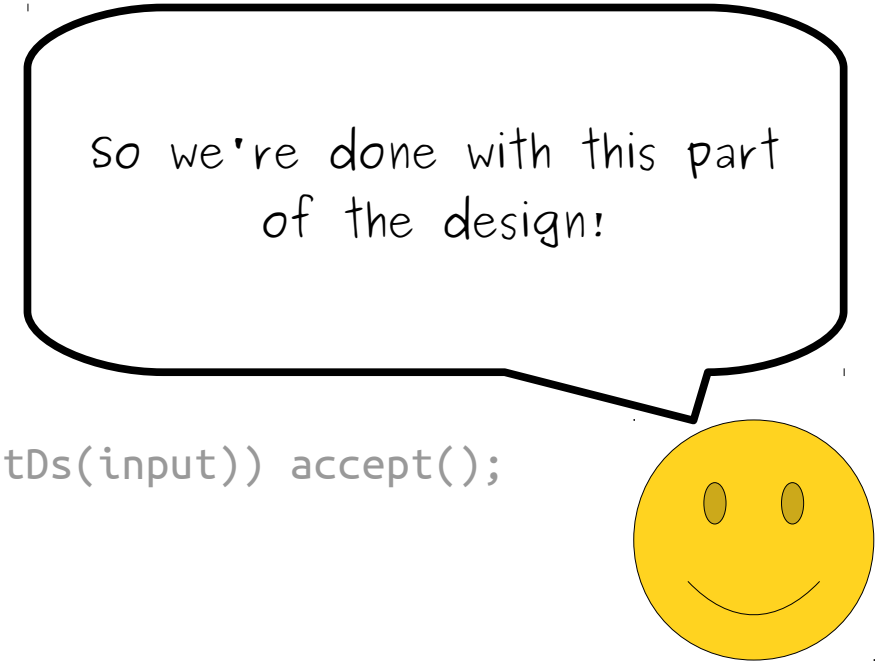

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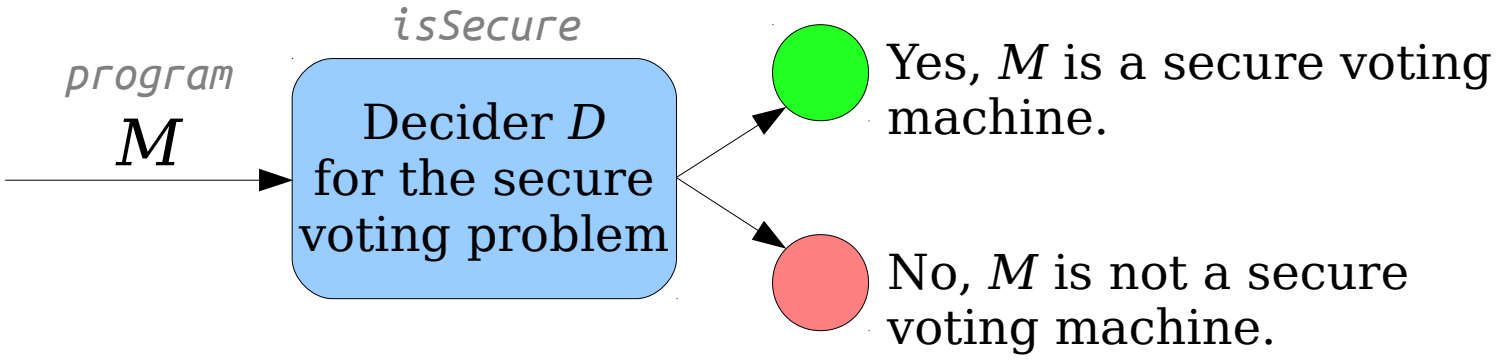


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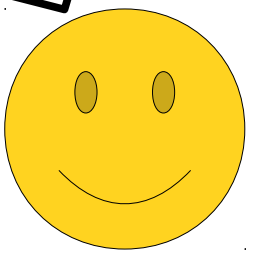
Putting it all together, take a look at what we accomplished. This program is a secure voting machine if and only if it isn't a secure voting machine!



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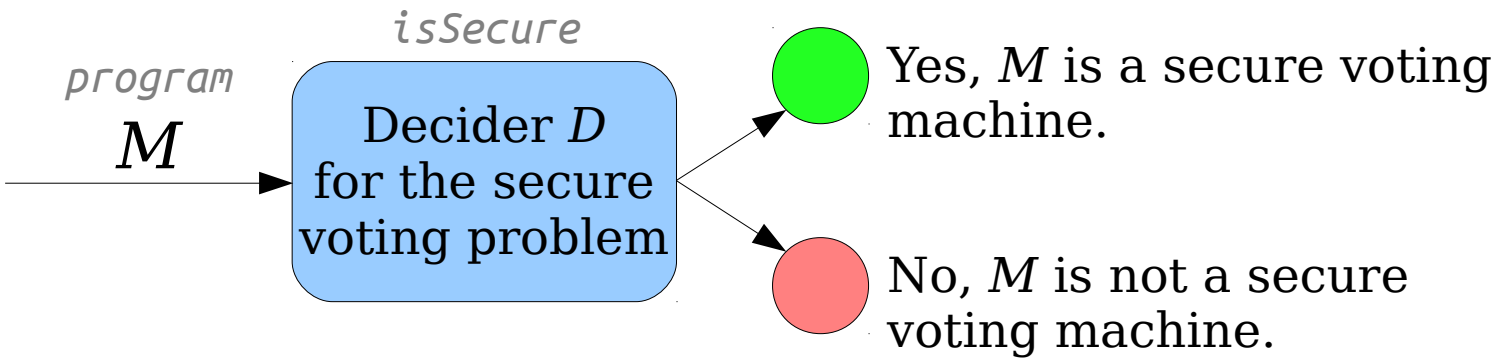


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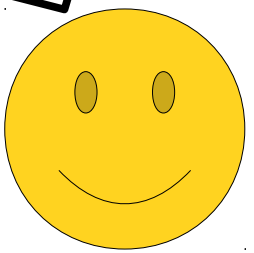
That gives us the contradiction that we needed to get.



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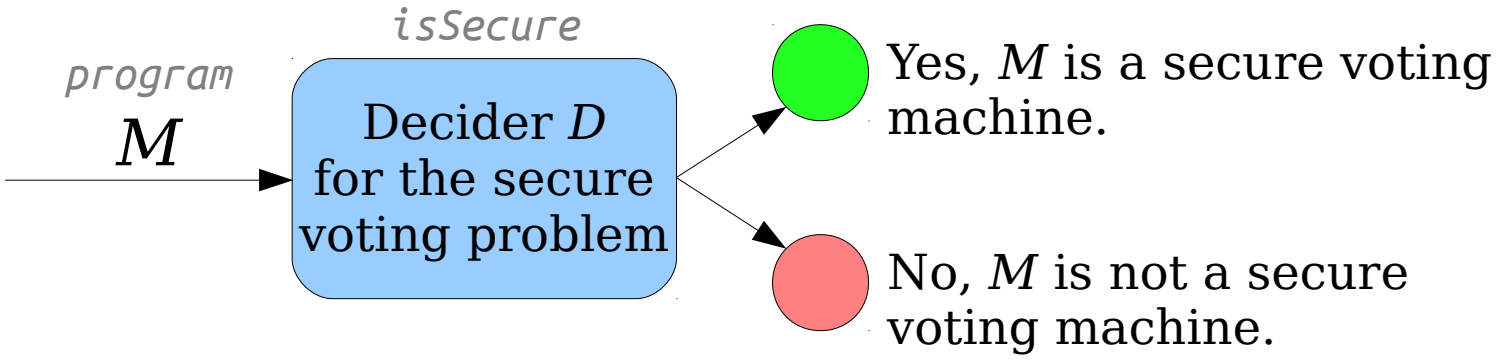


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```
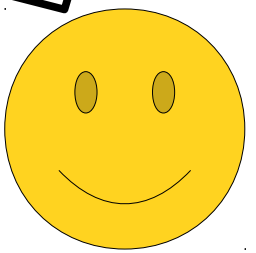

We're done! We've shown that starting with the assumption that the secure voting problem is decidable, we reach a contradiction.



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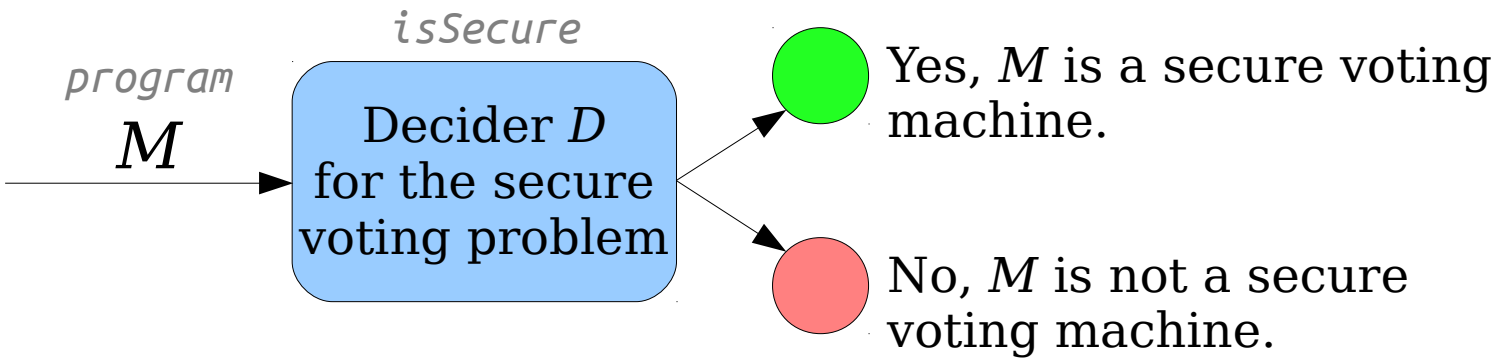


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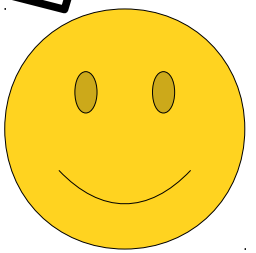
You might have noticed that this program isn't the one we used in lecture. But that's okay!



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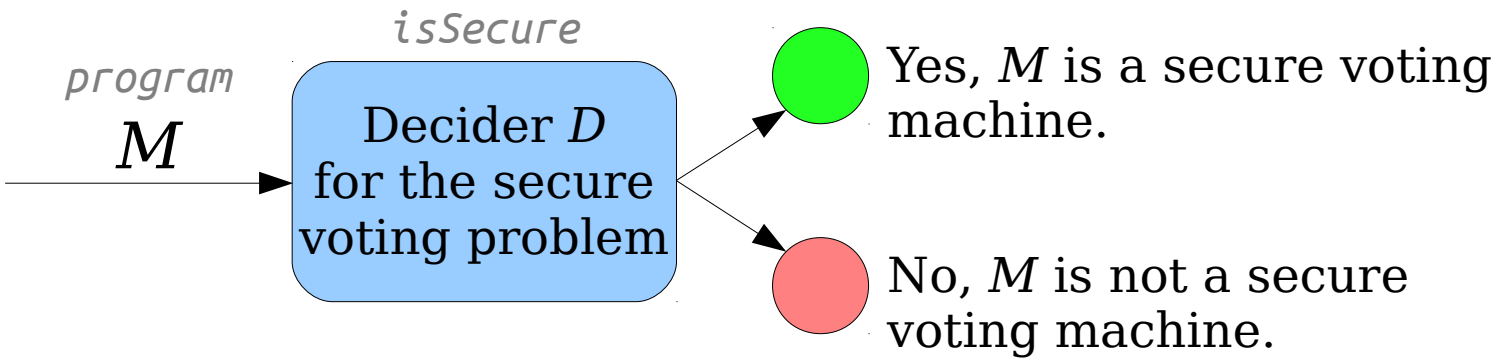


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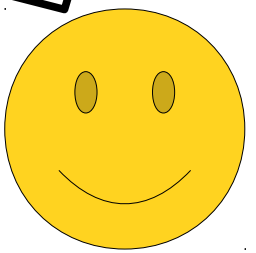
There can be all sorts of programs that meet the design specification we set out above.



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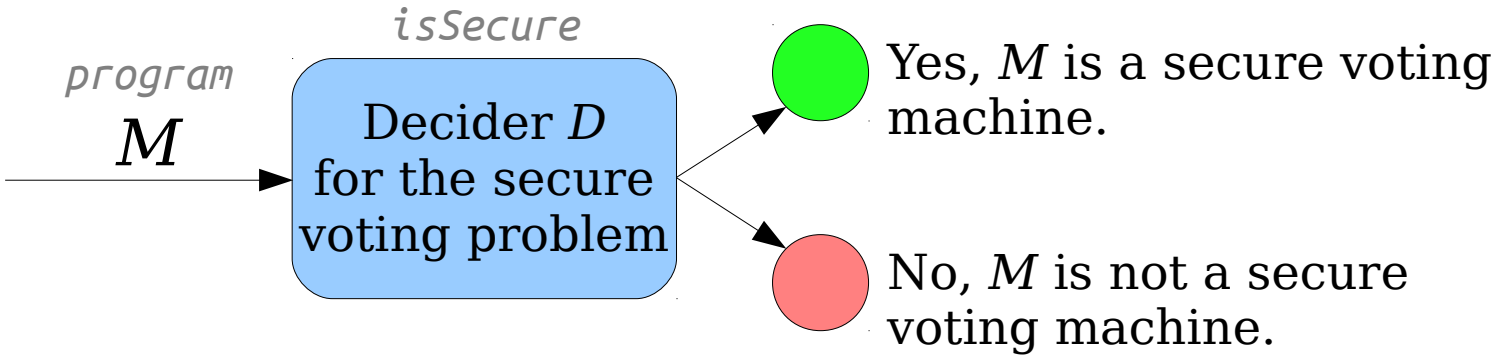


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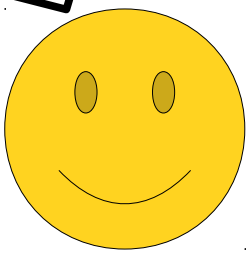
That's great news for you, because it means that these sorts of proofs aren't about finding a needle in a haystack.



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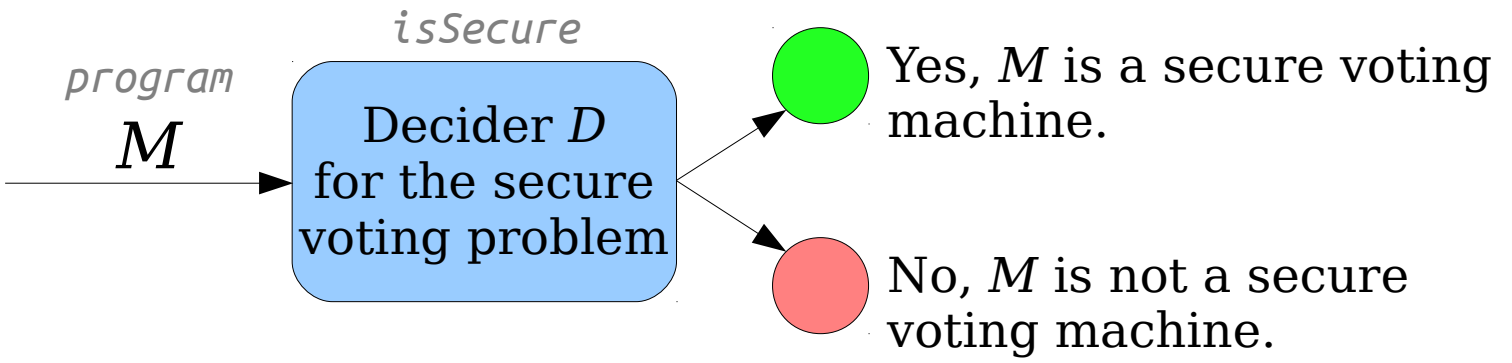


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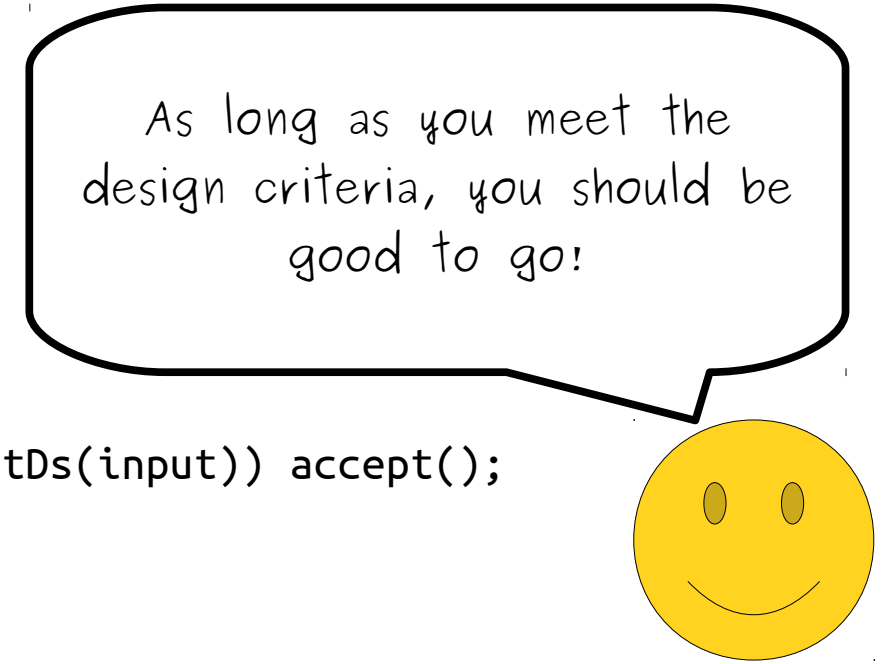

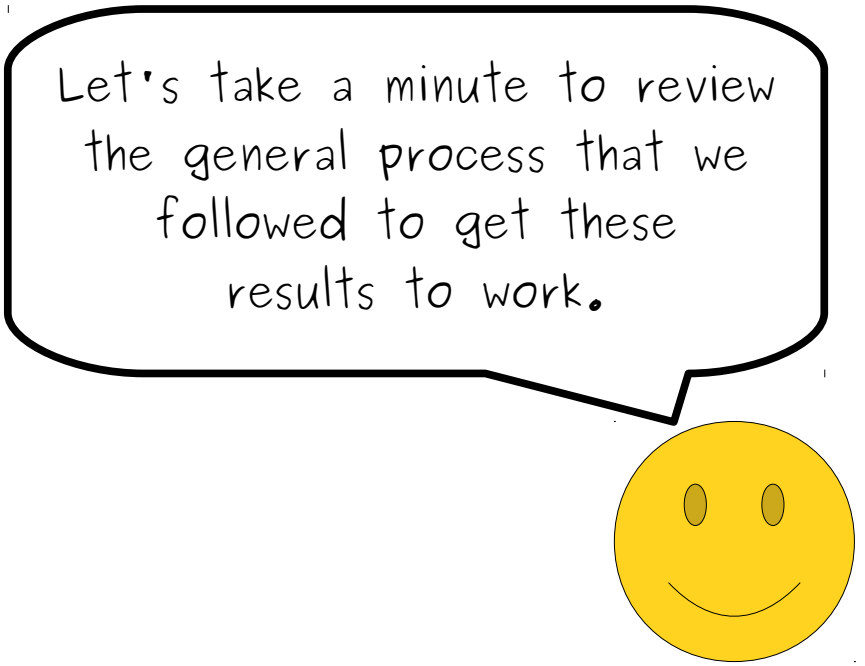
Let's take a minute to review the general process that we followed to get these results to work.


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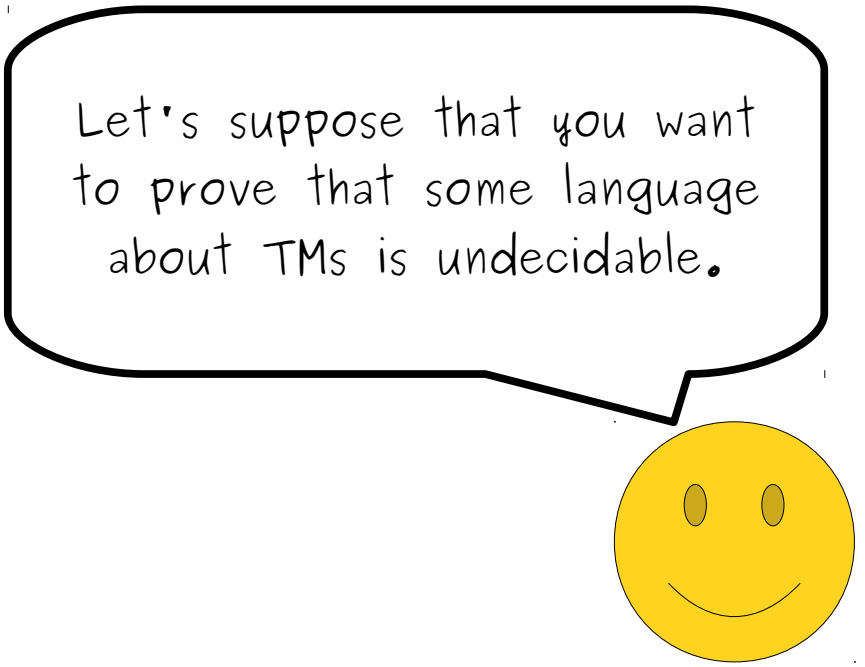
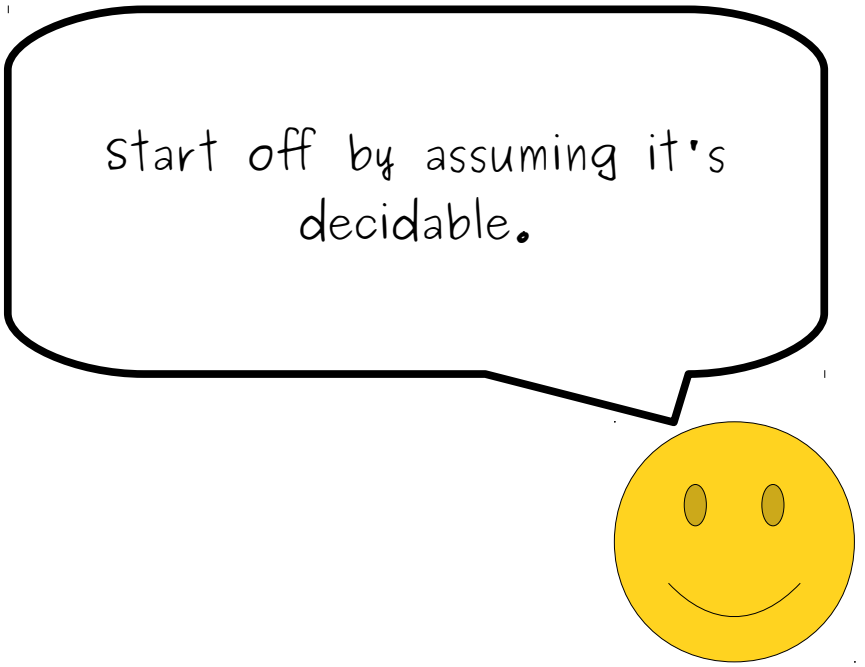
That other guy is going to tell you a general pattern to follow. You might want to take notes.


Let's suppose that you want to prove that some language about TMs is undecidable.

The problem in question is decidable



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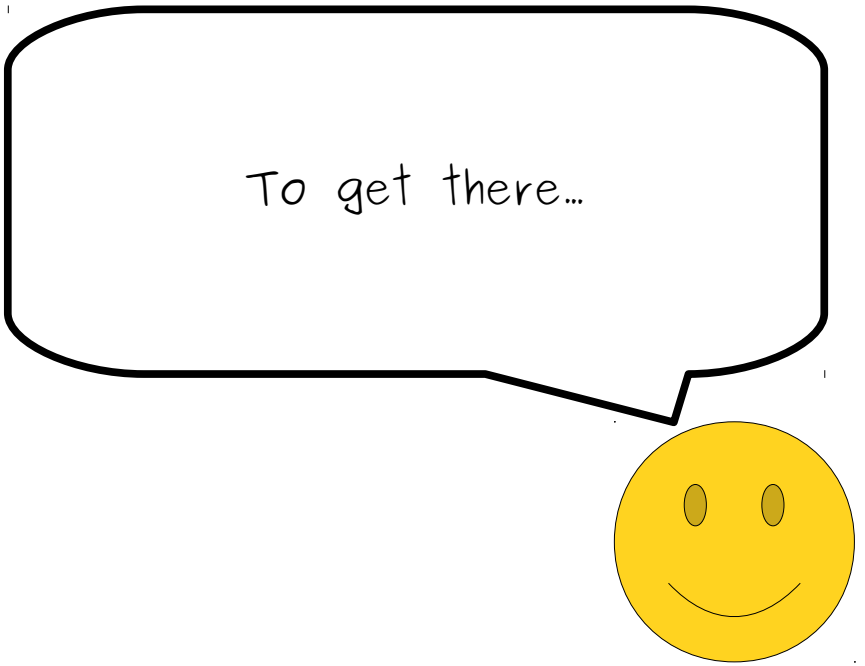

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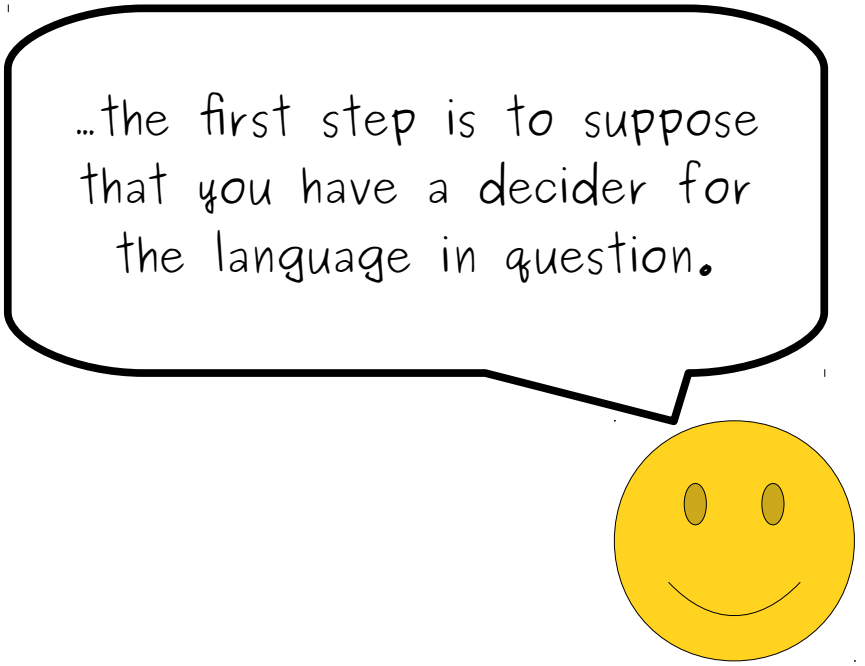

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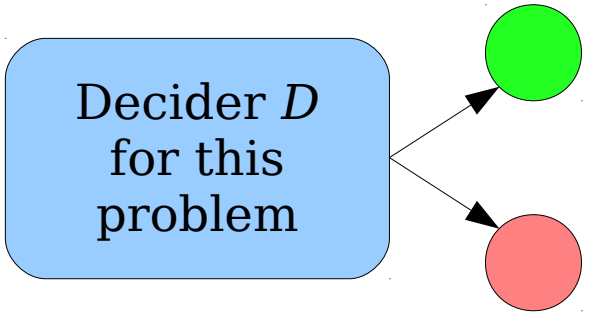

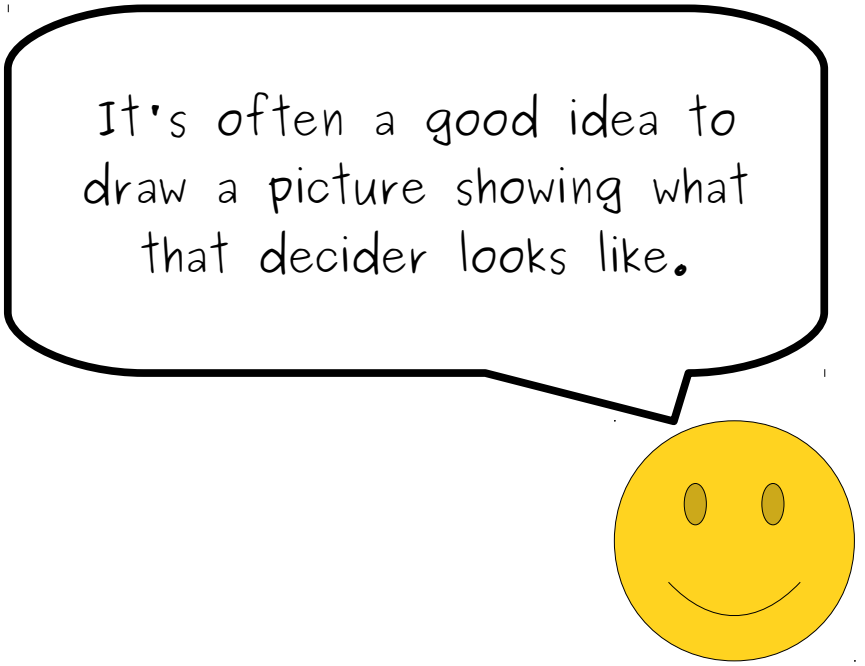

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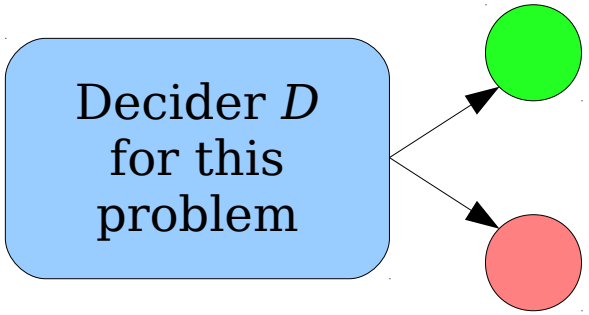

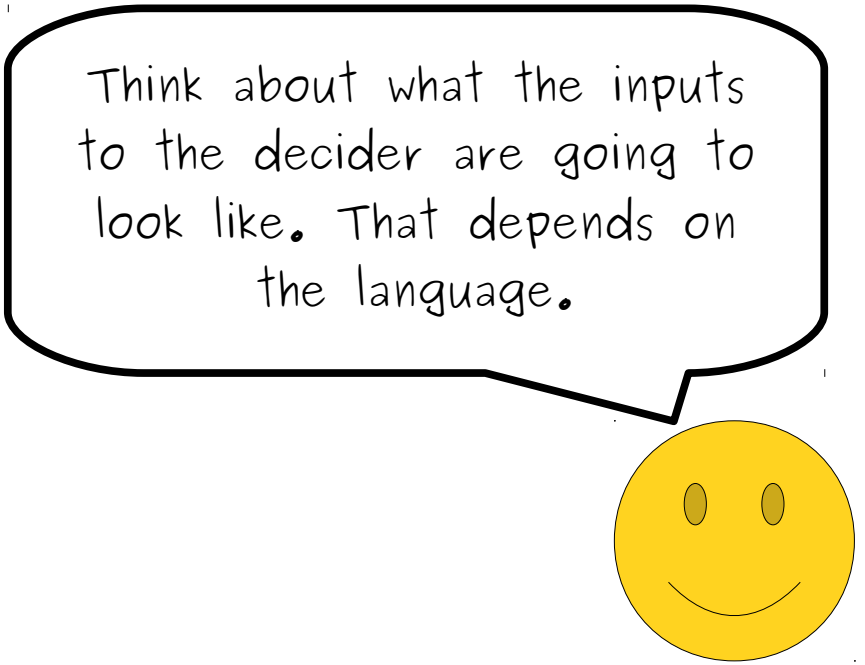

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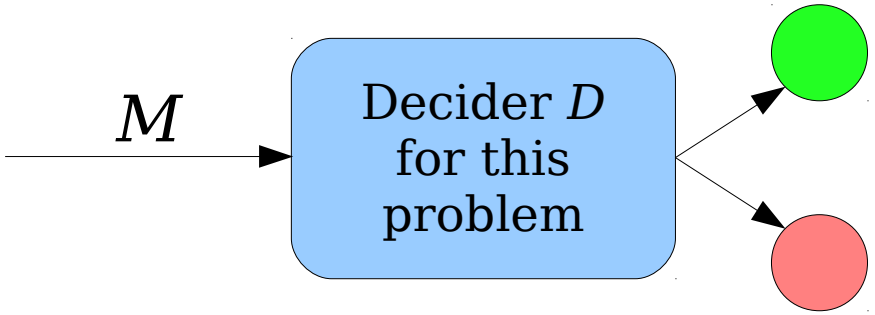

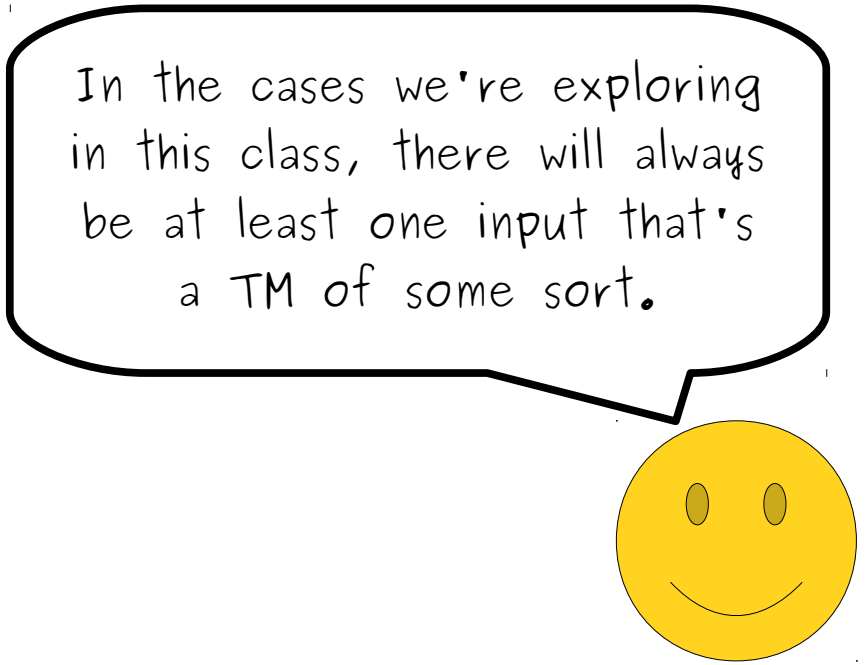

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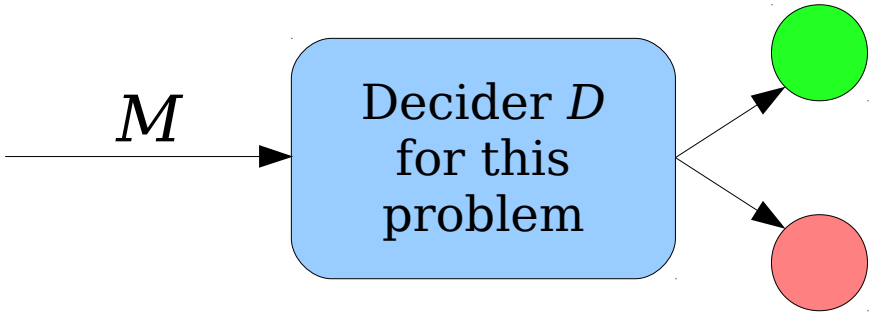

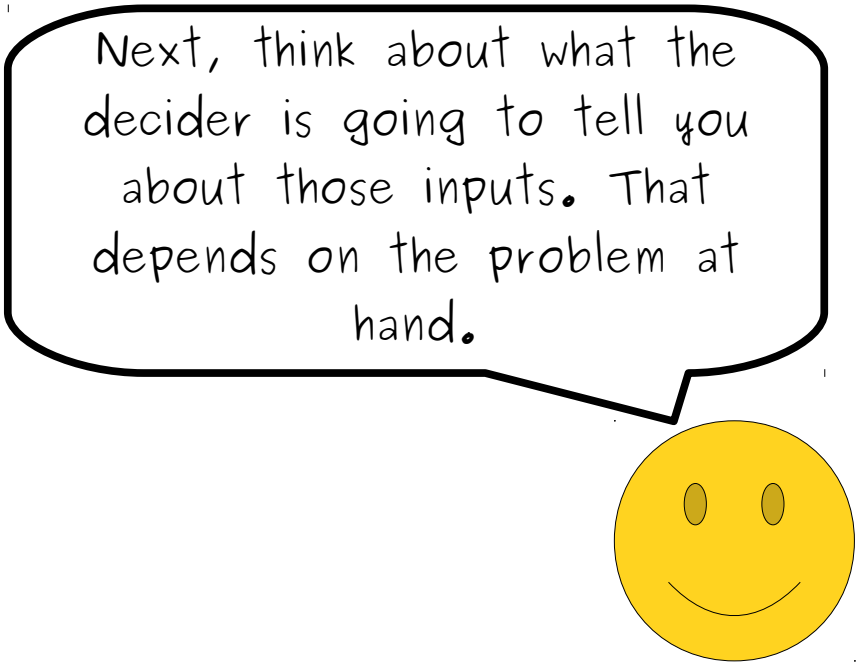

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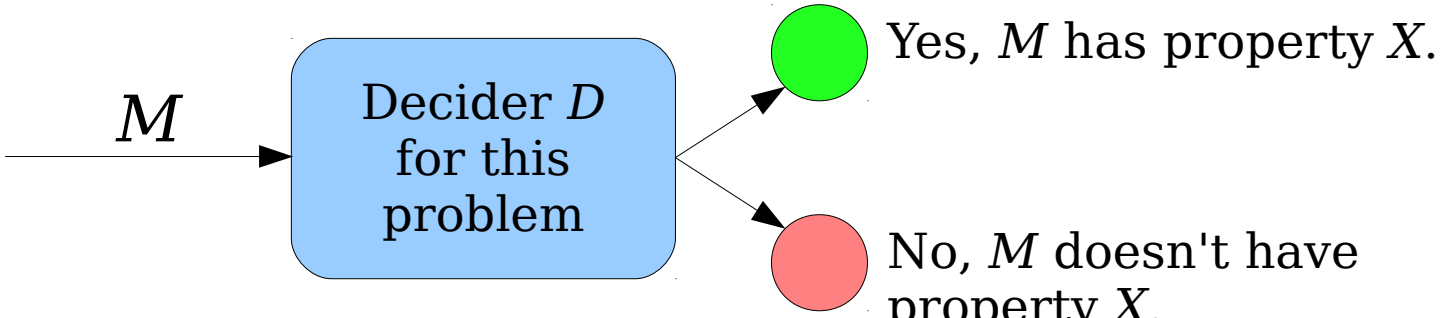

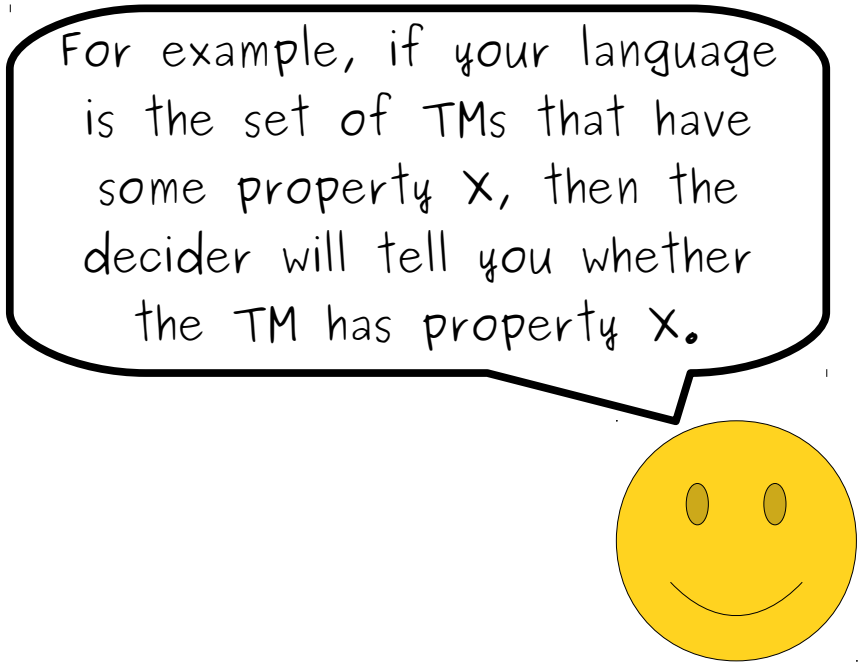

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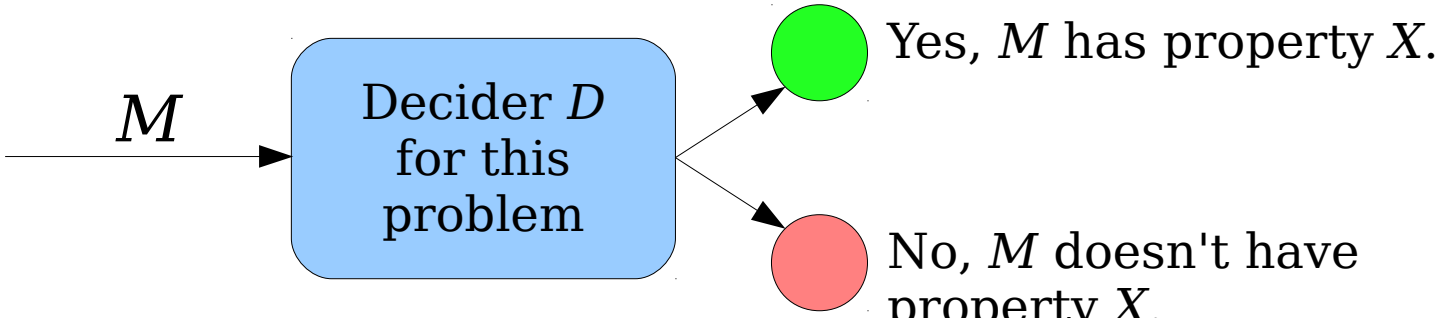

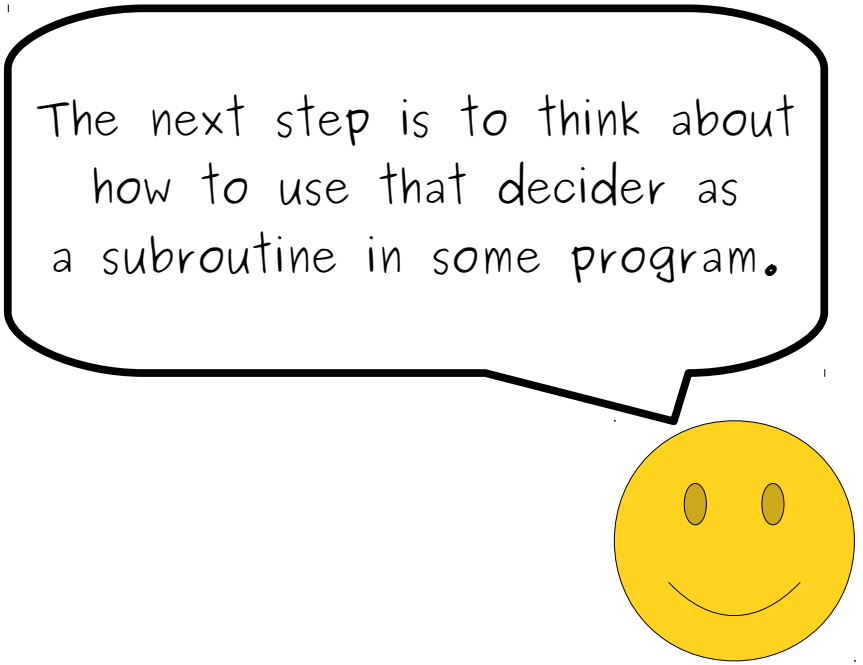

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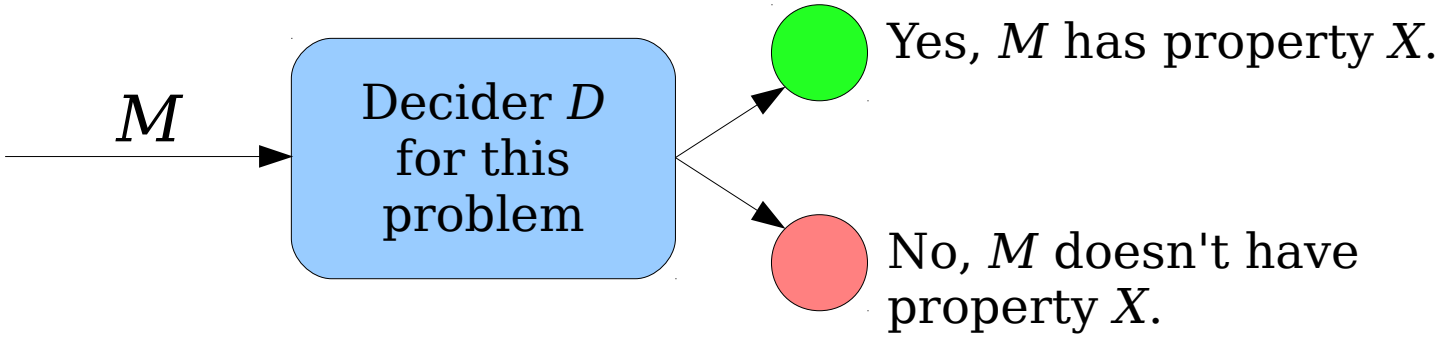

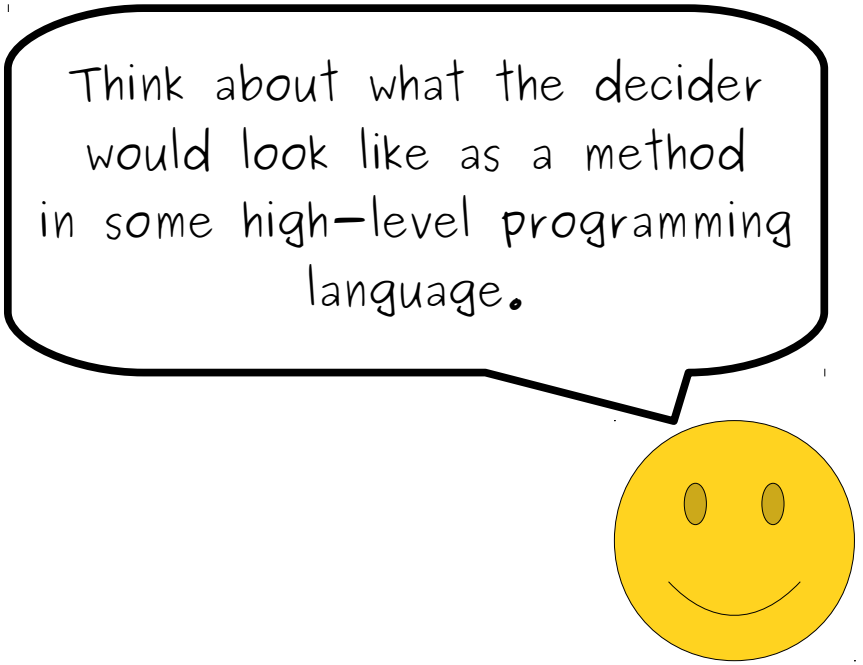

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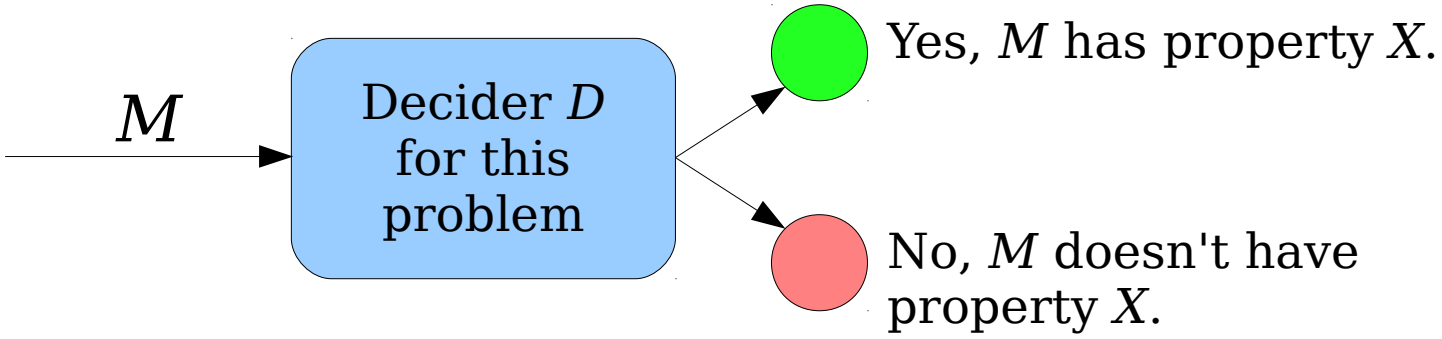

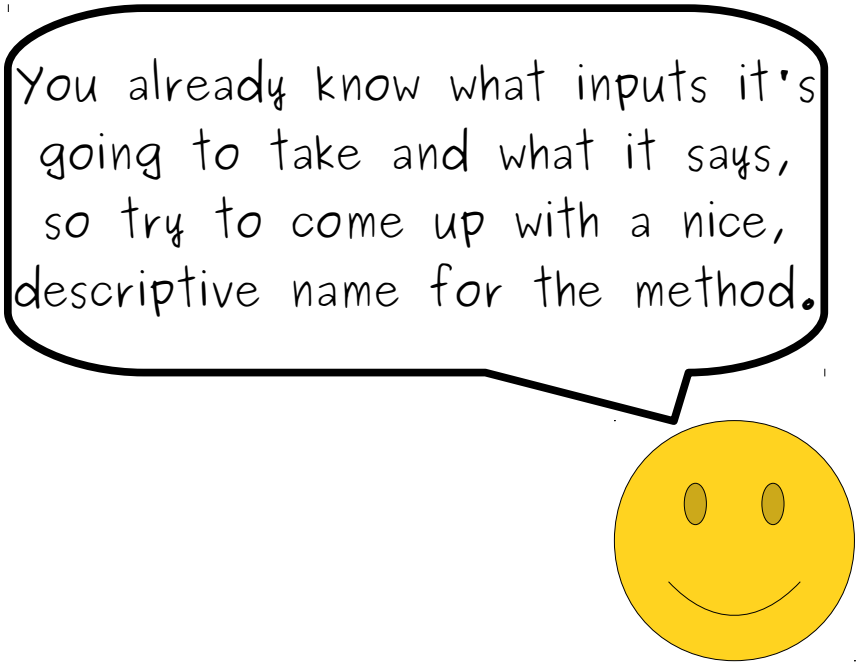

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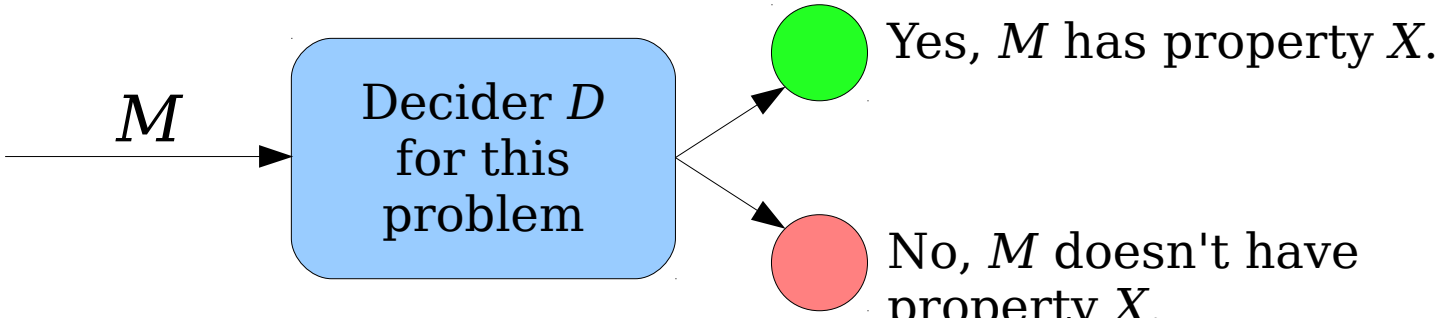


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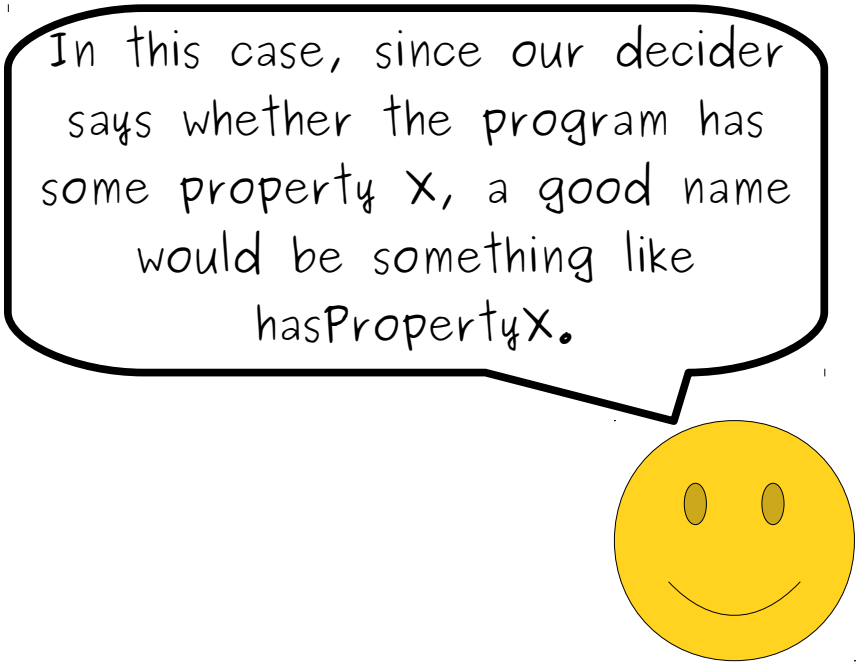

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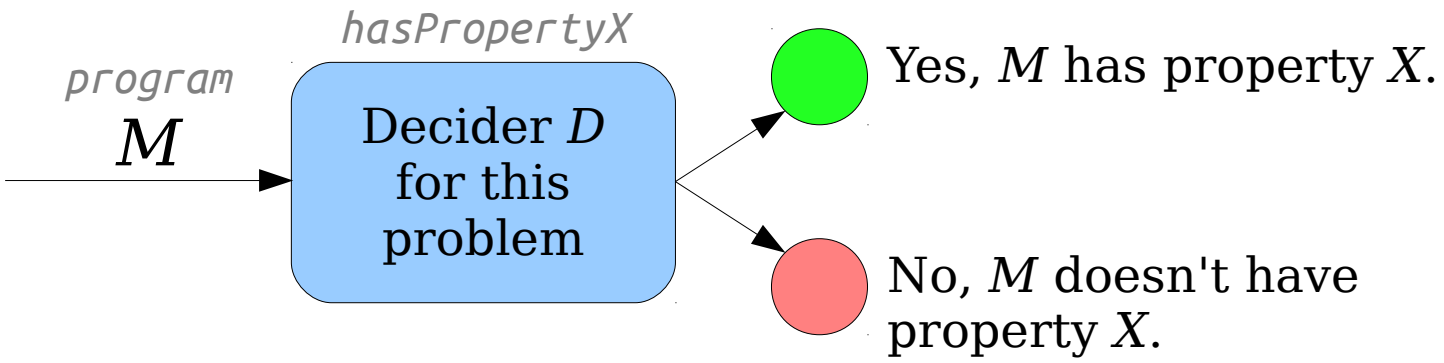


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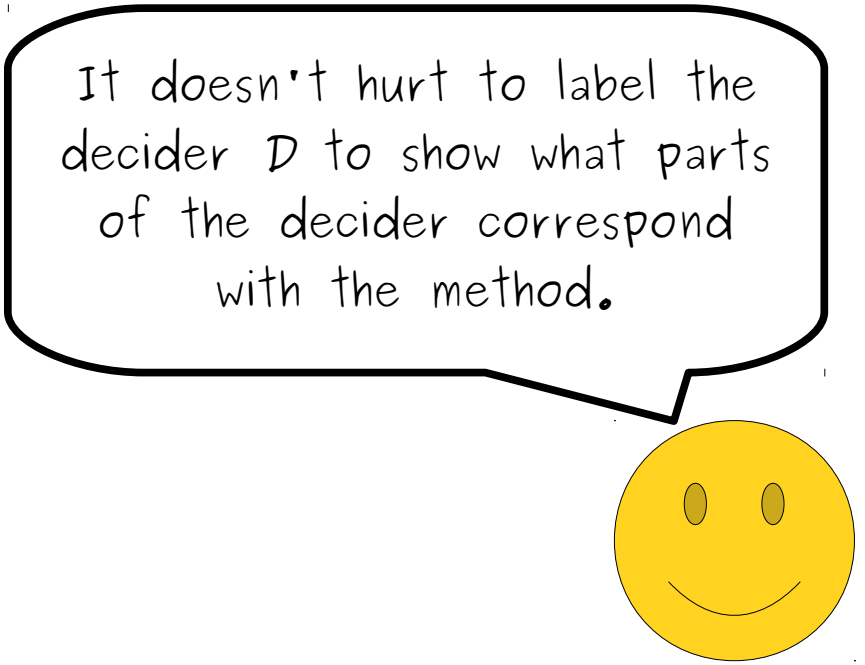

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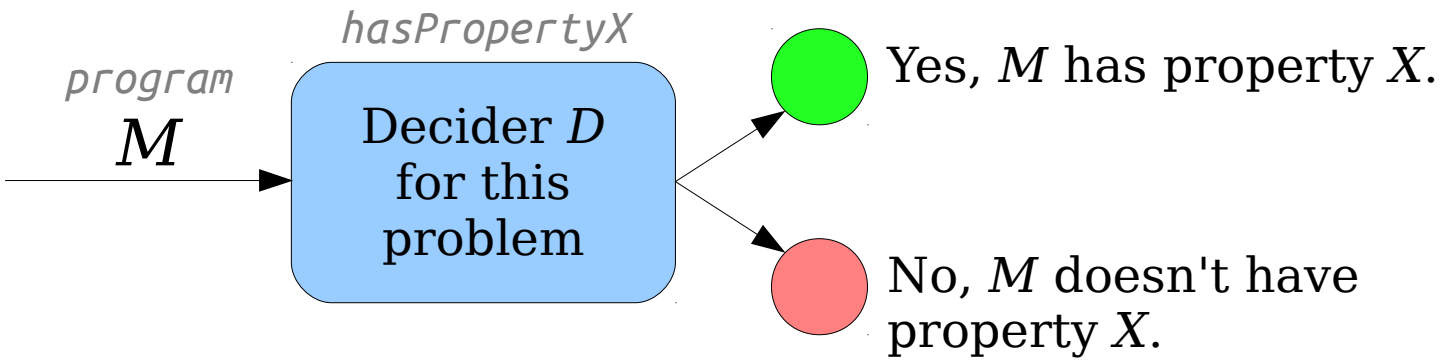


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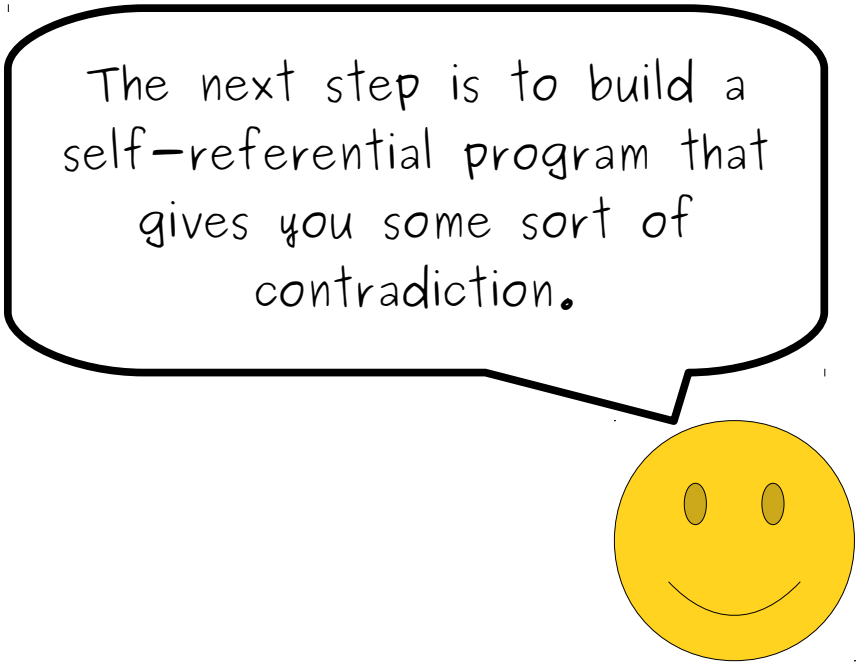

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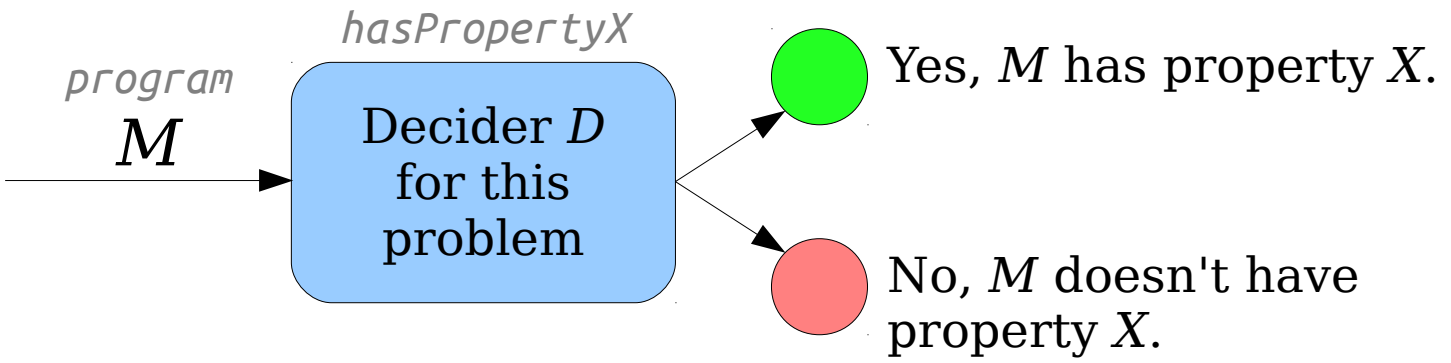


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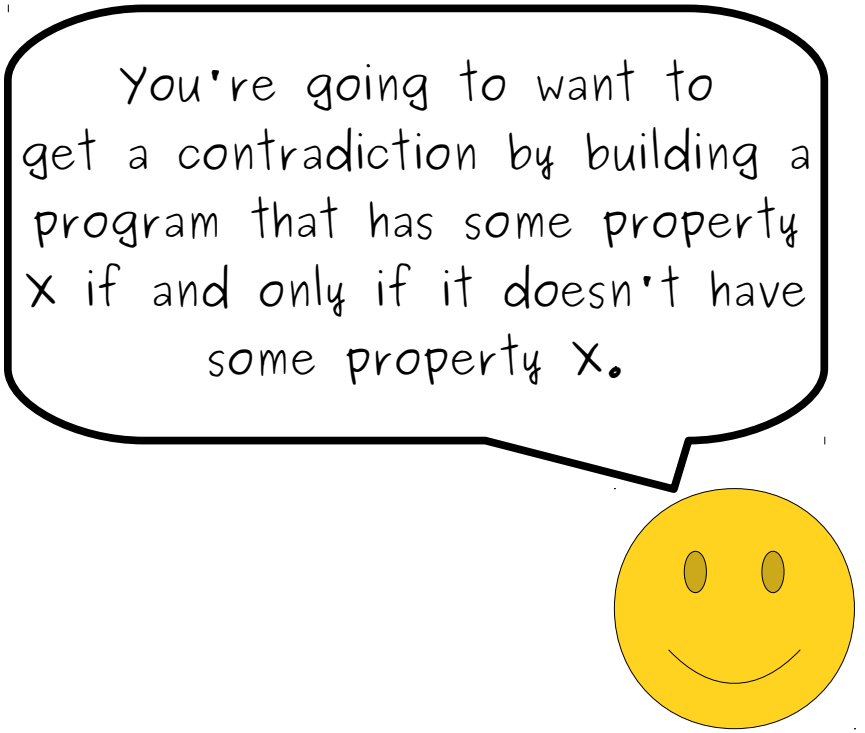

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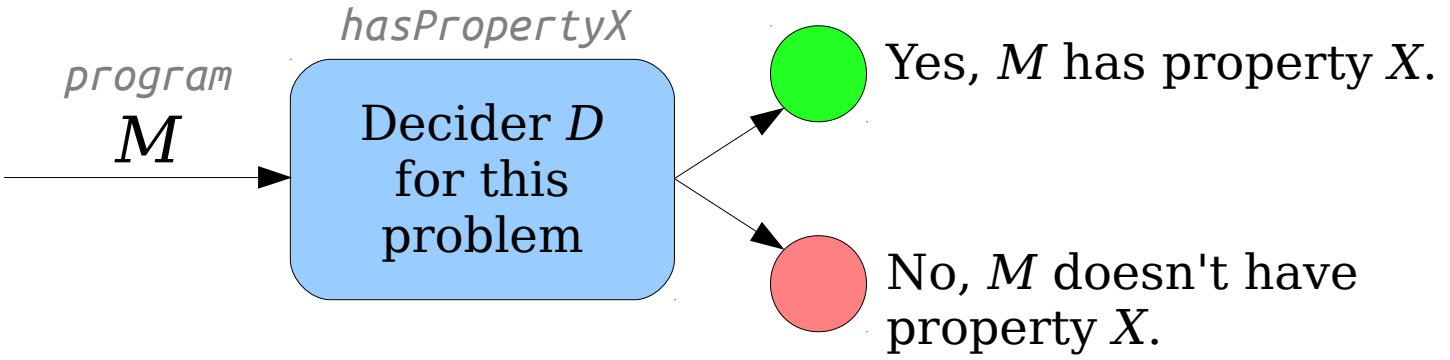


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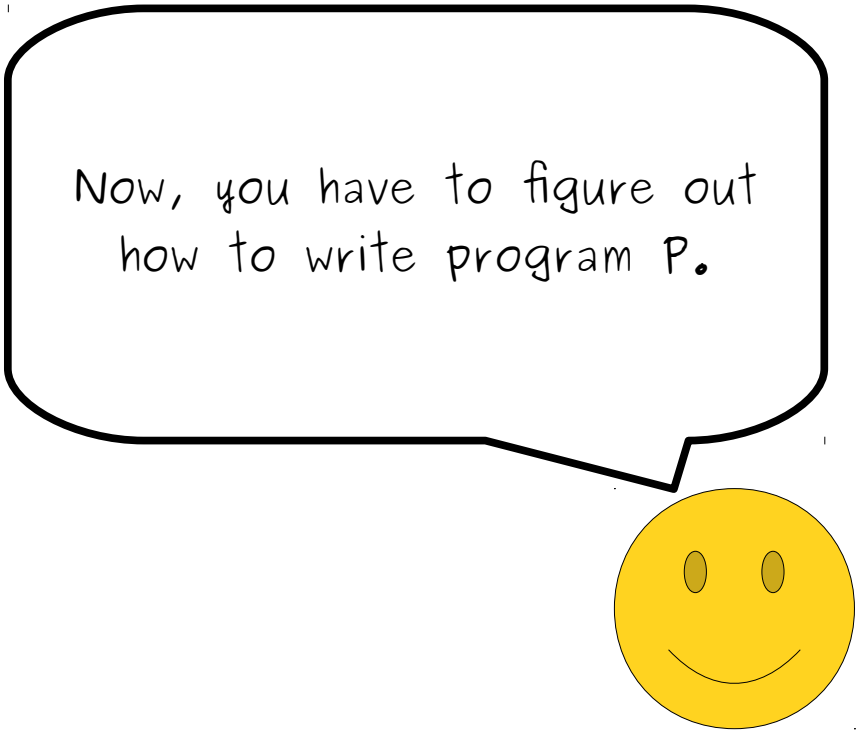

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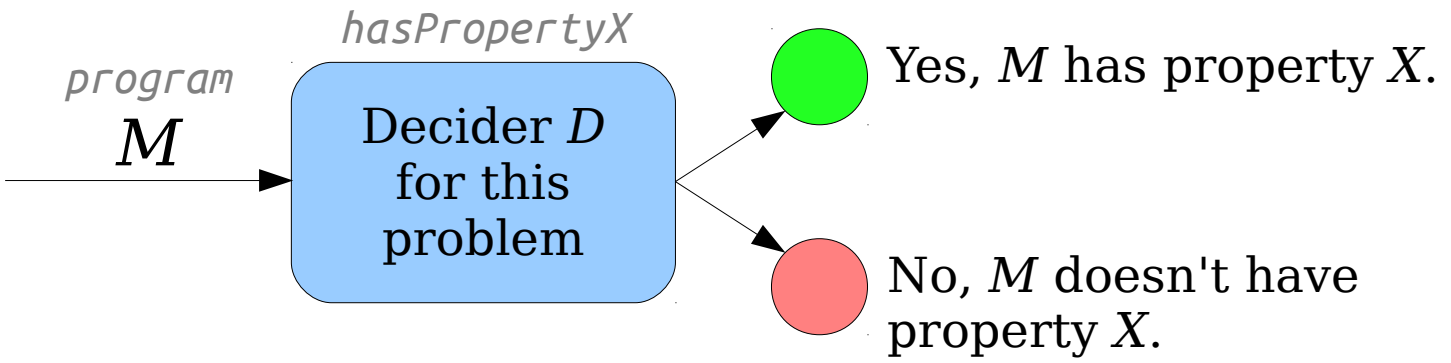


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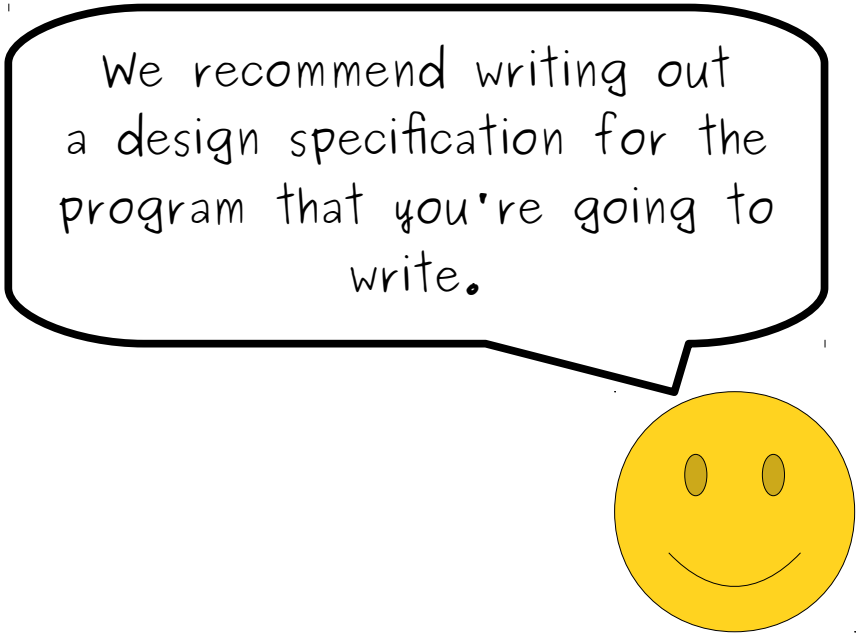

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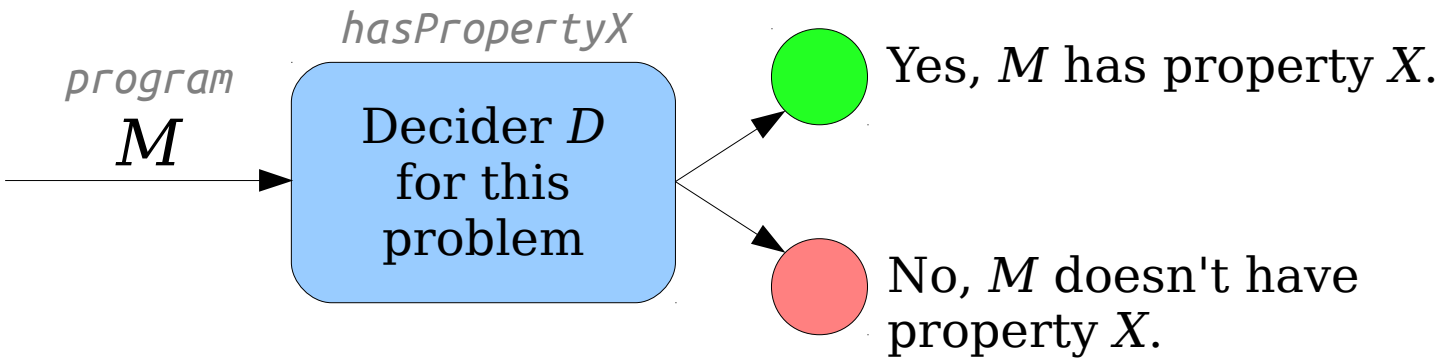


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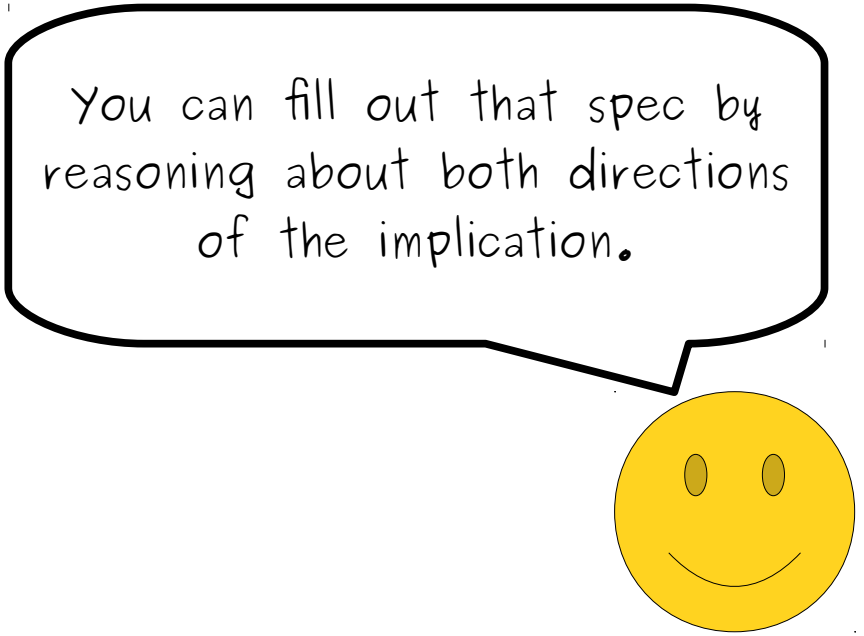

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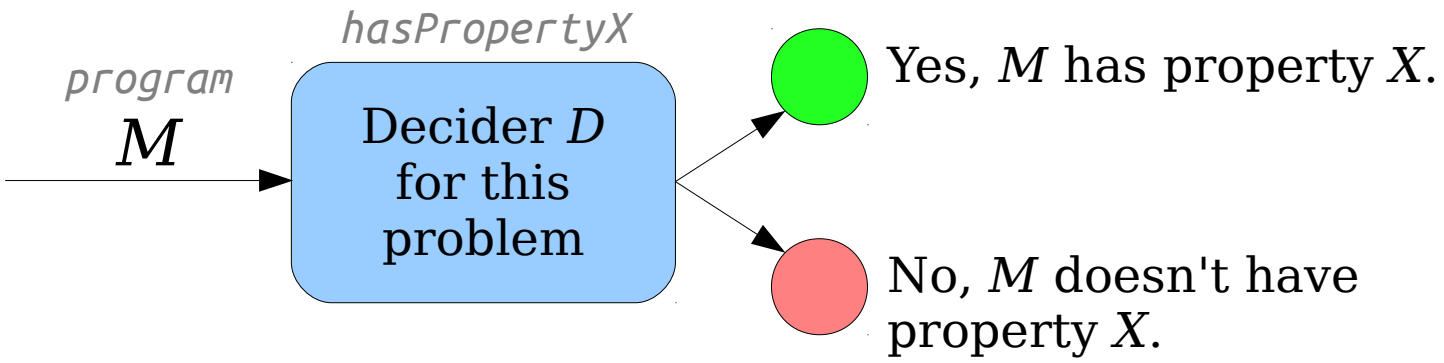


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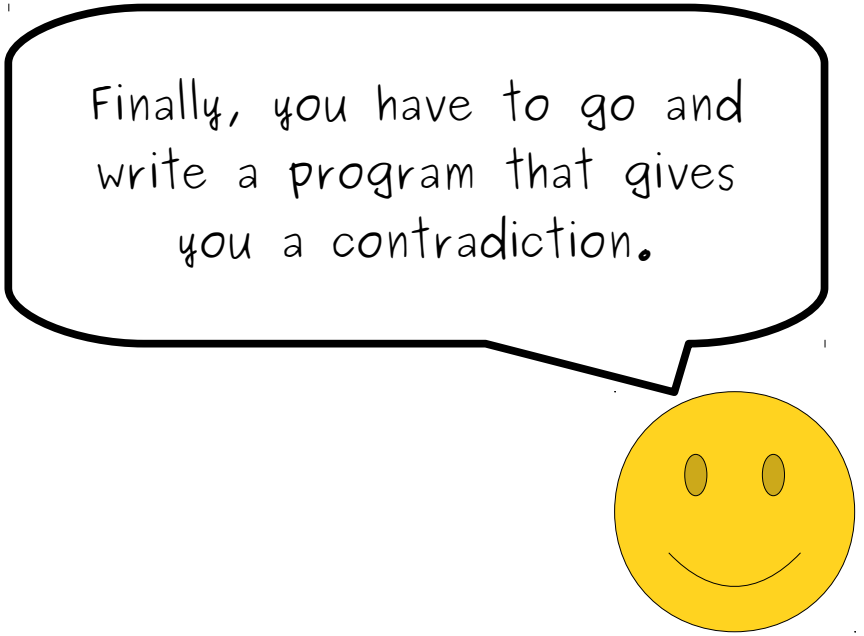

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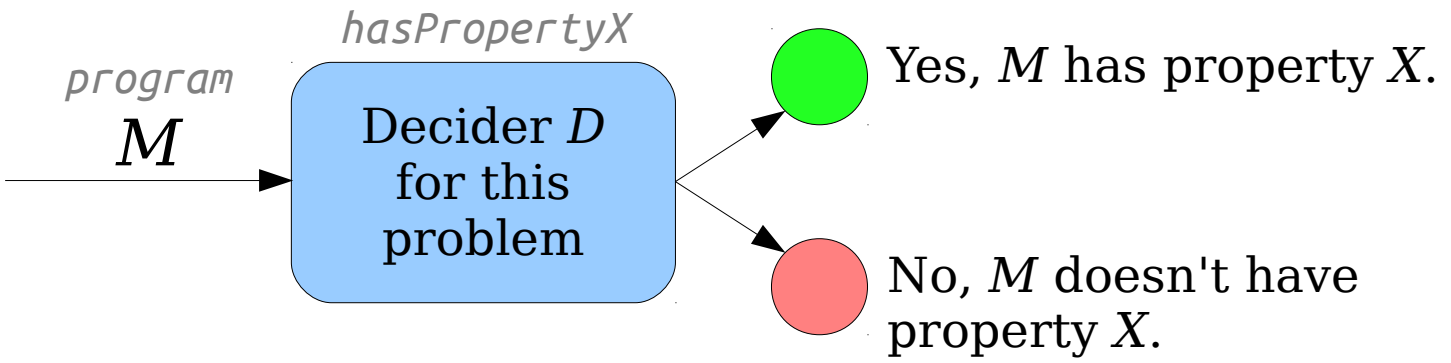


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  string me = mySource();

  if (hasPropertyX(me)) {
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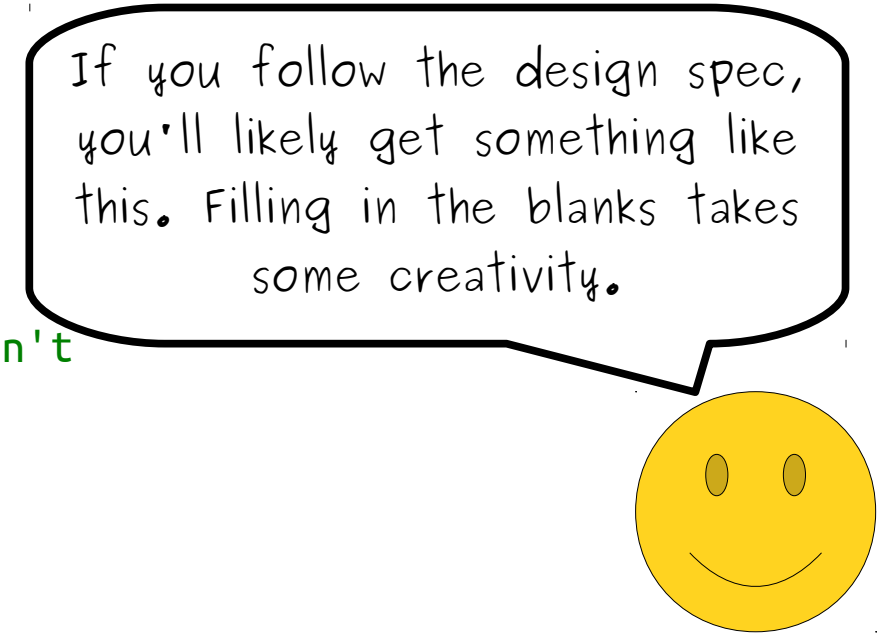

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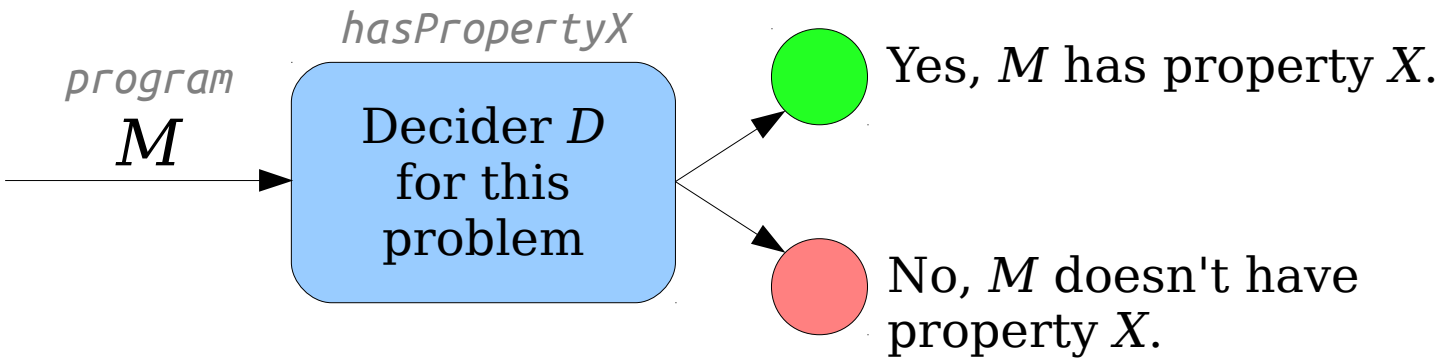


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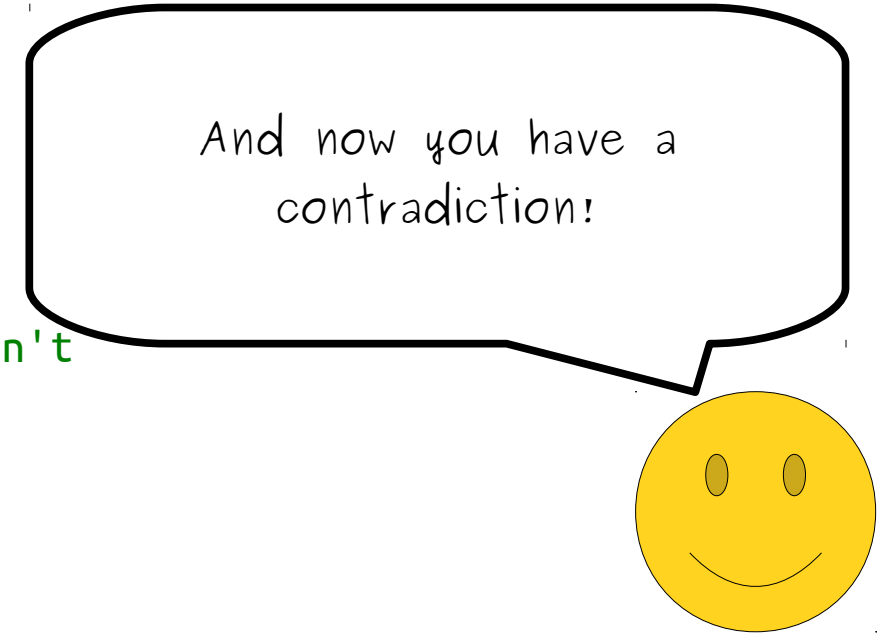


Hope this helps!
Please feel free to ask questions if you have them.


Did you find this useful? If so, let us know! We can go and make more guides like these.